

Chapter 1

Introduction

This monograph is about probabilistic inequalities from the theoretical point of view, however with lots of implications to applied problems. Many examples are presented. Many inequalities are proved to be sharp and attained. In general the presented estimates are very tight.

This monograph is a rare of its kind and all the results presented here appear for the first time in a book format. It is a natural outgrowth of the author's related research work, more precisely see [18]-[90], over the last twenty one years.

Several types of probabilistic inequalities are discussed and the monograph is very diverse regarding the research on the topic.

In Chapters 2-4 are discussed univariate and multivariate Ostrowski type and other general probabilistic inequalities, involving the expectation, variance, moment generating function, and covariance of the engaged random variables. These are applications of presented general mathematical analysis classical inequalities.

In Chapter 5 we give results related to the probabilistic version of Grothendieck inequality. These inequalities regard the positive bilinear forms.

Next, in Chapters 6-15 we study and present results regarding the Csiszar's f -Divergence in all possible directions. The Csiszar's discrimination is the most essential and general measure for the comparison between two probability measures. Many other divergences are special cases of Csiszar's f -divergence which is an essential component of information theory.

The probabilistic results derive as applications of deep mathematical analysis methods. Many Csiszar's f -divergence results are applications of developed here Ostrowski, Grüss and integral means inequalities. Plenty of examples are given.

In Chapter 16 we present the basic moment theory, especially we give the geometric moment theory method. This basically follows [229]. It has to do with finding optimal lower and upper bounds to probabilistic integrals subject to prescribed moment conditions.

So in Chapters 17-24 we apply the above described moment methods in various studies. In Chapters 17-19 we deal with optimal lower and upper bounds to the expected convex combinations of the Adams and Jefferson rounding rules of a nonnegative random variable subject to given moment conditions. This theory has

applications to political science.

In Chapters 20-22, we estimate and find the Prokhorov radius of a family of distributions close to Dirac measure at a point, subject to some basic moment constraints. The last expresses the rate of weak convergence of a sequence of probability measures to the unit measure at a point.

In Chapter 23 we apply the geometric moment theory methods to optimal portfolio management regarding maximizing profits from stocks and bonds and we give examples. The developed theory provides more general optimization formulae with applications to stock market.

In Chapter 24 we find, among others, estimates for the differences of general integral means subject to a simple moment condition.

The moment theory part of the monograph relies on [28], [23], [89], [85], [88], [90], [86], [44] and [48].

In Chapter 25 we develop Grüss type inequalities related to Stieltjes integral with applications to expectations.

In Chapter 26 we present Chebyshev-Grüss type and difference of integral means inequalities for the Stieltjes integral with applications to expectation and variance.

Chapters 27-28 work as an Appendix of this book. In Chapter 27 we discuss new expansion formula, while in last Chapter 28 we study the integration by parts on the multivariate domain.

Both final Chapters have potential important applications to probability.

The writing of the monograph was made to help the reader the most. The chapters are self-contained and several courses can be taught out of this book. All background needed to understand each chapter is usually found there. Also are given, per chapter, strong motivations and inspirations to write it.

We finish with a rich list of 349 related references. The exposed results are expected to find applications in most applied fields like: probability, statistics, physics, economics, engineering and information theory, etc.