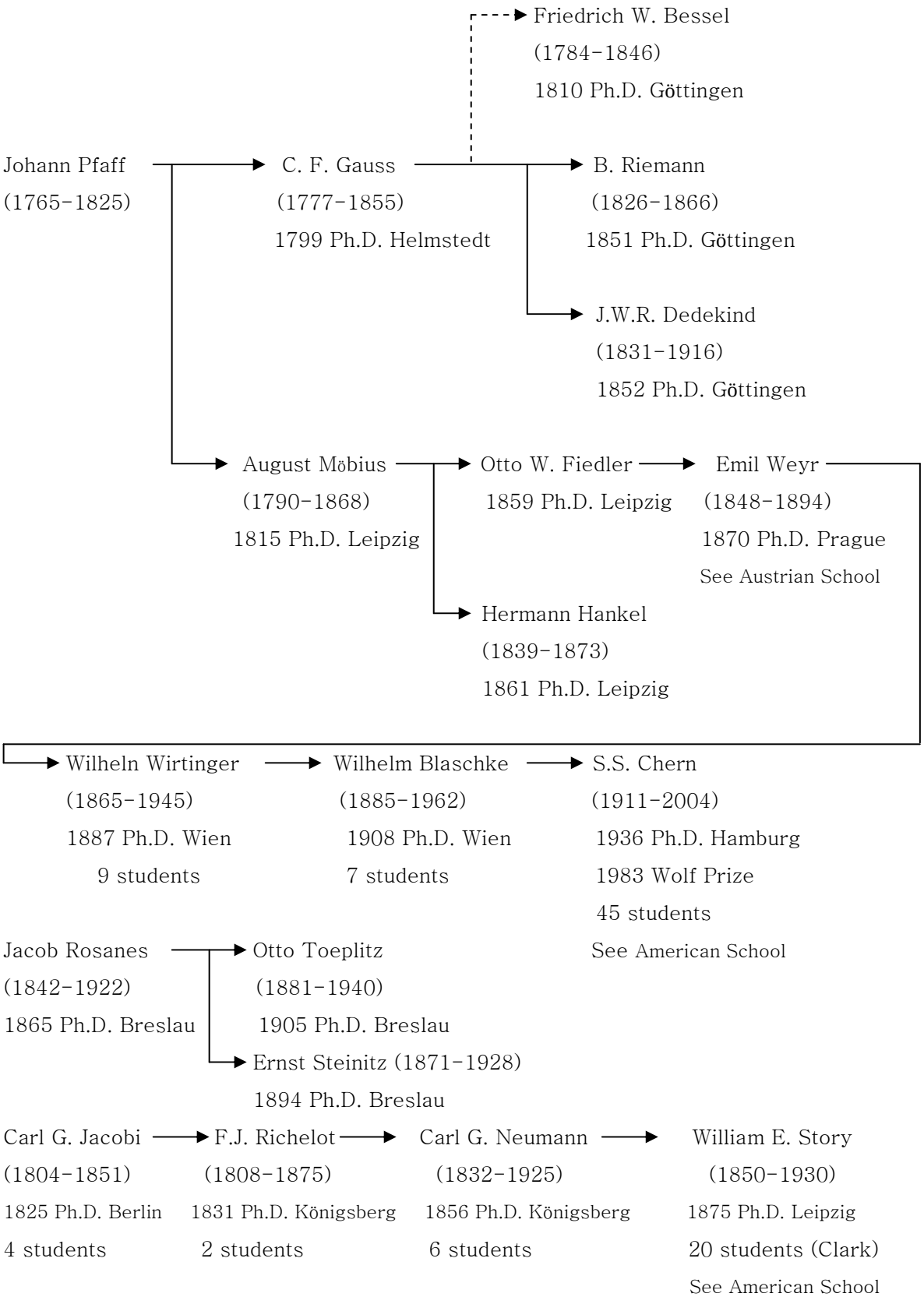
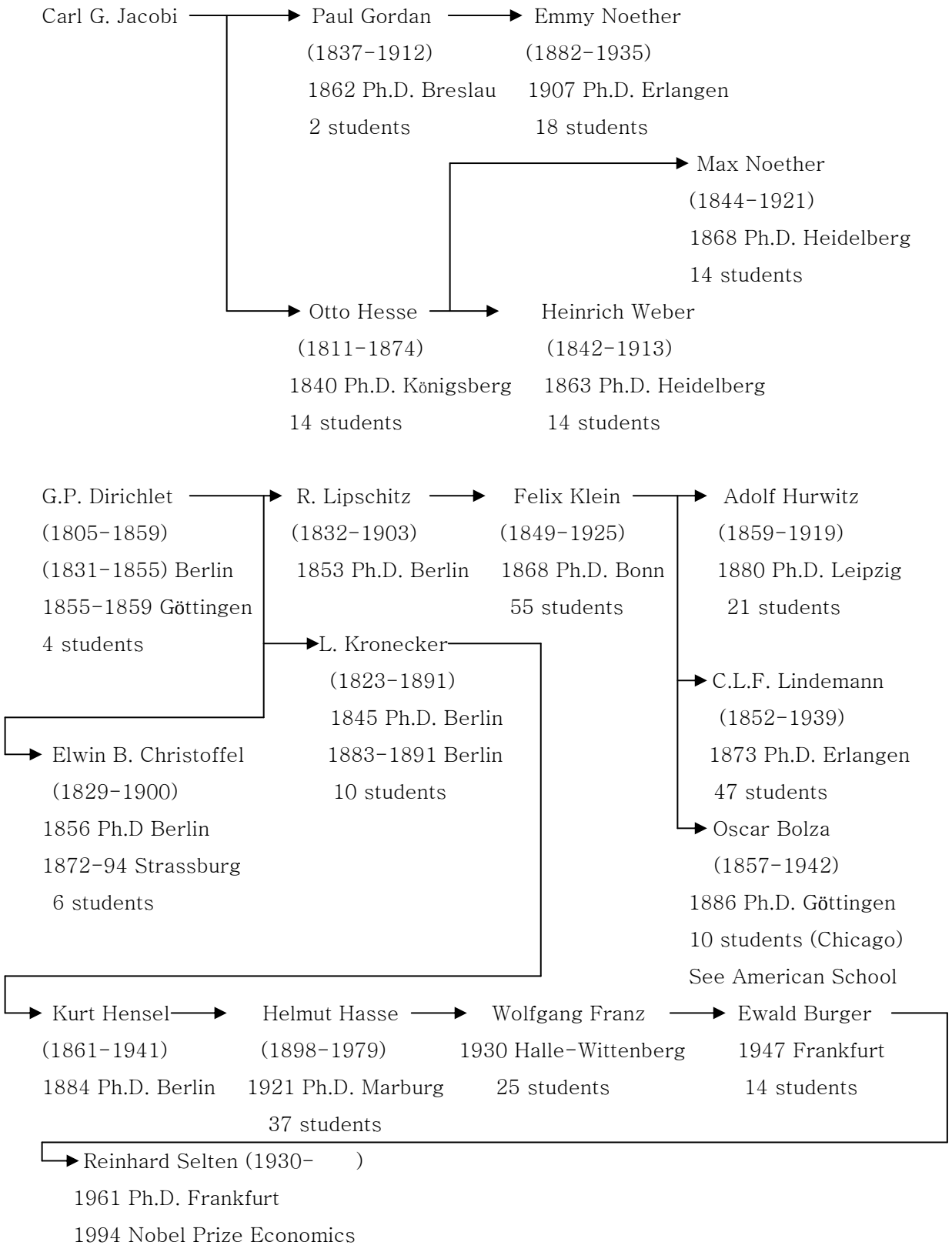
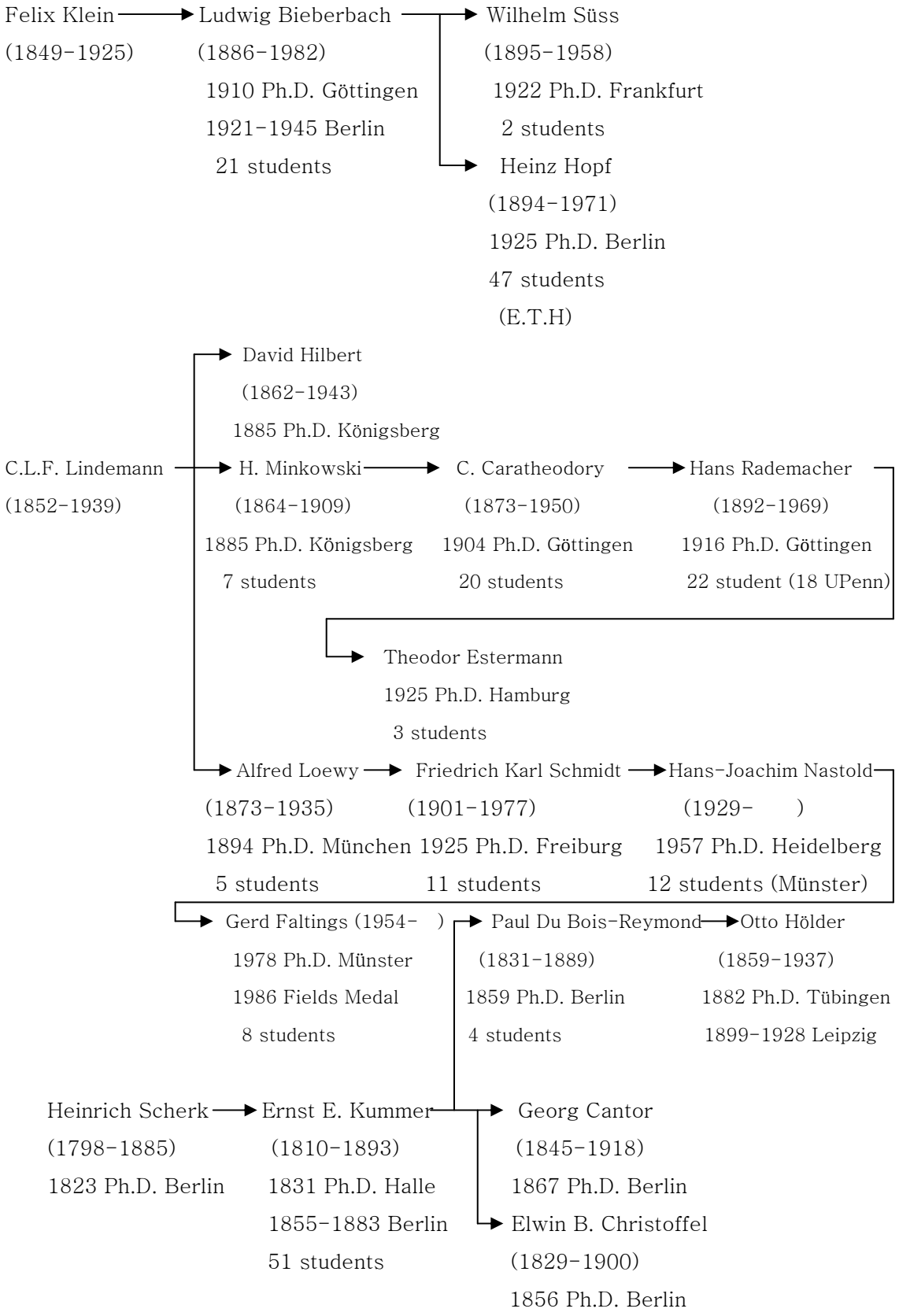
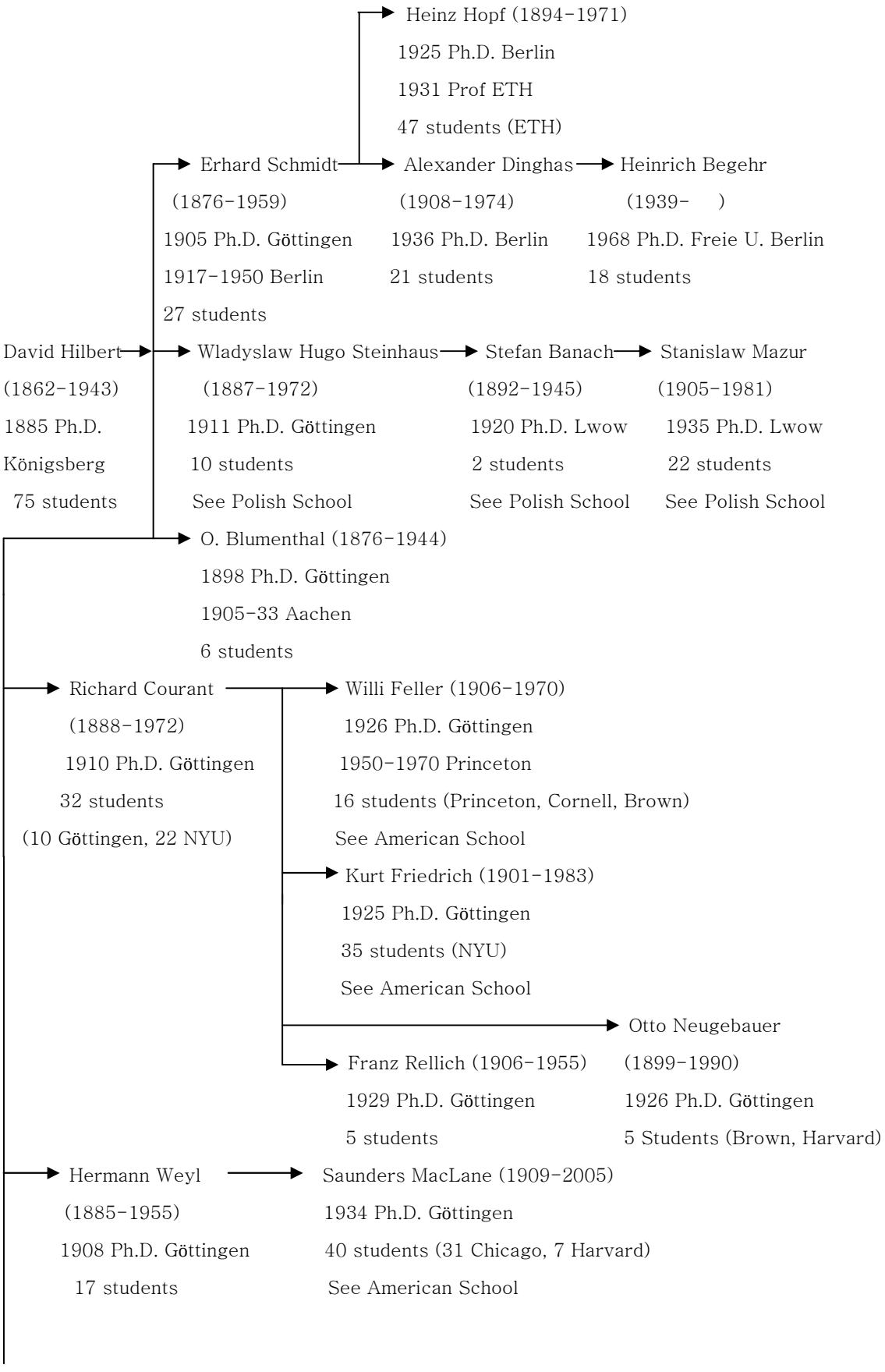
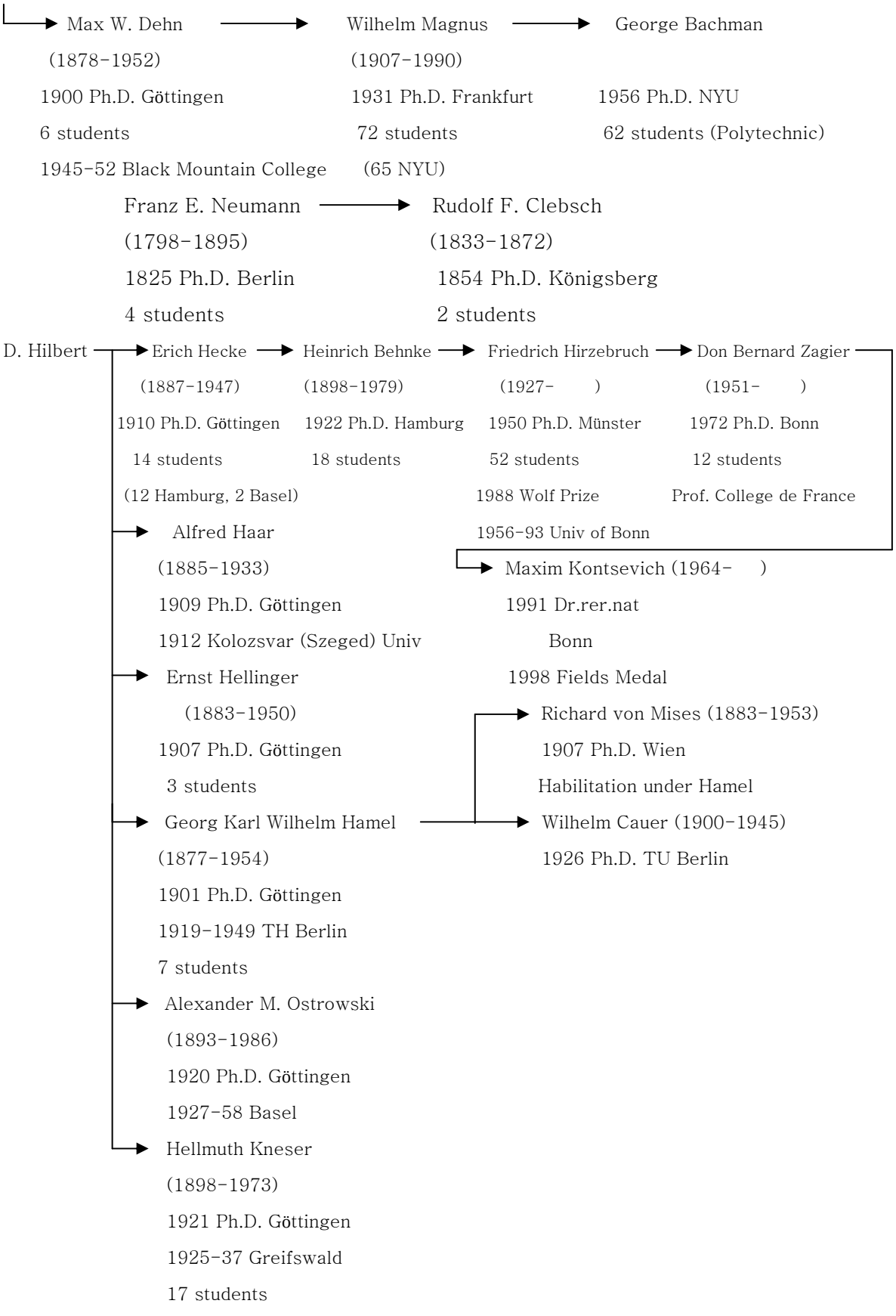
German School

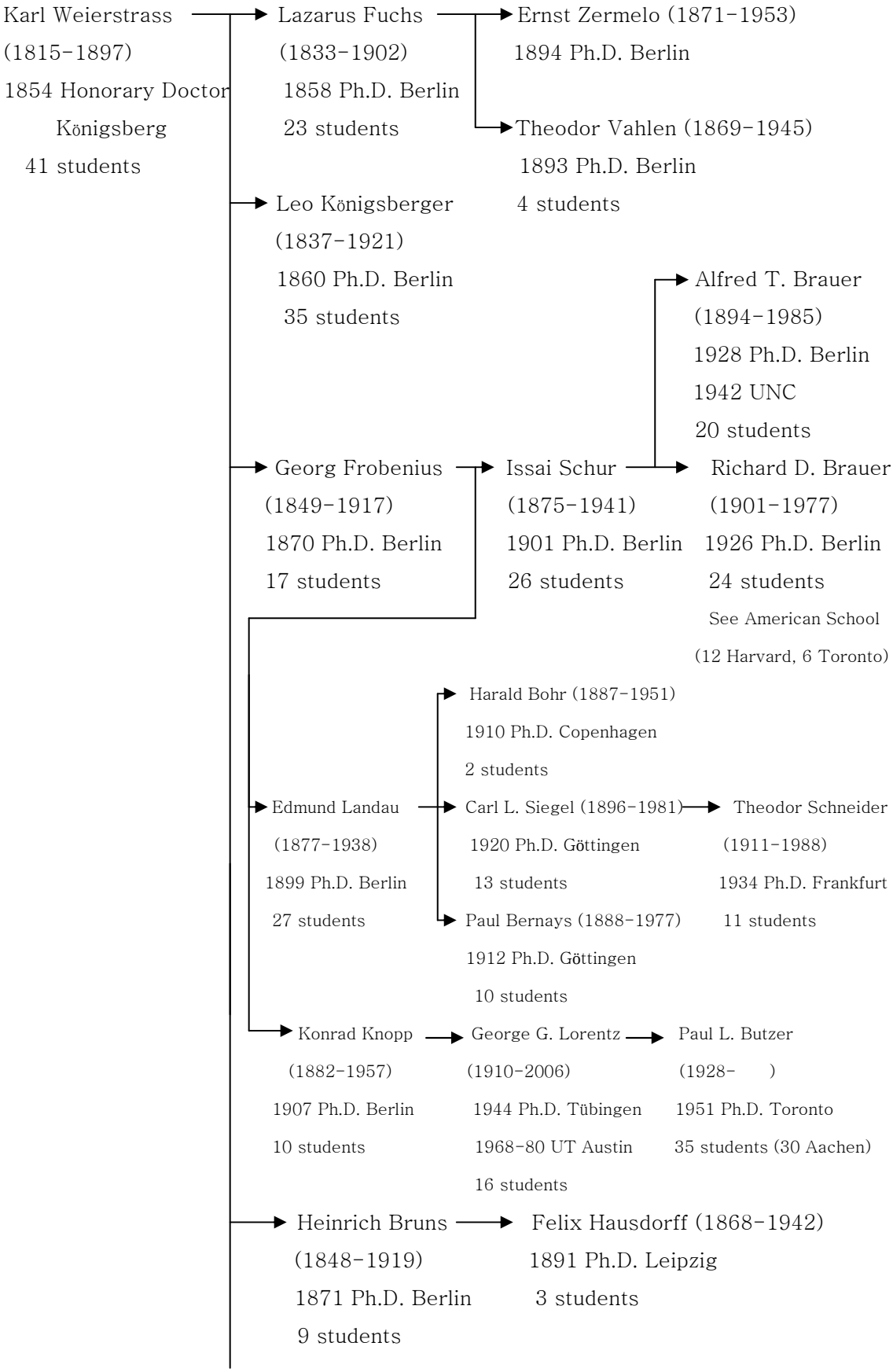


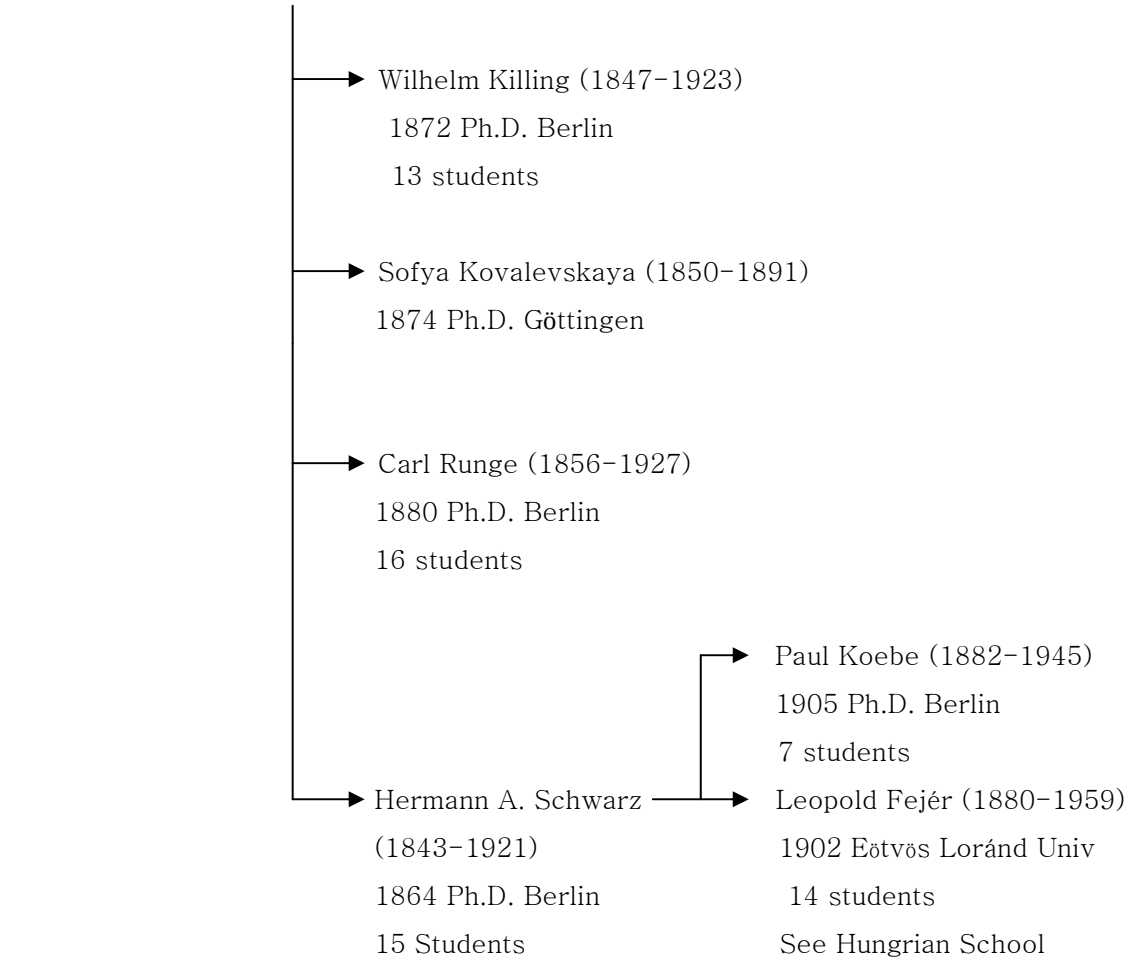












Hugo von Seeliger (1849-1924)
1872 Ph.D. Leipzig
15 students (Munich)

→ Gustav Herglotz (1881-1953)
1900 Ph.D. München
27 students
(25 Leipzig)

→ Emil Artin (1898-1962)
1921 Ph.D. Leipzig
1925 Prof Hamburg
34 students
(Princeton, Hamburg)
See American School

Wilholm Franz Meyer
1899-1924 Königsberg

→ Theodor Kaluza (1885-1954)
1907 Ph.D. Königsberg
1935-54 Göttingen

German School

Although Germany was unified in 1869, Prussia, Bayern, Hessen, Hannover, Baden, Sachsen, Mecklenburg and Württemberg are all included when we discuss German universities. Until the first half of the 20th Century, the standard of education in German gymnasiums (high school) had been very high. After studying six semesters at any university, students could submit a doctoral dissertation to a university. Students were free to move around universities.

A committee of three or four professors examine the dissertation and the candidate takes an oral examination. At some universities, the candidate was required to defend his dissertation in front of many students. Universities gave no examination until the final doctoral examination. Consequently there were no bachelors or masters degrees. Some universities award Ph.D and magister (master) degrees at the same time.

But students at Technische Hochschules (TH) must receive Diplom degrees first and THs were allowed to award doctoral degrees in 1899.

In 1925, there were 23 universities and 10 Technische Hochschules. The total number of doctorates awarded in 1925 was 4698.

The following table shows Ph.D. degrees and habilitations awarded by German universities in mathematics.

Universities	Ph.D.	Hab	Period	1893-1910
Berlin	145	1	1835-1893	29
Bonn	51	0	1853-1891	7
Breslau	55	10	1853-1890	27
Erlangen	33	3	1855-1891	16
Freiburg	15	3	1862-1890	5
Giessen	25	5	1863-1892	22
Göttingen	160	1	1851-1892	76
Greifswald	15	0	1860-1887	21
Halle	93	9	1851-1892	27
Heidelberg	22	7	1851-1892	9
Jena	109	6	1863-1893	13
Kiel	23	1	1857-1892	20
Königsberg	34	1	1854-1892	13
Leipzig	79	16	1867-1892	37
Marburg	104	5	1851-1893	11
München	37	5	1855-1893	41
Münster	18	0	1850-1893	11
Rostock	70	0	1868-1893	30
Strassburg	22	0	1876-1892	58

Tübingen	34	0	1865-1890	22
Würzburg	7	1	1878-1893	5
Sum	**1151**	**74**		**500**

Source: Mathematik in Berlin Geschichte und Dokumentation, Erster Halbband, herausgegeben von Heinrich Begehr, Shaker Verlag, Aachen 1998, p. 185

In order to teach at universities, a habilitation thesis must be accepted. Then he or she can be appointed as a privatdozent which is a lecturer that is not paid by the university but receives fees from the students.

After spending several years as privatdozent, one can be appointed as extraordinary (associate) professor or ordinary professor. Promotion to ordinary (full) professor is not automatic. One must apply for that position in a nationwide search.

Extraordinary professors now are called C-3 professors and ordinary professors are called C-4 professors.

Only C-4 professors can be appointed Dean (Dekan) or Rector of a university.

Number of Doctorates in Mathematics in Germany from 1907 to 1945.

Universities/TH	Domestic Men	Domestic Women	Foreign Men	Foreign Women
Aahen TH	3	1		
Berlin TH	16	1	6	
Berlin Univ	85	9	13	
Bonn Univ	41	14	3	
Braunschweig TH	1			
Breslau TH	7	1	1	
Breslau Univ	47	9	1	
Danzig TH	1			
Darmstadt TH	17		2	
Dresden TH	58	7		
Erlangen Univ	21	2		
Frankfurt/M	29	7	2	
Freiburg Univ	21	3	1	
Giessen Univ	48	5	5	
Göttingen Univ	119	7	37	1
Greifswald Univ	20			
Halle Univ	56	9	2	
Hamburg Univ	58	5	5	
Hannover TH	5	1		
Heidelberg Univ	32	3	6	
Jena Univ	41	1	2	
Karlsruhe TH	5			

Kiel Univ	33	2	2	
Köln Univ	8	1	1	
Königsberg Univ	37	5		
Leipzig Univ	68	3	10	1
Marburg Univ	31	2	5	1
München TH	45	1	3	
München Univ	53	4	16	1
Münster Univ	60	9		
Rostock Univ	35	1		
Strassburg Univ	30	1	3	
Stuttgart TH	5			
Tübingen Univ	49	1	1	
Würzburg Univ	30	1	4	
	1215	116	131	4

Source: Renate Tobies

www.mathematik.uni-bielefeld/DMV/archiv/dissertation.html

Johann Friedrich Pfaff

Johann Friedrich Pfaff was born on 22 December 1765 in Stuttgart, Germany. He had studied mathematics on his own and began to study the works of Euler. He also spent two years studying mathematics and physics at the University of Göttingen.

Pfaff held the chair of mathematics at the University of Helmstedt from 1788 until 1809. Helmstedt was dissolved under the French occupation in 1809. Napoleon closed 22 universities in Germany including Köln, Ingolstadt, Trier, Mainz, Dillingen, Helmstedt, Rinteln, Altdorf, Fulda, Passau, Lemberg, Bonn and Strassburg.

Pfaff held the chair of mathematics at the University of Halle-Wittenberg from 1810 to 1825. He is best known for his 1813 memoir on the integration of systems of differential equations. This posed the Pfaffian problem of classifying certain differential expressions; this was treated as a problem involving alternating bilinear forms with congruent variables.[2]

Gauss went to Helmstedt in 1798 to attend Pfaff's lectures and even lived in his house.[3] Gauss always retained a friendly memory of Pfaff both as a teacher and as a friend. Gauss received his Ph.D. at Helmstedt in 1799.

Pfaff died on 21 April 1825 in Halle, Saxony.

Carl Friedrich Gauss

Carl Friedrich Gauss was born on 30 April 1777 in Brunswick (Braunschweig), Germany to a very poor, uneducated family. His father was a gardener and brick layer, but his mother was eager to educate her son. When he was 14 years old, his skill at computing impressed the Duke of Brunswick who supported Gauss generously with a grant so that he could study at the Brunswick Collegium Carolinum in Hanover.

Gauss studied at the University of Göttingen from 1795 to 1798. Gauss's teacher there was Abraham G. Kästner (1719–1800), whom Gauss often ridiculed. Gauss obtained his doctorate with a thesis entitled *New Demonstration of the Theorem that Every Rational Integral Algebraic Function in one Variable can be Resolved into Real Factors of First or Second Degree* in 1799 at the University of Helmstedt.[2]

Gauss married his childhood friend Johanna Ostoff on 9 October 1805 but Johanna died after giving birth to their second son in 1809. Gauss married for a second time to Frederica Wilhelmine (Minna) Waldeck, the daughter of a law professor at the University of Göttingen. After bearing three children between 1811 and 1816, Minna became a permanent invalid. She died in 1831.

Gauss's daughter Therese stayed with her father and kept house for him until his death in 1855. Two sons, Eugene and Wilhelm, emigrated to the United States. The eldest son Joseph, from the first marriage, became a railroad engineer.

Gauss' father died in 1808, and his mother died in 1839 at the age of ninety-seven. Gauss was appointed director of the Göttingen Observatory and professor of mathematics in 1807. He did not like to teach. Between 1810 and 1820 much of his time was spent building and outfitting a new observatory. During the 1820's he was charged

Figure 1. Johann F. Pfaff

Figure 2. Carl F. Gauss

with the geographical survey of the Kingdom of Hannover, which opened a new direction in differential geometry and geodesy.

In 1831 Wilhelm Weber was appointed professor of physics at the University of Göttingen. Gauss and Weber worked together for six years building a primitive telegraph device which could send messages over a distance of 1500 meters.

There is a statue of Gauss and Weber in Göttingen. Gauss became a good friend of Hungarian Farkas Bolyai. In 1831 Farkas Bolyai sent his son Janos Bolyai's work on non-Euclidean geometry to Gauss. Gauss replied "to praise it would mean to praise myself." Bolyai resented it very much.

The collected works of Gauss include over 300 papers, many written in Latin. He taught himself to read and write in Russian.

He was elected a foreign member of the Royal Society of London in 1804, mainly based on his calculations of the orbits of the asteroids Ceres and Pallas, and he was awarded the Copley Medal in 1838. He also became a Geheimrat (Privy Councilor). Gauss was featured on the 10 Deutsche Mark note.

He died on 23 February 1855 in Göttingen, Germany.

Friedrich Wilhelm Bessel

Friedrich Wilhelm Bessel was born on 22 July 1784 in Minden, now Nordrhein· Westfalen, Germany. In January 1799, at the age of 14, he left school to become an apprentice to the commercial firm of Kulenkamp in Bremen[3]. The company was involved in the import-export business so Bessel became interested in navigation and finding the position of a ship at sea. This in turn led him to study astronomy and mathematics.

In 1804 Bessel wrote a paper on Halley's comet which was sent to Heinrich Olbers (1758-1840). Olbers was so impressed that he arranged its publication in Monatliche Correspondenz and proposed Bessel as assistant at the Lilienthal Observatory. After only four years at Lilienthal, the Prussian government requested that he construct the first big observatory in Königsberg (now Kalingrad). In 1810 at the age of 26, Bessel was appointed professor of astronomy at Königsberg.

It was not possible for Bessel to be appointed professor without a doctoral degree. A doctorate was awarded by the University of Göttingen in 1810 on the recommendation of Gauss, who had met Bessel in 1807 in Bremen.

When he was at the University of Königsberg, Carl Jacobi and Franz Neumann were also professors there. Many great mathematicians and physicists came from Königsberg

including Carl Neumann, Otto Hesse, David Hilbert, Hermann Minkowski, Rudolf Clebsch, Theodor Kaluza, Gustav Kirchhoff and Arnold Sommerfeld.

In 1812 Bessel was elected to the Berlin Academy and in 1825 he received the honor of being elected as a Fellow of the Royal Society. Bessel determined the positions and proper motions of over 5000 stars which led to the discovery in 1838 of the parallax of 61 Cygni as 0.314". The correct value is 0.292".[3]

Bessel died of cancer on 17 March 1846 in Königsberg.

Figure 3. Friedrich W. Bessel

Figure 4. Julius W. R. Dedekind

Julius Wilhelm Richard Dedekind

Richard Dedekind was born on 6 October 1831 in Brunswick (Braunschweig) where Gauss had been born in 1777. Dedekind's father was a professor at the Collegium Carolinum where Gauss studied. Dedekind also studied two years at the Collegium Carolinum before entering the University of Göttingen in 1850.

After only four semesters in Göttingen, he received his doctorate supervised by Gauss in 1852 on the theory of Eulerian integrals. However he was not well trained in advanced mathematics and fully realized the deficiencies in his mathematical education.[3] He spent two more years learning the latest mathematical developments and working for his habilitation. In 1854 both Dedekind and Riemann were awarded their habilitation degrees which qualified Dedekind as a University Lecturer.

In 1858 Dedekind was appointed professor of mathematics at Polytechnikum in Zürich (now E.T.H). While in Zürich, Dedekind discovered the idea of a Dedekind cut. The idea was that every real number r divides the rational numbers into two subsets, namely those greater than r and those less than r.

The Collegium Carolinum in Brunswick had been upgraded to the Brunswick Polytechnikum (now TU Braunschweig) by the 1860's, and Dedekind was appointed to the Polytechikum in 1862. He remained there for the rest of his life retiring on 1 April 1894. He was never married and lived with sister Julie who also remained unmarried.

Dedekind edited Dirichlet's Lectures on number theory and published these as *Vorlesungen über Zahlen theorie* in 1863. In 1879 he published *Über die Theorie der ganzen algebraischen Zahlen.*

He died on 12 February 1916 in Braunschweig where he was born.

Georg Friedrich Bernhard Riemann

Bernhard Riemann was born on 17 September 1826 in Breselenz, Hannover. His father was a Lutheran minister. While Riemann was a student in the Johanneum Gymnasium in Lüneburg, he read Legendre's book on the theory of numbers (900 pages) in six days.[3]

In 1846 Riemann enrolled at the University of Göttingen as a theology student then later transferred to the Faculty of Philosophy to study mathematics.

Until the early 20th century, mathematics and physics belonged the Faculty of Philosophy in German Universities. Riemann took courses in mathematics from Moritz Stern and Gauss. Then Riemann moved to the University of Berlin in 1847 to study under Jacob Steiner, Carl Jacobi, Peter Lejeune Dirichlet and Gotthold Eisenstein. But Dirichlet was the most influential professor to Riemann in Berlin.

In 1849 Riemann returned to Göttingen and received his Ph.D. supervised by Gauss in 1851. His thesis *Grundlagen für eine allgemeine Theorie der Funktionen einer veränderlichen Komplexe grösse* (Foundations for a General Theory of Functions of One Complex Variable).[4]

Riemann's thesis studied the theory of complex variables and what we now call Riemann surfaces. It introduced topological methods into complex function theory and examined geometric properties of analytic functions, conformal mappings and the connectivity of surfaces.

After two years of intense work Riemann presented his habilitation thesis in December 1853. It translates as "On the representability of function by a trigonometric series." In the thesis he introduced the type of integral we know as the Riemann integral.

In his inaugural lecture on 10 June 1854 as privatdozent (lecturer), he presented on the hypotheses that lie at the foundations of geometry. Of those in the audience at the lecture, only Gauss would have been able to appreciate the depth of Riemann's thinking. Riemann raised the question of physical space. His results were so significant that Bertrand Russell described him as "logically the immediate predecessor of Einstein."

Without the concepts of the curvature of a Riemann space or manifold, the theory of general relativity could not have been formulated.

Riemann worked for 18 months as an assistant to physicist Wilhelm Weber and he gained a strong background in theoretical physics.

After Gauss died in 1855, Dirichlet succeeded him, but Dirichlet died soon after on 5 May 1859. Riemann was appointed to the chair of mathematics (Erster Lehrstuhl) on 30 July 1859.

In June 1862 Riemann married Elise Koch and they had one daughter Ida.

Riemann sent a report "On the number of primes less than a given magnitude" to the Belin Academy of Sciences. In this paper Riemann examined the zeta function

$$\zeta(s) = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + ... + \frac{1}{n^s} + ...$$

where $s = \sigma + i\tau$ is a complex variable.

The Riemann conjecture is that all of the imaginary zeros of the zeta function have real part $\sigma = \frac{1}{2}$. Mathematicians have not yet been able to prove or disprove this famous conjecture.

Figure 5. Georg F. B. Riemann

Figure 6. August Möbius

Riemann died of tuberculosis on 20 July 1866 in the small village of Selasca, at the northern end of Lake Maggiore in Italy. He was less than forty years old.

August Ferdinand Möbius

August Möbius was born on 17 November 1790 in Schulpforta, near Naumburg, Saxony. In 1809 he enrolled at the University of Leipzig to study mathematics, astronomy and physics. In those days German students freely moved from one university to another, so he studied astronomy under Gauss for two semesters at the University of Göttingen.

Then he went to Halle where he studied under Johann Pfaff. In 1815 Möbius wrote his doctoral thesis on *The occultation of fixed stars* at the University of Leipzig.

He was appointed extraordinary professor (Associate Professor) in 1816 at Leipzig and remained there until his death on 26 September 1868.[1]

The Möbius strip, presented in a paper discovered after his death, was devised by him to illustrate the properties of one-sided surfaces, and consists of a length of paper with connected ends and a half-twist in the middle.[5]

Johann Benedict Listing (1808-1882) also discovered the strip independently at the same time.

Hermann Hankel

Hermann Hankel was born on 14 February 1839 in Halle. He studied at the Universities of Leipzig, Göttingen and Berlin. Hankel received his doctorate at Leipzig for a thesis *Über eine besondere Klasse der symmetrischen Determinanten*[3] (On the Special Class of Symmetrical Determinants) in 1862.

After one semester as an extraordinary professor at Leipzig, he was appointed ordinary (full) professor at Erlangen in 1867. He married Marie Dippe in Erlangen and moved to University of Tübingen in 1869. Hankel was there only four years before he died on 29 August 1873.

The Hankel functions provide a solution to the Bessel differential equation, which was originally produced in connection with astronomical studies.

Wilhelm Wirtinger

Wilhelm Wirtinger was born on 15 July 1865 in Ybbs, a town on the Danube River about half way between Vienna and Linz. He received his doctorate in 1887 and his habilitation in 1890 at the University of Vienna.

After ten years at the University of Innsbruck he returned to a chair at the University of Vienna in 1905.

In 1896 he published a work of major importance on the general theta function which brought him recognition as a leading mathematician.

He was awarded the Sylvester Medal of the Royal Society of London in 1907.

Wirtinger died on 15 January in Ybbs, Austria.

Figure 7. Hermann Hankel

Figure 8. Wilhelm Wirtinger

Wilhelm Johann Eugen Blaschke

Wilhelm Blaschke was born on 13 September 1885 in Graz, Austria. He studied architectural engineering at the Technische Hochschule in Graz for two years before going to Vienna. In 1908 he received a doctorate supervised by Wirtinger at the University of Vienna. He went to Bonn where he worked with Eduard Study (1862–1930) and submitted his habilitation thesis in 1910.[3]

After working at TH Prague, Leipzig, and Königsberg, he was appointed in 1919 to a chair at the University of Hamburg from where he retired in 1953.

Blaschke wrote an important book on differential geometry *Vorlesungen Über Differential Geometrie* (1921–1929). He wrote two more major texts in 1938 and 1955.

In 1936 Blaschke joined the Nazi Party and took a leading role in German mathematics until the end of World War Ⅱ. In September 1945 the allies dismissed Blaschke from his chair at Hamburg but he was reinstated in October 1946.

He died on 17 March 1962 in Hamburg, Germany.

Figure 9. Wilhelm J. E. Blaschke

Figure 10. Otto Toeplitz

Otto Toeplitz

Otto Toeplitz was born on 1 August 1881 in Breslau, Germany. Breslau is now Wroclaw, Poland.

His father and grandfather both taught mathematics in a Gymnasium. Toeplitz was awarded his doctorate in 1905 at the University of Breslau.

Composer Johannes Brahms received an honorary doctorate at the same University in 1879, composing the Academic Festival Overture.

In 1906 Toeplitz went to Göttingen and completed his habilitation in 1907. He was a personal assistant of Felix Klein but was greatly influenced by Hilbert's works. Toeplitz discovered the basic idea of what is now called the 'Toeplitz Operator'.

Toeplitz, Max Born, Ernst Hellinger and Richard Courant all came from Breslau. Born and Hellinger came to Göttingen in 1904, Toelitz in 1906 and Courant in 1907.

Although Courant was younger than the other three, they all became good friends.

After seven years in Göttingen, Toeplitz was appointed extraordinary professor at the University of Kiel in 1913 and then was promoted to ordinary professor at Kiel in 1920.[3] From 1928 to 1935 he was at the University of Bonn. Because he was Jewish, Teoplitz was dismissed from his chair by the Nazis in 1935 and emigrated to Palestine in 1939.

He died on 15 Februnary 1940 in Jerusalem.

Carl Gustav Jacob Jacobi

Carl Jacobi was born of Jewish parents at Potsdam on 10 December 1804. His father was a banker and his family was prosperous. Jacobi was a child prodigy and he was awarded his doctorate in 1825 at the University of Berlin. His Ph.D thesis was an analytical discussion on the theory of partial fractions. Jacobi presented his habilitation thesis in 1825 to became a privatdozent (lecturer).

In 1827 he was appointed extraordinary professor at the University of Königsberg (now Immanuel Kant State University in Kaliningrad, Russia) and was then promoted to ordinary professor two years later.

Friedrich Bessel and Franz Neumann were professors at Königsberg at that time and many great mathematicians and physicists had studied there previously including David Hilbert, Hermann Minkowski, Otto Hesse, Alfred Clebsch, Carl Neumann, Gustav Robert Kirchhoff, and Arnold Sommerfeld.

Jacobi was one of the great mathematical teacher of his generation and he stimulated and influenced his pupils. Jacobi's most celebrated investigations were those on elliptic functions, and the modern notations on elliptic functions are due to him. He introduced elliptic functions into the number theory and into the theory of integration, which in turn connected with the theory of differential equations.[5]

Sylvester has given the name "Jacobian" to the functional determinant in order to pay resepect to Jacobi's work on algebra and elimination theory.[6]

In 1831 Jacobi married Marie Schwink. Around 1843 he became ill and was diagnosed with diabetes. He took a year's sick leave in Italy and returned to Germany in 1844. Because of the more severe climate of Königsberg he decided to move to Berlin. Since he had been elected to the Berlin Academy he was entitled to give lectures at the University of Berlin. Jacobi became a good friend of Dirichlet.

Early in 1851 Jacobi caught influenza, followed by smallpox, and this led to his death on 8 February 1851. He left a wife, five sons and three daughters.[4]

His great friend Dirichlet delivered a memorial lecture at the Berlin Academy of Science on 1 July 1852, in which he described Jacobi as "the greatest mathematician among members of the Academy since Joseph Lagrange."

Carl G. Neumann

Carl Neumann was born on 7 May 1832 in Königsberg, the son of Franz Neumann and Bessel's sister-in-law. He received his doctorate in 1856 at Königsberg and his habilitation in 1858 at the University of Halle-Wittenberg. He was appointed extraordinary professor at Halle in 1863 after five years as a lecturer.

After two years in Basel and three years in Tübingen he was appointed a chair at the University of Leipzig in 1868 succeeding A.F. Möbius.

He married Mathilde Elise Kloss in Basel but she died in 1875.

In 1868 he and Alfred Clebsch founded *Mathematische Annalen* and he became an editor.

During his Leipzig years, Felix Klein, Sophus Lie and Felix Hausdorff were in the same department.

Neumann's name is honored in the Neumann boundary condition and the Bessel function of the second kind is called the Neumann function.

He died on 27 March 1925 in Leipzig, Germany.

Figure 11. Carl G. J. Jacobi

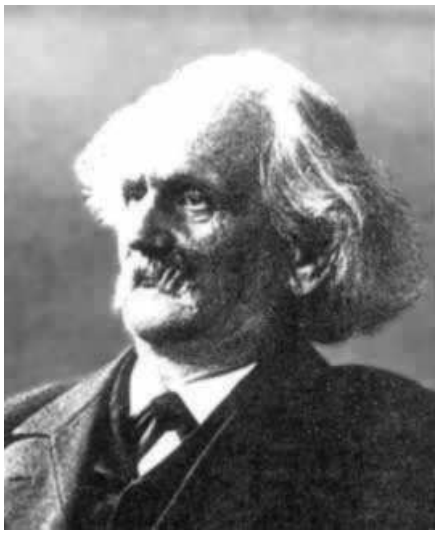


Figure 12. Carl G. Neumann

Paul Albert Gordan

Paul Gordan was born on 27 April 1837 in Breslau, Germany (now Wroclaw, Poland). He studied at the Universities of Berlin and Königsberg under Jacobi and he received his doctorate in 1862 at the University of Breslau. His dissertation was on geodesics of spheroids. He received his habilitation in 1863 at the University of Giessen supervised by Alfred Clebsch. Gordan was appointed extraordinary professor at Giessen in 1865.

In 1869 Gordan married Sophie Deuer, the daughter of a professor of law at Giessen.

The Clebsch–Gordan coefficients used in quantum mechanics were introduced by them while working on *"Theorie der Abelschen Funktionen"* in 1866.

The topic for which Gordan is most famous is invariant theory, a topic to which Clebsch introduced him in 1868.[3]

Gordan moved to Erlangen in 1875 and retired there in 1910. His predecessor was Felix Klein. One of Gordan's student was Emmy Noether, the daughter of Max Noether who was Professor of Mathematics at Erlangen from 1888 to 1919.

Gordan died on 21 December 1912 in Erlangen, Germany.

Emmy Amalie Noether

Emmy Noether was born on 23 March 1882 in Erlangen. Her father was a distinguished professor of mathematics at Erlangen. Her mother was Ida Kaufmann and both Emmy's parents were of Jewish origin. She was the eldest of their four children.

Figure 13. Paul A. Gordan

Figure 14. Emmy A. Noether

In 1904 she became the first woman to enroll at the University of Erlangen and received her doctorate in 1908 under Paul Gordan. Her dissertation was *On complete systems of invariants for ternary biquadratic forms.*

Hilbert and Klein invited Emmy to Göttingen but many professors objected to granting her habilitation. Hilbert provoked his famous outburst at a faculty meeting: "I do not see that the sex of the candidate is an argument against her admission as Privatdozent. After all, we are a University, not a bathing house."[4]

In 1919 she finally became a privatdozent. In 1922 she was appointed *"nicht beamte ausserordentliche Professor"* (an unofficial associate professor) with no salary. There is a Göttingen saying to the effect that "an extraordinary professor knows nothing ordinary and an ordinary professor knows nothing extraordinary."

Noether's theorem (1918) demonstrated the intimate relationship between the symmetry properties of the dynamical equations (i.e., their invariance under infinitesimal Lie transformations) and the conservation laws recognized to be valid in the given dynamics.[7]

In 1924 B.L. van der Waerden came to Göttingen to spend a year studying with Noether. After returning to Amsterdam he wrote "Modern Algebra" in two volumes. The major part of the second volume consists of Noether's work.[3]

She received, jointly with Emil Artin, the Alfred Ackermann-Teubner Memorial Prize for the Advancement of Mathematical Knowledge in 1932.

In 1933 the Nazis government dismissed her from the University of Göttingen because she was Jewish. She accepted a visiting professorship at Bryn Mawr College in Pennsylvania, USA. After surgery for a uterine tumor, she died suddenly in 14 April 1935 in Bryn Mawr.

Hermann Weyl said at her funeral: "The memory of her work in science and of her personality among her fellows will not soon pass away. She was a great mathematician, the greatest, I firmly believe, her sex has ever produced, and a great woman."[4]

Ludwig Otto Hesse

Otto Hesse was born on 22 April 1811 in Königsberg. He taught physics and chemistry at a trade school before he received his doctorate at Königsberg in 1840. His thesis was *De ecto punctis intersectionis trium superficium secundi ordinis.*[3] In the same year he submitted his habilitation thesis and was appointed as a privatdozent.[1] Hesse married Marie Sophie Emilie Dulk in 1841, and they had one son and five daughters.

In 1845 Hesse was appointed extraordinary professor and stayed in that position until 1856. The years he was at Königsberg were very productive, and published most of his work in Crelle's Journal. Hesse had many outstanding students including Gustav Kirchhoff, Carl Neumann, Alfred Clebsch and Rudelph Lipschitz.

After one year at Halle, Hesse was appointed ordinary professor at Heidelberg in 1856. At Heidelberg he also had many famous students including Adolph Mayer, Heinrich Weber and Max Noether.

He then moved to Technische Hochschule München in 1868 and stayed there until his death.

Hesse introduced the "Hessian Determinant" in 1842 during an investigation of cubic and quadratic curves.[3]

He also wrote two textbooks in mathematics. Some recent research has shown that Hesse not only improved Jacobi's results but also made many original contributions.

Hesse died on 4 August 1874 in Munich, and he was buried in Heidelberg.

Figure 15. Ludwig O. Hesse

Figure 16. Heinrich M. Weber

Heinrich Martin Weber

Heinrich Weber was born on 5 May 1842 in Heidelberg. Weber was awarded a doctorate in 1863 supervised by Otto Hesse at the University of Heidelberg three years after enrollment. He went to Königsberg for his habilitation work under Franz Neumann and Friedrich Richelot, but his thesis was submitted to Heidelberg and was granted in 1866.

In 1869 he was appointed as extraordinary professor at Heidelberg. After only one year at Heidelberg, he went to Zurich Polytechnikum (1870–1875), Königsberg (1875–1883), Technische Hochschule Chalottenburg (now TU Berlin) (1883–1884), Marburg (1884–1892), Göttingen (1892–1895) and Strassburg (1895–1912).[1] He is best known for his textbook *"Lehrbuch der Algebra"* published in 1895.

Weber died on 17 May 1913 in Strassburg, Germany (now Strasbourg, France).

Max Noether

Max Noether was born on 24 September 1844 in Mannheim, Baden. He suffered an attack of polio when he was 14 years old which left him handicapped for the rest of his life.[3]

He obtained a doctorate under Otto Hesse in 1868 at Heidelberg. From 1870 to 1874 he was Privatdozent and in 1874 he was appointed extraordinary professor.

Noether moved to Erlangen in 1875 and was then appointed ordinary professor in 1888. He retired in 1919.[1] In 1882, his daughter Emmy Norther was born.

He was one of the leaders of 19th century algebraic geometry. In 1873 he proved an important result on the intersection of two algebraic curves.[3]

He died on 13 December 1921 in Erlangen.

Figure 17. Max Noether

Figure 18. J. Peter G. Lejeune Dirichlet

Johann Peter Gustave Lejeune Dirichlet

Lejeune Dirichlet was born on 13 February 1805 in Düren, halfway between Aachen and Köln.

In 1805, Düren was a part of France, but it belongs to Germany now. He attended the Jesuit college in Köln (secondary school) and there one of his teacher was Georg Ohm (1789–1854), discoverer of the Ohm's law.

His family originally came from Belgium town of Richelet. His name means "Le jeune de Richelet" (Young from Richelet).[3]

Dirichlet went to Paris in May 1822 to attend lectures at the Collége de France and the Sorbonne Faculté des Sciences. He was fortunate to contact Biot, Fourier, Laplace, Legendre and Poisson. From the summer of 1823 Dirichlet was employed as a tutor by general Maximilien Sebastien Foy (1775–1825), living in his house in Paris.[3] When the general died in 1825, Dirichlet returned to Germany. In 1825 he presented his first paper to the French Academy of Sciences.

To be a professor he needed a doctorate and habilitation, but he had none. Fortunately an honorary doctorate was given at the University of Bonn in 1827.[10] Several authors claim it was the University of Cologne, but it was closed by Napoleon and reopened in 1919. Dirichlet submitted his habilitation thesis at the University of Breslau in 1827 to become a privatdozent. He was appointed extraordinary professor at Breslau in 1828 when he was 23 years old. He was extraordinary professor at the University of Berlin (1831–1839) and ordinary professor (1839–1855).

He became a full member of the Prussian Academy of Science at the age of 27 and a corresponding member of the Academie des Sciences at the age of 28, as well as of the Academy of Sciences of Russia at 32.[117]

After Gauss died, Dirichlet succeeded him at Göttingen in 1855. Dirichlet considered Gauss as his teacher. When he went to France, Dirichlet carried with him Gauss's *Disquisitiones arithmeticae* (Arithmetic Inquiry).

In 1831 Dirichlet married Rebecca Mendelssohn, one of the composer Felix Mendelssohn's two sisters. In the summer of 1858 he suffered a heart attack in Montreux, on the lake of Genera. Not long afterwards his wife Rebecca died after a stroke and Dirichlet also died on 5 May 1859 in Göttingen at the age of 54.[4]

In 1837 he suggested a very broad definition of function and he gave also the first rigorous proof of the convergance of Fourier series for a function subject to certain restrictions, known as Dirichlet's conditions. His name arises as Dirichlet boundary condition in thermodynamics and electrodynamics.

He was a good friend with Carl Jacobi who said: "Dirichlet alone, not I, nor Cauchy, nor Gauss knows what a completely rigorous proof is. Rather we learn it first from him. When Gauss says he has proved something it is very clear; when Cauchy says it, one can wager as much pro as con; when Dirichlet says it, it is certain."[4]

In his address on the occasion of the Dirichlet centenary in Göttingen in 1905, Minkowski listed Ferdinand Eisenstein, Bernhard Riemann, Leopold Kronecker, Richard Dedekind and Rudolf Lipschitz as students of Dirichlet. At the University of Berlin, Dirichlet was a formal first or second referee for doctoral dissertations of 14 students including Kronecker and Lipschitz. Dirichlet also supervised or influenced five doctoral dissertations at Göttingen.[117]

Elwin Bruno Christoffel

Elwin Christoffel was born on 10 November 1829 in Monschau near Aachen, Germany. After one year of military service he continued his study at the University of Berlin where he recevied his doctorate in 1856. His dissertation was on the motion of electricity in homogeneous bodies (De motu permaneti electricitatis in corporibus homogeneis).[10] Although the referees of his dissertation were Martin Ohm, Ernst Kummer and Gustav Magnus, Christoffel was a student of Dirichlet.[117]

After three years taking care of his mother in Monschau, his habilitation thesis was approved in 1859. He served as privatdozent 1859–1862. From 1862 to 1868 he was at the Polytechnicum in Zurich (now E.T.H) succeeding Dedekind.

Then he moved to the Gewerbeakademie in Berlin (now TU Berlin) in 1869 and stayed there until 1872. He and his colleague Siegfried Aronhold tried to attract high quality students but they could not compete with the prestigions University of Berlin with Weiesstrass, Kummer and Kronecker.

After three years in Berlin he was offered the chair of mathematics at the University of Strassburg, where he retired in 1892.

Christoffel died on 15 March 1900 in Strassburg. He was a lonely man, shy, distrustful, unsociable, irritable and brusque.[3]

He supervised six doctoral candidates at Strassburg. Rikitaro Fujisawa (1861–1933) received his Ph.D. in 1886 to become the Japanese father of mathematics and Paul Epstein (1871–1939) received his doctorate in 1895. Epstein committed suicide after dismissed from his chair at the University of Frankfurt in 1939.

Christoffel generalized Gauss's method of quadrature and expressed the polynomials which are involved, as a determinant. This is now called Christoffel's theorem.[3] His best paper *"Über die transformation der homogen differentialausdrücke zweiten grades"* published in 1869 introduced the symbols that later became known as Christoffel symbols of the first and second order.

Rudolf Otto Sigmund Lipschitz

Rudolf Lipschitz was born on 14 May 1832 in Königsberg. He started his university education at the University of Königsberg then moved to Berlin where he received his doctorate in 1853 supervised by Dirichlet. After teaching at gymnasiums he submitted his habilitation thesis at the University of Bonn to become a privatdozent there in 1857.

In 1862 Lipschitz was appointed extraordinary professor at the University of Breslau and stayed there two years. In 1864 he was appointed ordinary professor at Bonn where he spent the rest of his career.

For an ordinary differential equation of the first order

$$x' = f(t,x)$$

If there exists a constant $k>0$ such that for every (t,x_1) and (t,x_2) in domain D

$$| f(t,x_1) - f(t,x_2) | \leq k | x_1 - x_2 |$$

This is the Lipschitz condition for uniqueness of solution for the differential equation. k is called the Lipschitz constant.

Lipschitz died on 7 October 1903 in Bonn, Germany.

Figure 19. Elwin B. Christoffel

Figure 20. Rudolf O. S. Lipschitz

Felix Christian Klein

Felix Klein was born on 25 April 1849 in Düsseldorf. Klein received his doctorate in 1868 at the University of Bonn when he was only nineteen years old. His dissertation was *Über die Transformation der allgemeinen Gleichung des zweiten Grades zwischen Linien-Koordinaten auf eine kanonische Form* supervised by Julius Plücker (1801-1868) and examined by Rudolf Lipschitz.[3]

Klein submitted his habilitation thesis at Göttingen in 1871 to became a privatdozent. Klein was appointed ordinary professor in 1872 at the age of 23 at the University of Erlangen. He was perhaps the youngest German professor of mathematics.

Klein was the successor of Hermann Hankel who had left for Tübingen in 1869. Klein delivered his famous antrittsrede (inangural speech) *"Vergleichende betrachtungen über neuere geometrische Forschungen"* (Comparative review of recent researches in geometry), which later known as the Erlangen Program.

Klein was unable to build a school because Erlangen was a small university. Even in 1925 there were only 220 students there in mathematics and natural sciences.

After three years at Erlangen, he was at Technische Hochschule München from 1875 to 1880. Then Klein was appointed chair of geometry at the University of Leipzig in 1880 where Carl Neumann was a professor. When Klein left for Göttingen in 1886 his successor was Sophus Lie.

Felix Klein stayed in Göttingen for 27 years and established a mathematical research center which became a model throughout the world. He brought David Hilbert, Hermann Minkowski, Heinrich Weber and Edmund Landau to Göttingen.

Many excellent students came to Göttingen to learn from these mathematicians. Klein was a good friend of Friedrich Althoff (1839-1908), director of higher education in the Prussian Government, Althoff stayed in that position for 27 years and he had control over budget allocation for universities.

Most universities had two or three chairs of mathematics, but Berlin and Göttingen had four. While Klein was in Leipzig, the effort of trying to compete with Poincaré imposed a great strain on his mental health.[4] He had a nervous breakdown and his career as a research mathematician was essentially over around 1884. Mathematicians in Berlin; Weierstrass, Kronecker and Kummer, considered Klein as superficial and sometimes a charlatan.

In the 1890's he worked on mathematical physics and engineering, and wrote a textbook on the theory of gyroscope with Arnold Sommerfeld who was Klein's assistant

at Göttingen. Through the effort of Klein, Technische Hochschules were allowed to award doctorate degrees beginning in 1899.

Klein was elected a Fellow of the Royal Society in 1885 and received the Copley Medal in 1912.

He died on 22 June 1925 in Göttingen and was buried in the Stadt Friedhof in Göttingen.

Figure 21. Felix C. Klein

Figure 22. Adolf Hurwitz

Adolf Hurwitz

Adolf Hurwitz was born on 26 March 1859 in Hildesheim, Niedersachsen, Germany into a Jewish family. He entered the Andreanum Gymnasium in Hildesheim in 1868 where he was taught mathematics by Hermann C.H. Schubert (1848–1911). Hurwitz and Schubert wrote a joint paper when Hurwitz was a pupil at Andreanum.

Hurwitz received his Ph.D. in 1881 at the University of Leipzig with his dissertation *Grundlagen einer independenten Theorie der elliptischen Modulfunktionen und Theorie der Multiplikator-Gleichungen 1. Stufe.* His adviser was Felix Klein.

Hurwitz was at Göttingen in 1882–84 as a privatdozent. In 1884 he was appointed extraordinary professor at the University of Königsberg where he stayed for eight years. In Königsberg, he became a lifelong friend with his students David Hilbert, 22 years old, and Hermann Minkowski, 20 years old. Although Hurwitz was their teacher, he was only 25 years old. The three young men met at 5:00 PM each evening for a walk "to the apple tree" in Königsberg discussing mathematics.

After Minkowski left for Bonn in 1885, Hurwitz and Hilbert continued to take the walks. Whenever Minkowski returned to Königsberg, he joined their daily walk. In Königsberg Hurwitz married Ida Samuel, and they had three children.

Königsberg had only one ordinary professor of mathematics until 1899 when another ordinary professor was appointed. But Königsberg produced many excellent mathematicians.

Hurwitz accepted a chair at Eidgenössische Polytechnicum Zürich in 1892 and remained there for the rest of his life.

He played piano very well but continually suffered from ill health.

Hurwitz published in 1895 the criterion for the equation

$$F(s) = a_n s^n + a_{n-1} s^{n-1} + ... + a_1 s + a_0 = 0,$$

where s is a complex variable to have all roots to lie in the left-half plane of the s-plane. The necessary and sufficient condition is that Hurwitz determinants D_k, $k = 1,2,\cdots,n$ must all be positive.

E.J. Routh (1831-1907) later published a similar but easier method, so it is now known as Routh-Hurwitz Criterion in automatic control theory.

Hurwitz invited George Polya to ETH in 1914 where Polya stayed until 1940. Hurwitz played music with his daughter Lisi and Albert Einstein. Hurwitz died on 18 November 1919 in Zürich.

Carl Louis Ferdinand von Lindemann

Ferdinand von Lindemann was born on 12 April 1852 in Hannover. He received his doctorate in 1873 at the University of Erlangen supervised by Felix Klein who was only three years older than Lindemann.

His dissertation was *Über unendlich kleine Bewegungen under über Kraftsysteme bei allgemeiner projektivischer Massbestimmung.*

Lindemann was a privatdozent at the University of Würzburg in 1877. In the same years he was appointed extraordinary professor at the University of Freiburg and he was promoted to ordinary professor in 1880.

In 1883 he became a professor at the University of Königsberg where he stayed for ten years. He was lucky to have David Hilbert, Hermann Minkowski and Arnold Sommerfeld as his students. Lindemann was the first to prove in 1882 that π is transcendental, that is π is not the root of any algebraic equation with rational coefficients.[3]

Figure 23. Carl L. F. von Lindemann

Figure 24. Leopold Kronecker

Lindemann was appointed a chair at the University of München in 1893 and retired there in 1923. While he was in München, Werner Heisenberg asked him to be his adviser, but Lindemann did not accept him because Heisenberg told him he read Weyl's Raum-Zeit-Materie. Heisenberg instead became a student of Sommerfeld who was a pupil of Lindemann.

Some people confuse British physicist Frederick Alexander Lindemann (1886-1957) with the German mathematician Lindemann.

Lindemann died on 6 March 1939 in München.

Leopold Kronecker

Leopold Kronecker was born on 7 December 1823 in Liegnitz, Prussia (now Legnica, Poland) into a wealthy Jewish family.

Kronecker was taught mathematics at the local gymnasium by Ernst Kummer who later became a professor at the University of Berlin.

In 1845 he was awarded a Ph.D at Berlin with a thesis on the complex roots of unity. His adviser Dirichlet said that "Kronecker demonstrated unusual penetration, great assiduity, and an exact knowledge of the present state of mathematics."[4] Kronecker was a good athlete; a gymnast, swimmer and mountain climber. He was also a good pianist and vocalist. However, Kronecker was very small in stature and was very self-conscious of his height.

After receiving his doctorate Kronecker spent many years to manage his uncle's extensive farming enterprises. He married Fanny Prausnitzer, his late uncle's daughter and they had six children.

Even though he did not hold a university chair, he published many papers on the number theory, elliptic functions and algebra. He was nominated to the Berlin Academy by his teacher Kummer in 1860. Carl Borchardt and Karl Weierstrass seconded the nomination, and in 1861 Kronecker was elected to the Berlin Academy. Members of the Berlin Academy were entitled to lecture at the University of Berlin. He therefore lectured there from 1861 to 1883 without a university appointment. When Kummer retired in 1883 Kronecker was appointed ordinary professor at Berlin.

Kronecker criticized Weierstrass' mathematics although they were close friends in their early years. He particularly disliked Georg Cantor's revolutionary ideas on the transfinite set theory and tried to prevent or at least delay publication of Cantor's work.

The Deutsche Mathematiker-Vereinigung was founded in 1890 and Cantor invited Kronecker to address the first meeting.

He was elected to the Paris Academy in 1868 and to the Royal Society of London in 1884.

δ_{mn} is called the Kronecker delta.

His wife died on 23 August 1891 after injury in a climbing accident and Kronecker also died on 29 December 1891 in Berlin.

Helmut Hasse

Helmut Hasse was born on 25 August 1898 in Kassel, Germany. He was in the German Navy during World War I, Hasse received his doctorate in 1921 at the University of Marburg with his dissertation *Über die Darstellbarkeit von Zahlen durch quadratische Formen im Körper der rationalen Zahlen.* He also obtained his habilitation at Marburg in 1922 and he moved to the University of Kiel later that year to become a privatdozent there. In 1925 Hasse was appointed ordinary professor at Halle.

After staying five years at Halle, he succeeded his teacher Kurt Hensel at Marburg where he stayed for four years.

When Hermann Weyl left Göttingen for the United States in 1933, Hasse was appointed to his chair. Initially there was fierce opposition for his appointment from the Nazis in the Mathematics Institute in Göttingen. Hasse tried not to compromise his mathematics for political reasons, but he made no secret of his strong approval of many of Hitler's policies.

From 1939 until 1945 Hasse was on leave from Göttingen to work in Berlin on problems in ballistics. He returned to Göttingen in 1945 but he was dismissed by the British occupation forces. After one years at Humboldt University (former University of Berlin) in East Berlin, he was appointed in 1950 to a chair at the University of Hamburg and he retired in 1966.

He is best known as the editor of Crelle's Journal for 50 years and author of his textbook *"Zahlentheorie"*.

Hasse died on 26 December 1979 in Ahtensburg near Hamburg, Germany.

Figure 25. Helmut Hasse

Figure 26. Ludwig G. E. M. Bieberbach

Ludwig Georg Elias Moses Bieberbach

Ludwig Bieberbach was born on 4 December 1886 in Goddelau near Darmstadt, Germany. He received a doctorate in 1910 at the University of Göttingen with his thesis *Zur Theorie der automorphen Funktionen* supervised by Felix Klein. Bieberbach submitted his habilitation thesis in 1911 at the University of Königsberg on his solution of the first part of Hilbert's eighteenth problem. After staying two years as a privatdozent in Königsberg, he was appointed full professor at the University of Basel in Switzerland in 1913. In 1915 he moved to Frankfurt and stayed there for six years. When Bieberback left for Berlin in 1921 his successor was Max Dehn.[1]

Bieberbach was at the University of Berlin from 1921 to 1945. His colleagues in Berlin were Issai Schur, Erhard Schmidt and Richard von Mises.

After Hitler came to power on 30 January 1933, Bieberbach was converted to the views of the Nazis and persecuted his Jewish colleagues. Von Mises left Berlin for Istanbul in 1934 and Schur was forced to retire in 1934 and emigrated to Palestine in 1939.

Bieberbach wore the Nazi uniform during doctoral oral examinations after 1933.

He was dismissed from his professorship in 1945 and died on 1 September 1982. Before he died he lived in the house of one of his sons.

Bieberbach is best remembered for the Bieberbach conjecture (1916) which stated a necessary condition on a holomorphic function to map the open disk of the complex plane injectively to the complex plane.[8] He also wrote an important book with Issai Schur titled *Über die Minkowskische Reduktiontheorie der Positiven quadratischen Formen* in 1928.

Biebach and another anti-semite Theodore Vahlen (1869-1945) founded the "Deutsche Mathematik" journal.

Heinz Hopf

Heinz Hopf was born on 19 November 1894 in Gräbschen near Breslau, Germany (now Wroclaw, Poland). His studies in Breslau was interrupted by the outbreak of World War I in 1914. He fought on the Western front as a lieutenant, wounded twice and received the iron cross (first class) in 1918.

Figure 27. Heinz Hopf

Figure 28. Wilhelm Süss

Hopf received his Ph.D in 1925 at the University of Berlin with his thesis *Über zusammenhänge zwischen Topologie and Metrik von Mannigfaltigkeiten* (Connections between topology and metric of manifolds). His advisers were Ludwig Bieberbach and Erhard Schmidt. Hopf stayed in Berlin as a privatdozent from 1926 to 1931 when he was appointed full professor at ETH in Zürich succeeding Hermann Weyl. He spent the 1927–28 academic year at Princeton University on a Rockefeller fellowship.

Hopf married Anja von Mickwitz in October 1928.[8] He and Russian mathematician Pavel Aleksandrov (1896–1982) published a book on topology in 1935 originally planned in three volumes.

Wilhelm Süss

Wilhelm Süss was born on 7 March 1895 in Frankfurt am Main. He fought for three years during World War I. Süss received his Ph.D in 1920 at Frankfurt with his thesis *Begründung der Inhaltslehre in Raum ohne Benetzung von Stetigkeitsaxiomen.*[10]

From 1923 to 1928 he taught in Kagoshima, Japan.

He submitted his habilitation thesis at the University of Greifswald to became a privatdozent. Süss became a lifelong friend of Hellmuth Kneser (1898–1973) at Greifswald. Süss was appointed President of the German Mathematical Society in 1937 and joined the Nazi party in 1938.

In August 1944 Süss created the *Mathematisches Forschungsinstitut Oberwolfach* in the tiny village of Oberwolfach in the Black Forest and became its first director.

In 1945 Oberwolfach was occupied by the Moroccan army who tried to use the books as fuel. It was mathematician John Todd who prevented the destruction of the books. Todd was a Lieutenant Commander in the British Navy who arrived Oberwolfach in July 1945.

Süss died on 21 May 1958 in Freiburg im Breisgau, Germany.

Friedrich Ernst Peter Hirzebruch

Friedrich Hirzebruch was born on 17 October 1927 in Hamm between Dortmund and Münster, Germany. He received his Ph.D from the University of Münster in 1950 for his thesis *Über vierdimensionale Riemannsche Flächen Mehrdeutiger analytischer Funktionen von zwei komplexen Veränderlichen.* He also studied the algebraic topology and algebraic geometry with Heinz Hopf at ETH Zürich from 1949 to 1950.

Hirzebruch spent the two years 1952–54 at the Institute for Advanced Study in Princeton with Armand Borel, Kunihiko Kodaira and D.C Spencer.[3] He married Ingeborg Spitzley in 1952 and they had one son and two daughters.

His most famous achievement, the Hirzebruch–Riemann–Roch theorem, appeared in his 1954 paper. After spending one year at Münster and one year at Princeton University he was appointed ordinary professor at the University of Bonn 1956 where he retired in 1993.

He founded the *Max–Planck–Institut für Mathematik* in Bonn in 1980 and served as director of the institute from 1980 to 1995.[3] Hirzebruch was President of the German Mathematical Society in 1961–62 and again in 1990.[3]

Hirchbruch received many prizes including the Wolf Prize, the Lobachevsky Prize, the Lomonosov Gold Medal, the Albert Einstein Medal, the Stefan Banach Medal and Georg Cantor Medal. He also received thirteen honorary doctorates.

Figure 29. Friedrich E. P. Hirzebruch

Figure 30. Maxim Kontsevich

Maxim Kontsevich

Maxim Kontsevich was born on 25 August 1964. After attending Moscow State University he received his Ph.D (Dr.rer.nat) at the University of Bonn under Don Bernard Zagier in 1992. His thesis *Intersection Theory on the Moduli Space of Curves and the Matrix Airy Function* proves a conjecture by Edward Witten that two quantum gravitational models are equivalent.

He has been influenced by the work of Richard Feynman and Edward Witten. Kontsevich is an expert in the string theory and quantum field theory.

Another result of Kontsevich relates to the knot theory. He is a professor at the Institute des Hautes Etudes Scientifique (IHES) in France and Visiting Professor at Rutgers University, New Jersey in the United States. Kontsevich was awarded the Fields Medal in 1998 for his work in mathematical physics, algebraic geometry and topology opening up new approaches in research.

David Hilbert

David Hilbert was born on 23 January 1862 in Königsberg, Prussia (now Kaliningrad, Russia). His father Otto was a judge and his mother, Maria Theresa Erdtmann, was interested in mathematics. He received his doctorate in 1885 for a thesis *Über invariante Eigenschaften specieller binärer Formen, insbesondere der Kugelfunktionen.*[10] In Königsberg he became a good friend of Adolf Hurwitz and Hermann Minkoski. Hilbert was a privatdozent at Königsberg (1886-1892), extraordinary professor (1892-1893) and ordinary professor (1893-1895).

In 1895 he was appointed to the chair of mathematics at Göttingen succeeding Heinrich Weber. In 1892 Hilbert married Käthe Jerosh and their only son Franz was born in 1893. Franz suffered from mental illness from the age of twenty-one and never lived a normal life.[4]

Hilbert completed *"Zahlbericht"* in 1897 a brilliant synthesis of the number theory works of Kummer, Kronecker and Dedekind also containing a wealth of Hilbert's own ideas. He published *"Grundlagen der Geometrie"* in 1899. Paul Bernays published the 10th edition of the book in 1999.

Hilbert proposed 23 problems to be solved in 20th century at the Second International Congress of Mathematicians in Paris in 1900. Mathematicians who solved these problems gained great reputation among their colleagues.

His name is best remembered through the concept of Hilbert space. Richard Courant and Hilbert published *"Methoden der Mathematischen Physik"* in 1924 which became very useful in quantum mechanics after 1925.

He retired in 1929, although he continued to lecture occasionally. His house was at 29 Wilhelm Weber Strasse in Göttingen. A street in Göttingen was named after Hilbert. In 1930 Königsberg made him an honorary citizen. In his address at the ceremony *"Naturerkennen und Logik"*, he said *"Wir müssen wissen, wir werden wissen"* (We must know, we shall know). These words are engraved in his tombstone in StadtFriedhof, Göttingen.

He was well known for his forgetfulness. Once he invited guests to his house, but Mrs. Hilbert noticed that he needed to change his shirts. He went upstairs to his bedroom, took off his shirts and went to bed.

After many Jewish professors and liberal non-Jewish professors were expelled from Göttingen he was seated at a banquet next to Bernhard Rust, the Nazis Minister of Education. The minister asked Hilbert whether the Mathematical Institute had suffered from the removal of those professors. Hilbert replied bitterly, 'It hasn't suffered, Herr Minister. It just doesn't exist any more.'

Hilbert died on 14 February 1943 in Göttingen, and was buried in the Stadt Friedhof in Göttingen.

Figure 31. David Hilbert

Figure 32. Hermann Minkowski

Hermann Minkowski

Hermann Minkowski was born on 22 June 1864 in Alexotas, Russia (now Kaunas, Lithuania) but his parents were German. When Hermann was eight years old the family moved to Königsberg.

Minkowski received the Grand Prize in Mathematics by the Paris Academy of Sciences in 1883 jointly with Henry Smith (1826-1883) for a paper on the theory of quadratic forms with integral coefficients.

Minkowski received his Ph.D in 1885 at Königsberg with his thesis *Untersuchungen über quadratische Formen Bestimmung der Anzahl Verschiedener Formen, welche ein*

gegebenes Genus enthält. He presented his habilitation thesis *Raumliche Anschauung und Minima positive definiter quadratischer Formen* in 1887 to the University of Bonn.

Minkowski became Heinrich Hertz's assistant and intended to be a physicist. When Hertz died in 1894, Minkowski returned to mathematics. After spending five years as a privatdozent, he was appointed extraordinary professor in 1892 at Bonn. In 1895 Minkowski succeeded Hilbert as ordinary professor at Königsberg.

Minkowski married Auguste Adler in Strassburg in 1897; they had two daughters.[3] After only one year in Königsberg, he accepted a chair at the Eidgenössische Polytechnikum Zurich (now ETH) where he became a colleague of his former teacher and friend Hurwitz who had come to Zürich in 1892. Einstein took several courses from Minkowski at Zürich and the two would become interested in the relativity theory.

Klein and Hilbert created a new chair at Göttingen with Althoff's help, and Minkowski was appointed to the new chair in 1902. Therefore Klein, Hilbert, Minkowski, Carl Runge and Felix Bernstein were all at the Göttingen mathematics department in the first decade of 20th century.

In September 1908 at the annual meeting of the Society of German Scientists and Physicians in Köln, Minkowski lectured in "Space and Time." He said, "Henceforth space by itself, and time itself are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality." His theory of four-dimensional manifold was necessary for the working out of the general theory of relativity. His most original achievement was his *"Geometric der Zahlen"* (geometry of numbers) first published in 1896. It was reprinted in 1953 and 1968 by Chelsea, New York.

Figure 33. Constantin Carathéodory

Figure 34. Hans Rademacher

Minkowski died in 12 January in Göttingen from a ruptured appendix. His successor at Göttingen was Edmund Landau.

Constantin Carathéodory

Constantin Carathéodory was born on 13 September 1873 in Berlin. His father, Stephanos Carathéodory, was First Secretary at the Ottoman Embassy in Berlin. Two years later Stephanos was appointed the Ottoman Ambassador in Belgium, so the family went to Brussels where Constantin attended secondary school and trained as an engineer. He worked on the construction of the Assiut dam in Egypt until April 1900.

He entered the University of Berlin in May 1900 where he learned from Frobenius and Schwarz. Caratheodory became a good friend with Hungarian Leopold Fejér, in Berlin.[3] Carathéodory moved to Göttingen where he received his Ph. D. in 1904 with his thesis *Über die diskontinuierlichen Lösungen in der Variationsrechnung.* He also wrote his habilitation thesis *Über die starken Maxima und Minima bei einfachen Integralen* in 1905. He was a privatdozent in Göttingen until 1908. After one year at Bonn, he was appointed ordinary professor in 1909 at the Technische Hochschule Hannover.

On 5 February 1909 Carathéodory married Euphrosyne Carathéodory in Istanbul who was his aunt and eleven years his junior.[3] They had two children.

Carathéodory moved to TH Breslau from 1910 to 1913 where his successor was Max Dehn. Then in 1913 he succeeded Felix Klein at Göttingen when Klein retired. In 1918 Carathéodory moved to Berlin to succeed Georg Frobenius.

Carathéodory was appointed professor of Analytical and Higher Geometry at the University of Athens in 1920 and tried to establish another university in Smyrna. Because the Turks attacked Smyrna, which is now called Izmir in Turkey, the plan completely failed. He taught at Athens until 1924 when he moved to München to succeed Lindemann.

Carathéodory retired in 1938 at München. He made significant contributions to the calculus of variations, the theory of point set measure, and the theory of functions of a real variable.[3] He wrote many good books. Carathéodory died on 2 February 1950 in München.

Hans Rademacher

Hans Radenacher was born on 3 April 1892 in Hamburg, Germany. He served for two years in the German Army during World War I. He was awarded his doctorate in 1916 at Göttingen with his thesis *Eindeutige Abbildungen und Messbarkeit* (Single-valued mappings and measurability) under Caratheodory's supervision. Radermacher was a privatdozent at Berlin from 1919 to 1922 and an extraordinary professor at Hamburg from 1922 to 1925.[1]

In 1925 he was appointed ordinary professor at Breslau where he stayed until 1933. He was not Jewish but he was the Chairman of the local German Peace Society (Deutsche Friedens Gesellschaft) in Breslau which was not acceptable to the Nazis. He left Germany for the United States to be a visiting professor at the University of Pennsylvania in 1934. He had married Suzanne Gaspary during his Berlin years but the marriage ended in divorce in 1929 and he remarried Olga Prey. After divorcing Olga Prey in 1947 he married Irma Wolpe in 1949.

Although he was a full professor for eight years in Germany, the University of Pennsylvania appointed him assistant professor in 1935 which he gladly accepted. He supervised 18 doctoral students in Pennsylvania and four in Germany.

Rademacher was invited to deliver an address to the International Congress of Mathematicians in Cambridge, Massachusetts in 1950.[3] He wrote many fine books.

After retiring from the University of Pennsylvania in 1962, he taught at New York University and the Rockefeller University.[3]

He died on 7 February 1969 in Haverford, Pennsylvania.

Klaus Friedrich Roth

Klaus Roth was born on 29 October 1925 in Breslau, (now Wroclaw, Poland). He came to England when he was young and graduated from St. Paul's School in London in 1943. He received his BA degree in 1945 at Peterhouse, Cambridge where Harold Davenport was his teacher.

Roth was awarded his doctorate in 1950 under Theodor Eastmann's supervision at the University College London. He became a professor in 1961.

He solved the major open problem of approximating algebraic numbers by rationals in 1955. This work made Roth be awarded a Fields Medal at the International Congress of Mathematicians in Edinburgh in 1958.[3]

In 1952 Roth also proved a conjecture made by Erdös and Turán, in 1935.[3]
Roth moved to the chair of Pure Mathematics at Imperial College, London in 1966 and stayed there until 1988.

He was elected a Fellow of the Royal Society of London in 1960 and received the DeMorgan Medal in 1983 and the Sylvester Medal in 1991.

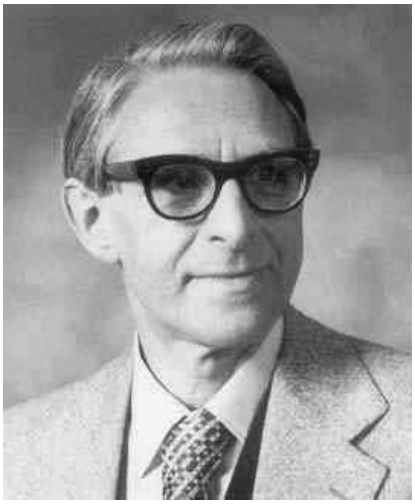


Figure 35. Klaus Friedrich Roth

Figure 36. Gerd Faltings

Gerd Faltings

Gerd Faltings was born on 28 July 1954 in Gelsenkirchen-Buer between Essen and Dortmund, Germany. He received his doctorate (Dr. rer. nat) in 1978 at the University of Münster where he also received his habilitation in 1981.

After two years as a professor at the University of Wuppertal (1982-1984), he was appointed professor at Princeton University in 1985.

In 1986 he was awarded a Fields Medal at the International Congress of Mathematicians at Berkeley for his proof of the Mordell Conjecture.

Louis Mordell (1888–1972) in 1922 stated that a given set of algebraic equations with rational coefficients defining an algebraic curve of genus greater than or equal to 2 must have only finite number of rational solutions.

In 1995 he moved to the University of Bonn and received the Gottfried Wilhelm Leibniz Prize in 1996.

Ernst Eduard Kummer

Eduard Kummer was born on 29 January 1868 in Sorau, Brandenburg, Prussia. He was awarded a doctorate in 1831 at the University of Halle-Wittenberg with his thesis *De cosniuum et sinuum potestatibus secundum cosinus et sinus arcuum multiplicium evolvendis.*[10]

He taught for ten years at the Gymnasium in Liegnitz, now Legnica in Poland where Kronecker was a student.

Kummer's paper on hypergeometric series published in Crelle's Journal in 1836 impressed Jacobi and Dirichlet. Kummer was elected to the Berlin Academy of Science in 1839 on Dirichlet's recommendation. In 1840 Kummer married a cousin of Dirichlet's wife, but she died in 1848.

In 1842 he was appointed ordinary professor at Breslau with strong support from Jacobi and Dirichlet.

When Dirichlet left Berlin to succeed Gauss at Göttingen in 1855, Kummer filled Dirichlet's chair in Berlin.

Kummer was a popular professor because of his charm and sense of humor. He was concerned with the well-being of his students and willingly aided them when financial difficulties arose.[3]

While he was in Berlin, Weierstrass and Kronecker were also teaching there. Although Kronecker was appointed to a chair in 1883 when Kummer retired, Kronecker taught from 1863 as a member of the Berlin Academy.

Kummer's successor was Lazarus Fuchs. The Paris Academy of Sciences awarded Kummer the Grand Prize in 1857 for his work relating to Fermat's Last Theorem. He supervised 51 doctoral students in Berlin.

He died on 14 May 1893 in Berlin.

Figure 37. Ernst E. Kummer

Figure 38. Georg F. L. P. Cantor

Georg Ferdinand Ludwig Philipp Cantor

Georg Cantor was born on 3 March 1845 in St. Petersburg, Russia. When Cantor was eleven years old the family moved to Germany. He became a good friend with Hermann Schwarz in Berlin while attending lectures by Kummer, Weierstrass and Kronecker. He was awarded a doctorate in 1867 at the University of Berlin with his thesis *De aequationibus secondi gradus indeterminatis.*[10]

In 1869 in Halle he received his habilitation with his thesis on number theory. He was a privatdozent for three years, extraordinary professor for seven years and ordinary professor for 34 years (1879–1913) at Halle.

In 1872 he began a friendship with Dedekind whom he had met in Switzerland. Cantor married Vally Guttmann in 1874. They had six children.

At the end of May 1884 Cantor had his first episode of depression.[3] The bitter antagonism between Cantor and Kronecker began in 1887 when Kronecker opposed Cantor's paper on dimension submitted to Crelle's Journal. The mathematical correspondence between Cantor and Dedekind was ended in 1882 as a result.

Cantor co-founded the Deutsche Mathematiker-Vereinigung in 1890. He was the first President then stepped down in 1893 and was succeeded by Paul Gordan in 1894.

His last major papers on the set theory were published in 1895 and 1897 in *Mathematische Annalen.*

Hilbert said, about Cantor's work "It is, I think, the finest product of mathematical genius and one of the supreme achievements of purely intellectual human activity."[9]

Cantor's mother died in 1876, and his younger brother and youngest son died in 1899. He developed depression and suffered from it for the rest of his life.

Cantor retired in 1913 and died of heart attack on 6 June 1918 in Halle, Germany.

Paul David Gustav du Bois-Reymond

Paul du Bois-Reymond was born on 2 December 1831 in Berlin. His parents had five children and they spoke French at home. He received a doctorate in 1853 at the University of Berlin for his thesis *De aequilibrio fluidorum.*

After teaching mathematics and physics at a secondary school for 12 years, he became a privatdozent at Heidelberg in 1865.[1] Three years later he was appointed extraordinary professor at Heidelberg.

In 1870 he was appointed ordinary professor at Freiburg where he stayed for three years.[1] He then moved to Tübingen in 1874 where he was until 1884. At Tübingen, he supervised a few doctoral students including Otto Hölder (1859–1937) who became professor at Leipzig (1899–1928). Paul du Bois-Reymond moved to TH Berlin in 1884 and stayed until 1889.

He published *Die allgemeine Funktiontheorie* in 1882 and many other important papers. He died on 7 April 1889 in Freiburg, Germany.

Figure 39. Paul D. G. du Bois-Reymond

Otto Ludwig Hölder

Otto Hölder was born on 22 December 1859 in Stuttgart, Germany. At the University of Berlin, he was a fellow student of Carl Runge and Hölder attended lectures by Weierstrass, Kummer and Kronecker. Hölder received a Ph.D. at the University of Tübingen in 1882 for his thesis *Beiträge zur Potentialtheorie* (Contributions to potential theory) supervised by Paul DuBois-Reymond.

Hölder received his habilitation in 1884 at the University of Göttingen and became a privatdozent there. He became an extraordinary professor at the University of Tübingen in 1889. In 1896, he succeeded Hermann Minkowski as an ordinary professor at the University of Königsberg, where he stayed for three years.[1] In 1899 he was appointed to a chair (Zweiter Lehrstuhl für Geometrie) at the University of Leipzig succeeding Sophus Lie.[1] He retired from Leipzig in 1928 where he had supervised 41 doctoral students.[10]

Hölder is famous for Hölder inequality (1889), the Jordan–Hölder theorem and he also proved that the Gamma function satisfies no algebraic differential equation.

The Hölder's inequality was first found by Leonard James Rogers (1862–1933), a graduate of Oxford University and professor at the Yorkshire College, later the University of Leeds. Rogers discovered the inequality in 1888, one year before Hölder.[105]

In the case of Euclidean space, for all x and y in R^n (or in C^n)[106]

$$\sum_{k=1}^{n}|x_k y_k| \le (\sum_{k=1}^{n}|x_k|^p)^{1/p}(\sum_{k=1}^{n}|y_k|^q)^{1/q}$$

For $p=q=2$, Hölder's inequality results in the Cauchy–Schwarz inequality.

Hölder died on 29 August 1937 in Leipzig, Germany.

Ludwig Otto Blumenthal

Otto Blumenthal was born in 20 July 1876 in Frankfurt am Main, Germany. When he was 18 years old, he converted from Judaism and became a Protestant.

He became the first doctoral student of Hilbert at Göttingen where he received a doctorate in 1898. He was much influenced by Arnold Sommerfeld (1868–1951) who was Klein's assistant at that time. His thesis was on the Stieltjes continued fraction expansions.[3]

Blumenthal received his habilitation at Göttingen in 1901 with his thesis *Über Modalfunktionen von mehreren Veränderlichen*. He stayed in Göttingen for three more years as a privatdozent.

Bludementhal was appointed ordinary professor at TH Aachen in 1905 and he was forced by the Nazis to retire in 1933.

In 1908 he married Mali Ebstein and they had two children, one daughter and one son. He became a member of the German League for Human Rights and the Society of Friends of the New Russia which would later be considered a crime by Nazis.[3]

He was the executive editor of the famous journal *Mathematischen Annalen* from 1905 to 1938. He was also editor of the *Jahresbericht der Deutschen Mathematiker-Vereinigung* (DMV). Blumenthal was the President of DMV in 1924.

The Blumenthals were forced to move to Utrecht, Netherlands in July 1939. With Dutch mathematician Julius Wolff Blumenthal conducted a weekly mathematical colloquium for two years in their private rooms until October 1942.[3]

The Blumenthals were sent to the concentration camp at Westerbork in the Netherlands where Mali died. Otto also died on 12 November 1944 in Theresienstadt (now in Czech Republic) after suffering from pneumonia, dysentery and tuberculosis.

Figure 40. Ludwig O. Blumenthal

Figure 41. Erhard Schmidt

Erhard Schmidt

Erhard Schmidt was born on 13 January 1876 in Dorpat, Germany (now Tartu, Estonia). He received his doctorate in 1905 at Göttingen with his thesis *Entwickelung willkürlicher Funktionen nach Systemen vorgeschriebener* which was on integral equations. He was awarded his habilitation in 1906 at Bonn where he stayed for two years as a privatdozent.

After two years as full professor in Zürich he was appointed ordinary professor in 1910 at Erlangen. From 1912 to 1917 he was at Breslau. And in 1917 he moved to the University of Berlin where he retired in 1950.

His colleagues in Berlin in the 1930's were Issai Schur, Richard von Mises and Ludwig Bieberwach.

After the end of World War Ⅱ, the University of Berlin which was located in East Berlin was renamed Humboldt University. Schmidt remained in East Berlin to become the first editor of *Mathematische Nachrichten.*

His main interest was in integral equations and the Hilbert space. He is best remembered for the Gram-Schmidt orthonormalisation process for constructing an

orthonormal set of functions from a linearly independent set. However, it should be noted that Laplace presented this idea before either Gram or Schmidt.[3]

Schmidt died on 6 December 1959 in East Berlin, East Germany.

Alexander Dinghas

Alexander Dinghas was born on 9 February 1908 in Smyrna (now Izmir), Turkey. He graduated from the Athens Technical University in 1930 majoring in electrical and mechanical engineering. He married Fanny Grafiadou in 1931.

Dinghas received his Ph.D. in 1936 at Berlin with his thesis *Beiträge zur Theorie der meromorphen Funktionen.*[10]

In 1939 he submitted his habilitation thesis to obtain *venia legendi* (right to lecture) at the same university.

Being a non-German, Dinghas found it difficult to obtain a permanent university position during World War II. He became professor at the reopened University of Berlin 1947 which was renamed Humboldt University in 1949.

Many professors and students opposing Communist rule in East Germany (Deutsche Demokratische Republik) moved to West Berlin where a new university Freie Universität Berlin was founded in December 1948.

Dinghas was appointed professor and director of the Mathematical Institute in the new University.

He wrote three books and 121 papers.[3] Dinghas commanded the respect owed to some of his position, but he was also extremely hospitable and generous. He felt profound sympathy for less fortunate.[3]

He died on 19 April 1974 in West Berlin.

Richard Courant

Richard Courant was born to Jewish parents on 8 January 1888 in Lublinitz, Germany (now Lubliniec, Poland). When he was nine his family moved to Breslau where he attended the university for two years.

He arrived at Göttingen in November 1907 and obtained his Ph.D. in 1910 with his thesis *Über die Anwendung des Dirichletschen Prinzipes auf die Probleme der konformen Abbildung* under Hilbert's supervision.[3]

Courant submitted his habilitation thesis to Göttingen and he gave his inaugural lecture as a privatdozent "On Existence Proof in Mathematics" on 23 February 1912.[3] He married Nelly Neumann in 1912 and they divorced in 1915.

During World War I he was wounded on a battle field in September 1915. After the war Courant married Nerina (Nina) Runge on 22 January 1919. She was a daughter of Professor Carl Runge and granddaughter of Emil du Bois-Reymond, eminent physiologist and former Rector of the University of Berlin.

Courant was appointed ordinary professor at Göttingen in 1920 succeeding Erich Hecke who left for Hamburg. Courant was elected to the Göttingen City Council.

In the 1920's Göttingen did not have a separate building for mathematics. In fact mathematics, physics, philosophy, history, classics, and philology all belong to the Philosophical Faculty. Professors did not even have offices.

Courant decided to construct a new building for the Mathematics Institute and contacted the International Education Board in the United States. The Board provided $350,000 for the construction and equipment of a building for the Mathematical Institute and an addition to the building for the Physics Institute at Göttingen, with the understanding that the German government would provide $25,000 annually for the maintenance of the buildings.[3]

The formal dedication of the Mathematics Institute, which was the first such building in Germany, took place on 2 December 1929. It was a three-level T-shaped building. The spacious lobby containing a bust of Hilbert today is known as "the Hilbert space." Some students would ask, "How come Hilbert is not in the infinite dimensional space?"

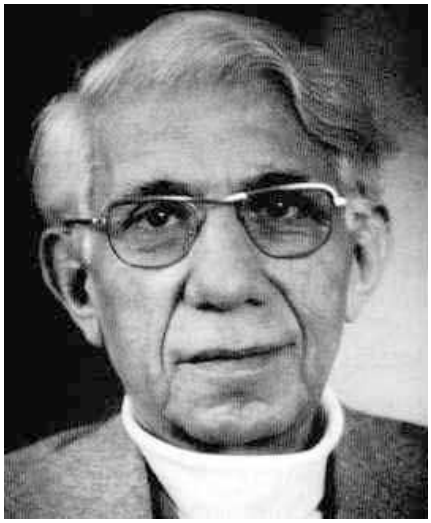


Figure 42. Alexander Dinghas

Figure 43. Richard Courant

In 1922 Courant published a book on the function theory co-written with Hurwitz although Hurwitz died in 1919. In 1924 he also published, jointly with Hilbert, *Methoden der mathematischen Physik* which was very useful in quantum mechanics.

After Hitler came to power in January 1933, all Jewish and liberal non-Jewish professors and lecturers were dismissed. Courant's family moved to New York on August 21, 1934.

Courant slowly built a graduate mathematics program at New York University attracting many German refugees including Friedrichs, Lewy, Busemann, Fritz John, and Willi Feller as well as American mathematicians.

From 1953 to 1958 he was the director of the new Institute of Mathematical Science at New York University, which in 1964 was re-named the Courant Institute.[3] His creation of the Courant Institute at NYU, starting with nothing was a great contribution to mathematics in the United States.

After suffering a stroke in November 1971, he died on 27 January in New Rochelle, New York.

Franz Rellich

Franz Rellich was born on 14 September 1906 in Tramin, South Tyrol, Austro-Hungarian Empire. He received a Ph.D. at the University of Göttingen in 1929 for his thesis *Verallgemeinerung der Riemannschen Integrationmethode auf Differentialgleichungen n-ter Ordnung in Zwei Veränderlichen* (Generalisation of Riemann's integration method on differential equations of n-th order in two variables) under Courant's supervision. He was habilitated in 1933 at Göttingen.[1] In 1934 he became a privatdozent in Marburg and in 1940 a professor at the Technische Hochschule Dresden, and in 1946 director of the Mathematical Institute in Göttingen. He was instrumental in rebuilding the mathematical tradition in Göttingen after many Jewish and liberal professors left.

Among his mathematical contributions, his works in perturbation theory of linear operators in Hilbert Space are important.

He also showed that the Monge-Ampère differential equation in the elliptic care, it can have at most two solutions.

In 1940, he proved that a differential equation W'= f(z,w) has at most countably many entire solutions W(z), if f(z,w) is a linear entire function in z.

This is now known as Rellich's theorem.[92]

He died on 25 September 1955.

Hermann Klaus Hugo Weyl

Hermann Weyl was born on 9 November 1885 in Elmshorn near Hamburg, Germany. He received his doctorate in 1908 at Göttingen with his thesis *Singluläre Integral-gleichungen mit besonder Berücksichtigung des Fourierschen Integraltheorems* under Hilbert's supervision. He submitted his habilitation thesis *Über gewöhnliche Differentialgleichungen mit Singularitäten und die zugehörigen Entwicklungen willkürlicher Funktionen* in Göttingen in 1910 and stayed there until 1913 as a privatdozent.[3]

Figure 44. Hermann K. H. Weyl

Weyl was an ordinary professor at Eidgenössische Technische Hochschule Zürich from 1913 to 1930 supervising 15 doctoral candidates. In 1918 he published *"Raum-Zeit-Materie"* (Space-Time-Matter) which was based in his lectures at ETH in the summer term of 1917.

In 1916 he published one of his best papers on number theory on the definition of the uniform distribution modulo 1, which was to be of great significance in the later work of G.H. Hardy and John Littlewood.[5]

He also published *Gruppentheorie und Quantemmechanik* in 1928. Weyl wrote 15 important books on mathematics and mathematical physics.

Weyl succeeded Hilbert at Göttingen 1930. In 1933 he was forced to leave Germany for the United States because his wife Helene (Hella) Joseph was Jewish. They had

married in 1913. In 1921 Erwin Schrodinger was appointed professor of physics at the University of Zürich and became a close friend of Weyl.

Hella Weyl was a beautiful philosopher and literatus. But Schrödinger's wife Annemarie (Anny) was almost an exact opposite of Hella Weyl. Anny Schrödinger was madly in love with Weyl, while Hella Weyl was infatuated with physicist Paul Scherrer (1890–1969).[3] Schrödinger invited "an old girl-friend in Vienna" to join him skiing in Arosa while his wife Annemarie remained home in Zürich.[11]

Weyl was appointed professor at the Institute for Advanced Study at Princeton from 1933 to 1952. His wife Hella died in 1948, and he married the sculptor Ellen Lohnstein Bär in 1950.[3] After his retirement he and his wife Ellen split their time between Princeton and Zürich. He was walking home in Zürich after mailing letters to those who had sent greeting for his 70th birthday when he collapsed and died on 9 December 1955.

Max Wilhelm Dehn

Max Dehn was born on 13 November 1878 in Hamburg. He obtained his doctorate in 1900 with his thesis *Die Legendreschen Sätze über die Winkelsumme in Dreieck* at Göttingen under Hilbert's supervision. He wrote his habilitation thesis at Münster in 1901 and stayed there for ten years. After two years (1911–1913) at Kiel and eight years (1913–1921) at TH Breslau, he was appointed ordinary professor at Frankfurt in 1921.

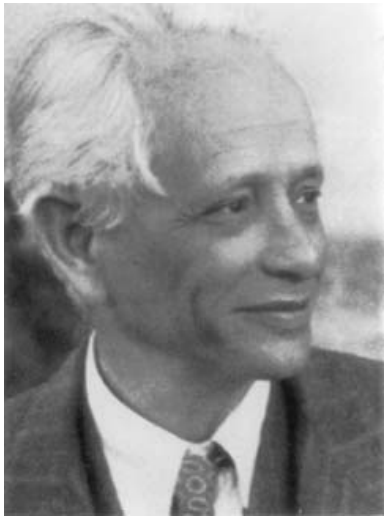


Figure 45. Max W. Dehn

Figure 46. Alexander M. Ostrowski

His colleagues at Frankfurt were Carl L. Siegel, Ernst Hellinger, Otto Szász and Paul Epstein. Together they created an important research center in mathematics.

Dehn was 22 years old when he solved Hibert's third problem on the congruence of polyhedra. He served in the German Army from 1915 to 1918.

After the Nazis came to power, the Frankfurt Mathematics Institute was completely destroyed. Siegel was not Jewish but because of the Nazis policies, he went to the United States followed by Dehn, Hellinger and Szász. Epstein committed suicide when he received a summons from the Gespato.

Dehn arrived in the United States in 1941 after traveling through Norway, Siberia, and Japan. He was an assistant professor at the University of Idaho, Illinois Institute of Technology and St. John's College in Annapolis, Maryland.

In 1945 he joined the faculty at Black Mountain College in North Carolina. It never had more than 90 students, but there was a large faculty, all of whom lived at the College.[12] Dehn taught Greek and philosophy in addition to mathematics.

He was the only mathematician at the college. Most of the mathematicians fleeing Europe were able to obtain prestigious academic position in the United States, but Dehn was unable to do so.

He was paid only $40 a month at Black Mountain College but he loved the dogwood forest nearby. One summer day in June 1952 he saw some loggers cutting down trees and he ran up a steep path to try to stop them. That effort probably caused the 'embolism' that killed Dehn the next day. He woke the next morning with severe chest pain and asked for aspirin. By the time his wife came back with the aspirin. He was dead on 27 June 1952 in Black Mountain.[12]

The Black Mountain College closed in 1956.

Alexander Markowich Ostrowski

Alexander Ostrowski was born into a Jewish family on 25 September 1893 in Kiev, Ukraine. While he was a teenager he participated B.N. Grave's seminar at the University of Kiev and wrote his first paper with Grave's assistance. Since he was a graduate of Kiev College of Commerce not a high school, he could not be admitted to Russian universities.

Kurt Hensel (1861–1941) invited Ostrowski to the University of Marburg in 1912. After the outbreak of World War I in 1914, Ostrowski was interned as a hostile foreigner. Fortunately he was allowed to use the library at the University of Marburg during that time so he could read through mathematical journals.

When the war ended in 1918, he went to Göttingen and he was awarded his doctorate in 1920 under Hilbert and Landau. His doctoral dissertation, his fifth paper, was published in *Mathematische Zeitschrift*.[3] The results of this work allowed Ostrowski to solve Hilbert's 18th Problem in part.

Ostrowski succeeded in proving that the Dirichlet zeta series

$$\zeta(x,s) = 1^{-s}x + 2^{-s}x^2 + 3^{-s}x^3 + \cdots\cdots$$

does not satisfy an algebraic partial differential equation.[30]

Ostrowski left Göttingen for Hamburg, where as an assistant of Erich Hecke he worked on his habilitation thesis. He was awarded habilitation in 1922, then he became a privatdozent at Göttingen from 1923 to 1927.

During the academic year 1925–26 he visited Oxford, Cambridge, and Edinburgh as a Rockefeller Research Fellow. He accepted the chair of mathematics at the University of Basel in 1927 where he remained until he retired in 1958. He married Margaret Sachs, a psychoanalyst from the school of Carl Gustav Jung in 1949.

Ostrowski wrote around 275 papers and books. His works were published in six volumes in 1983–85 by Birkhäuser.

He died on 20 November 1986 in Montagnola, Lugano, Switzerland.

Wilhelm Magnus

Wilhelm Magnus was born in Berlin on 5 February 1907. He received his doctorate in 1931 at Frankfurt with his thesis *Über Unendlich diskontinuierliche Gruppen von einerdefinirenden Relation (der Freiheitssatz).*

Magnus was a privatdozent at Frankfurt from 1933 to 1938. He refused to join the Nazi Party, and as a consequence, he was not allowed to hold an academic post during World War Ⅱ.[3]

From 1948 to 1950 he collaborated with Arthur Erdélyi on the project of organizing and publishing Harry Bateman's (1882–1946) manuscripts at the California Institute of Technology.[3]

He then spent 23 years at the Courant Institute of Mathematical Sciences at New York University and five years at the Polytechnic Institute of New York.

He supervised 72 doctoral candidates mostly at NYU where he received the Great Teacher Award. Magnus died on 15 October 1990.

He was a real expert on Rainer Maria Rilke.

Figure 47. Wilhelm Magnus

Rudolf Friedrich Alfred Clebsch

Alfred Clebsch was born on 19 January 1833 in Königsberg, Germany (now Kaliningrad, Russia).

He studied at the University of Königsberg where Otto Hesse, Friedrich Richelot and Franz Neumann taught. After receiving his Ph.D. in 1854 at Königsberg, he became a privatdozent at Berlin in 1857.

Clebsch was appointed to Chair of Mechanics and Synthetic Geometry in 1858 at TH Karlsruhe. He moved to Giessen in 1863 where he stayed for five years.

He succeeded Bernhard Riemann at Göttingen in 1868, and died of diphtheria there on 7 November 1872.

Although his Ph.D. thesis was on hydrodynamics, he changed his interests to study the calculus of variations and patial differential equations.[3]

Clebsch and Carl Neumann, the son of his teacher at Königsberg, founded *Mathematische Annalen*. The Clebsch-Gordan coefficients used in quantum mechanics were introduced by them while working on *"Theorie der Abelschen Funktionen"* in 1866.

Figure 48. Rudolf F. A. Clebsch

Figure 49. Alfréd Haar

Erich Hecke

Erich Hecke was born on 20 September 1887 in Buk, Posen, Germany (now Poznan, Poland). He was awarded his Ph.D. at Göttingen for a dissertation *Zur Theorie der Modulfunktionen von Zwei Variablen und ihrer Anwendung auf die Zahleatheorie* under Hilbert's supervision. He stayed three years at Göttingen as a privatdozent (1912–1915). Hecke was appointed to a chair at the University of Basel, Switzerland, in 1915. Then he succeeded Carathéodury at Göttingen in 1918.

He moved to the University of Hamburg in 1919, the year it was founded so he thought he would have had the opportunity to influence the new institution.[3]

When Hecke was Hilbert's assistant he received 50 Marks ($12.50 at that time) a month. One day Hilbert decided that sum was inadequate. He went to meet with Wilhelm Althoff, who was in charge of universities at the Ministry of Culture. At the end of the conversation, he remembered there was another matter he had intended to discuss with Althoff. Hilbert stuck his head out of the window of the Ministry and shouted down to Mrs. Hilbert who was waiting for him outside. "Käthe, Käthe! What was that other matter I wanted to talk about?" "Hecke, David, Hecke!" Hilbert turned back to Althoff, and demanded that Hecke's salary be doubled.[9]

This episode demonstrates how the Ministry of Culture in Germany micromanaged university affairs at that time.

Hecke's most important work was in 1936 with his discovery of the properties of the algebra of Hecke operators and of the Euler products associated with them.[3]

Hecke was President of the Deutsche Mathematiker-Vereinigung (German Mathematical Society) in 1923.

Hecke died of cancer on 13 February 1947 in Copenhagen, Denmark.

Alfréd Haar

Alfréd Haar was born on 11 October 1885 in Budapest, Hungary to an owner of great vineyards.[9] After graduating from high school in Budapest he went to Göttingen in 1904. He received his doctorate in 1909 at Göttingen with his thesis *Zur Theorie der orthogonalen Funktionensysteme* under Hilbert's supervision. Haar was small, delicately built and had the charming quality of feeling at home anywhere in the world. He possessed the kind of extremely quick and precise talent.[9]

Haar was a privatdozent at Göttingen from 1910 to 1912. He was appointed as an extraordinary professor at the Franz Josef Royal Hungarian University in Kolozsvár (now Cluj in Romania) in 1912. Five years later he was appointed ordinary professor there. Since Hungary was defeated in the World War I, Kolozsvár became a part of Romania and the university was moved to Szeged 270 km west from Cluj in 1920. In 1922 Haar and Frigyes Riesz (1880-1956) set up the János Bolyai Mathematical Institute at the University of Szeged because Bolyai had been born in Kolozsvár.

The University of Szeged is now a prestigious institution in Hungary and Szeged is a sister city of Cambridge, England.

Haar and Riesz also founded the famous journal *Acta Scientiarum Mathematicarum* in 1930. Haar introduced an invariant measure on locally compact groups, now called the Haar measure.[3]

John von Neumann had tried to discourage Haar from seeking such a measure since he was certain that no such measure could exist.[3]

Haar died on 16 March 1933 in Szeged, Hungary.

Ernst David Hellinger

Ernst Hellinger was born on 30 September 1883 in Striegau, Silesia, Germany (now Strizegom, Poland). Hellinger grew up nearby in Breslau where he graduated from the Gymnasium in 1902. Otto Toeplitz, Max Born and Richard Courant all grew up in Breslau.

Hellinger received his Ph.D at Göttingen in 1907 for his thesis *Die Orthogonalinvarianten quadratischer Formen von unendlich vielen Variablen* under Hilbert's supervision. He introduced a new type of integral, the Hellinger integral in his doctoral thesis. He was a privatdozent at Marburg from 1909 to 1914.

In 1914 he was appointed extraordinary professor at the new University of Frankfurt. In 1921 Max Dehn, Paul Epstein and Otto Szász joined the faculty. One year later Carl Siegel also came to Frankfurt which became an important mathematical research center with these five mathematicians.

Hitler's regime forced Hellinger to retire in 1936 despite the fact that he served in the German Army during World War I. He was sent to the Dachau concentration camp in 1938. Fortunately, his friends were able to arrange a temporary job for him at Northwestern University in the United States. He was released and emigrated to the United States in late February 1939.

He became a U.S. citizen in 1944 and worked at Northwestern University until 1949 when he retired. He was one of the best-liked professors in the Northwestern Mathematics Department.[3]

With Toeplitz he wrote *"Integralgleichungen und Gleichungen mit unendlich vielen Unbekannten"* in the *Enzyklopädie der Mathematischen Wissenschaften* in 1927.[3]

Hellinger died of cancer on 28 March 1950 in Chicago, Illinois.

Figure 50. Ernst D. Hellinger

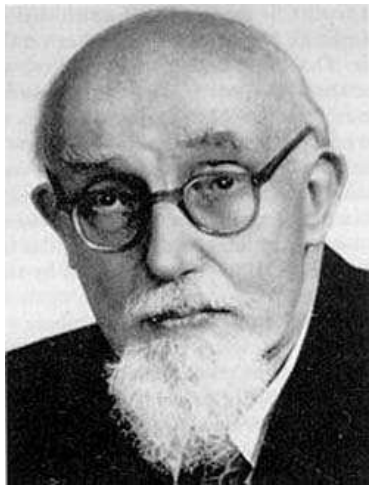


Figure 51. Georg K. W. Hamel

Georg Karl Wilhelm Hamel

Georg Hamel was born on 12 September 1877 in Düren near Aachen, Germany. He studied at TH Aachen, Berlin and Göttingen where he received his doctorate in 1901 for

his thesis *Über die Geometrien, in denen die Geraden die Kürzesten sind* under Hilbert's supervision.

He was a privatdozent at TH Karlsruhe from 1903 to 1905. Hamel was appointed professor of mechanics at the German Technical University of Brünn (now Brno, Czech Republic) in October 1905. In August 1909 he married Agnes Frangenheim in Köln. They had three daughters.[3]

After seven years at Brünn he moved to TH Aachen in October 1912. Then he was appointed in 1919 to a chair at TH Berlin from where he retired in 1949.

During the Third Reich Hamel was clearly associated with the Nazis. In 1935 he was appointed President of *Deutsche Mathematiker-Vereinigung*. He was made editor of the *Neue Deutsche Forschungen* in 1935. He is best known for the Hamel basis, published in 1905. He tried to construct a basis for the real numbers as a vector space over the rational numbers.

He died on 4 October 1954 in Landshut, Germany.

Figure 52. Richard von Mises

Richard von Mises

Richard von Mises was born on 19 April 1883 in Lemburg, Austria (Lwow, Poland and now Lvov, Ukraine). After graduating TH Vienna studying mathematics, physics and engineering, he became an assistant to Georg Hamel at the University of Brünn. Von Mises received his Ph.D. at Vienna and habilitation at Brünn under Hamel. Von Mises was appointed extraordinary professor at Strassburg in 1909.

He became a pilot and taught the first university course on powered flight in 1913 at Strassburg. During World War I he joined the Austro-Hungarian Army as a pilot.[3] He led a team that constructed a 600-horsepower plane for the Austrians in 1915.

In 1919 he moved to TH Dresden but he moved to Berlin one year later. He was appointed to the new Chair for Applied Mathematics (Lehrstuhl für Angewandte Mathematik) in 1920. It was the first Chair on applied mathematics in Germany.

His new Institute of Applied Mathematics became a center for research in probability, statistics, numerical solutions of differential equations, elasticity and aerodynamics. In 1921 he founded the journal *Zeitschrift für Angewandte Mathematik und Mechanick* and became the editor of the journal. He was an excellent lecturer.

After the Nazis came to power he had to give up his Berlin professorship because he fell under the definition of non-Aryan even though he was a Roman Catholic. His successor was the Nazis Theodor Vahlen who promised that von Mises would not lose his pension rights.

In 1934 he was appointed to a chair in Istanbul, Turkey. The mathematician Hilda Geiringer (1893-1973), von Mises lover, went with him to Istanbul in 1934. In 1939 von Mises left Turkey for the United States. He became a professor at Harvard University and was appointed Gordon-Mckay Professor of Aerodynamics and Applied Mathematics in 1944.

Geiringer also moved to the United States and they married in 1943. Before they were married there was a rumor that von Mises was hiding a mistress. He insisted that Geiringer addresse him as "Herr Professor."

Von Mises wrote a book on philosophy "A Study in Human Understanding" in 1951. He was also an international authority on the Austrian poet Rainer Maria Rilke.[3]

In 1950 von Mises was offered honorary membership of the East German Academy of Scince. He declined because it was the McCarthy era in the United States.

He died on 14 July 1953 in Boston, Massachusetts.

Wilhelm Cauer

Wilhelm Cauer was born in Berlin on 24 June 1900. He received a doctorate at TH Berlin under Hamel's supervision in 1926. His dissertation was *Die Verwirklichung von Wechstomwiderständen vorgeschriebener Frequenzabhängigkeit* (The Realisation of Impedance for Prescribed Frequency Dependence) which took the first step toward network synthesis.

The driving-point impedance for any passive electrical network is a rational function of complex variable. Complex variable s is called the complex frequency.

Cauer is best known for Cauer Form in network synthesis utilizing the continued fraction expansion which results in a ladder network. For a given driving-point impedance function, a pole at infinity is represented as a series inductor. This process is "remove a pole, invert and remove a pole," to produce a ladder network.

One of his doctoral students at Louvain, Vitold Belevitch, also made important contributions to the circuit theory. Cauer published *Theorie der linearen Wechselstromschaltangen* (Theory of Linear Alternating Current Circuits) Volume I in Leipzig in 1941.

Although he was a civilian and not a member of the Nazi Party, he was executed on 22 April 1945 by the Red Army soldiers of the Soviet Union after their entry into Berlin.[14]

Figure 53. Karl T. W. Weierstrass

Karl Theodor Wilhelm Weierstrass

Karl Weierstrass was born on 31 October 1815 in Ostenfelde, Bavaria. His father wanted Karl to study finance, law and economics despite Karl's love of mathematics.

At the University of Bonn he spent four years of intensive fencing and drinking,[3] and he studied mathematics on his own. In 1839 he enrolled at the Theological and Philosophical Academy of Münster (now University of Münster) to attend lectures by Christoph Gudermann (1798-1851).

In 1841 he passed the examination to teach at secondary schools. Weierstrass published *Zur Theorie der Abelschen Funktionen* in 1854 in Crelle's Journal. F.J. Richelot (1808-1875) at the University of Königsberg was impressed by the article and the university conferred an honorary doctoral degree on Weierstrass in March 1854. The Gewerbeakademie (Industry Institute in Berlin, later TH Berlin) offered him a chair in 1856 so that he could leave his secondary school after teaching there for 15 years.

In 1856 he was elected to the Berlin Academy which gave him the right to teach at the University of Berlin. Until 1864 he taught at the two universities in Berlin. He was finally offered a newly created chair (Dritter Lehrstuhl) at the University of Berlin in 1864, from where he retired in 1892.[1]

In 1861 his emphasis on rigour led him to discover a function that, although continuous, had no derivative at any point.[3]

Most mathematics students know the following two Weierstrass theorems[13]:

Let T be a closed bounded set of real numbers. Then every function $f(x)$, *continuous there, is the limit of a sequence of polynomials in* x *with real coefficients which is uniformly convergent on that set.*

Let $f(x)$ *be a continuous periodic function of period* 2π. *For every* $\varepsilon > 0$, *there exists a trigonometric polynomial* $T(x)$ *such that* $|f(x) - T(x)| < \varepsilon$ *for all* x.

Weierstrass never married and lived with his unmarried sisters. A lecture room at the University of Berlin (now Humboldt University) is named after him. He also served as Rector of the University of Berlin in 1873-74.

Weierstrass died on 19 February 1897 in Berlin.

Lazarus Immanuel Fuchs

Lazarus Fuchs was born on 5 May 1833 in Moschin, Germany (now Poznan, Poland). He received his doctorate at Berlin in 1858 with his thesis *"De Superficierum lineis Curveturae"* under Weierstrass' supervision. After teaching for seven years at a secondary school, he submitted his habilitation thesis in 1865 and he was appointed extraordinary professor at Berlin in 1866. In 1869 he moved to Greifswald, then to Göttingen in 1874 where he stayed only one year.

From 1875 to 1884 he held a chair at Heidelberg succeeding Leo Königsberger who was also a pupil of Weierstrass.

Figure 54. Lazarus I. Fuchs

Figure 55. Ernst F. F. Zermelo

In 1884 Fuchs succeeded Ernst Kummer at Berlin where he died in 1902.

Fuchs introduced an important class of linear differential equations in the complex domain with analytic coefficients, a class which is called the Fuchsian type.[3]

He died on 26 April 1902 in Berlin.

Ernst Friedrich Ferdinand Zermelo

Ernst Zermelo was born on 27 July 1871 in Berlin. After graduating from gymnasium he studied at Berlin, Halle and Freiburg. He was awarded a doctorate in 1894 at Berlin for his thesis *Untersuchungen zur Variationsrechnung*. He then became Max Planck's assistant. In 1897 he went to Göttingen where he completed his habilitation with his dissertation *Hydrodynamische Untersuchungen über die Wirbelbewegungen in einer Kugelfläche* in 1899.

He stayed in Göttingen for eleven years as a privatdozent performing significant work on the axiomatic set theory. In 1911 Göttingen awarded him 5000 Deutsche Marks for his major contributions to the set theory which enabled him rest to regain his health.

In 1910 he had been appointed to a chair at the University of Zürich. However, due to poor health he resigned his chair in Zürich in 1916 and moved to the Black Forest in Germany.[3]

Zermelo was appointed to an honorary chair at the University of Freiburg in 1921 but he renounced the chair in 1935.

He died on 21 May 1953 in Freiburg, Germany.

Leo Königsberger

Leo Königsberger was born on 15 October 1837 in Posen, Prussia (now Poznan, Poland). He became friendly with Lazarus Fuchs, who came from the same town. Königsberger was awarded his Ph.D. in 1860 at Berlin with his dissertation *De motu puncti versus duo fixa centra attracti.* His main research interest was on elliptic function influenced by his adviser Weierstrass.

After teaching three years at the Berlin Cadet Corps, he was appointed extraordinary professor at Greifswald in 1864. He was promoted to ordinary professor in 1866. In 1869 he moved to Heidelberg where he stayed six years. When he left Heidelberg for Dresden in 1875 his successor was Lazarus Fuchs.

After two years at TH Dresden he moved to Vienna in 1877, then returned to Heidelberg in 1884.

Despite many moves, he spent 36 years at Heidelberg. His colleagues at Heidelberg were Bunsen, Kirchhoff and Helmholtz.

When Königsberger was first appointed to a chair at Heidelberg, Sofia Kovalevskaya attended his lecture even though she was not permitted to be a formal student.

Königsberger died on 15 December 1921 in Heidelberg, Germany.

Figure 56. Leo Königsberger

Figure 57. Ferdinand G. Frobenius

Ferdinand Georg Frobenius

Georg Frobenius was born on 26 October 1849 in Berlin-Charlottenburg. He received his Ph.D. in 1870 at Berlin with his dissertation *De functionum analyticarum unius variab per series infin repraesentatione* supervised by Weierstrass.

After teaching for four years at a secondary school, he was appointed extraordinary professor at Berlin in 1874.

After only one year he was appointed ordinary professor at the Eidgenösische Polytechnikum Zürich in 1875 where he stayed for seventeen years.

Kronecker died on 29 December 1891 and Frobenius was appointed to his chair at Berlin in 1892, and he retired from there in 1916. He considered himself to be a scholar called to contribute to the knowledge of pure mathematics and he believed that applied mathematics belonged to the technical colleges. At Göttingen pure mathematics and applied mathematics were treated equally. Consequently Frobenius disliked Göttingen mathmaticians, Felix Klein in particular.

In 1892 Frobenius was elected to the Prussian Academy of Science. By studying the different representation of groups and their elements, he provided a firm basis for the solution of general problems in the theory of finite groups.[5]

The rank of a matrix was first defined in 1878 by Frobenius. He used matrices in his work on group representations. The power series approach in ordinary differential equations is called the Method of Frobenius, and the series solutions obtained are Frobenius series.

Issai Schur and Edmund Landau were the most important students of Frobenius.

Frobenius died on 3 August 1917 in Berlin.

Issai Schur

Issai Schur was born on 10 January 1875 in Mogilyov 180km east of Minsk, Belarus. After attending the gymnasium in Latvia, he entered the University of Berlin. He was awarded his Ph.D. in 1901 with his thesis *Über eine Klasse von Matrizen die sich einer gegeberen Matrix zuordnen lassen* supervised by Frobenius.

Schur was a privatdozent at Berlin from 1903 to 1909. From 1913 to 1916 he was an extraordinary professor at Bonn. In 1916 he moved back to Berlin and was promoted to ordinary professor. He succeeded Friedrich Schottky in 1921, and held this chair until the Nazis forced him to retire in 1935.

He was elected to the Prussian Academy of Science on Max Planck's recommendation but he was forced to resign from the Academy in 1938.

Schur is best known for his work on the representation theory of groups and Schur's Lemma.[3]

Students of Linear Algebra know that Schur decomposition of matrix $A = PTP^H$ where P is unitary and T is an upper-triangular matrix. And P^H is the Hermitian transpose of P.

His colleagues at Berlin were Ludwig Bieberbach, Erhard Schmidt and Richard von Mises. Bieberbach became a notorious Nazi and Schmidt collaborated with the Nazis. Schur and von Mises were dismissed because they were Jewish.

Schur was an excellent lecturer and supervised many outstanding doctoral candidates in Berlin. He left Germany for Palestine in 1939 suffering the final humiliation of having to find a sponsor to pay the Reichs flight tax that allowed him to leave Germany. He was forced to sell his beloved books to the Institute for Advanced Study in Princeton.[3]

Schur died on 10 January 1941 in Jerusalem, Palestine.

Figure 58. Edmund Landau

Figure 59. Issai Schur

Edmund Landau

Edmund Landau was born into a wealthy Jewish family on 14 February 1877 in Berlin. He received his Ph.D. in 1899 at the University of Berlin for a thesis on number theory supervised by Frobenius. He submitted his habilitation thesis on Dirichlet series, a topic in analytic number theory.[3] He taught at Berlin until 1909.

Minkowski died in January 1909 and Göttingen considered Oskar Perron and Edmund Landau as his successor. Klein preferred Landau because "it is better if we have a man who is not easy."[9] Despite his outstanding talents as both a teacher and researcher, Landau annoyed many of his colleagues with his arrogant manner. For example, if someone asked for his address in Göttingen, he replied "You'll find it easily; it is the most splendid house in the city."[3]

Landau's colleagues from 1909 to 1934 were Hillbert, Carathéodory, Hecke, Courant and Herglotz, Klein and Runge.

After the Nazis came to power in January 1933, Landau's lectures were boycotted by student members of the Sturmabteilung (storm troopers SA) led by Oswald Teichmüller (1913–1943). After receiving his Ph.D at Göttingen, Teichmüler was killed in action on the Russian front in 1943.

Landau published over 250 papers on number theory and his masterpiece of 1909 was the treatise *Handbuch der Lehre von der Verteilung der Primzahlen* a two volume book giving the first systematic presentation of the analytic number theory.

Landau's wife was Marianne, daughter of Paul Ehrlich, who was the 1908 Nobel laureate in medicine and physiology.

Landau officially retired on 7 February 1934. He received full pay until 1 July 1934, then a pension until he died of a heart attack in Berlin on 19 February 1938. Landau's successor in Göttingen was a Nazis party member Erhard Tornier (1894–1982).

Konrad Hermann Theodor Knopp

Konrad Knopp was born on 22 July 1882 in Berlin, Germany. He studied at the University of Berlin and received his doctorate in 1907 for his thesis *Grenzwerte von Reihen bei der Annäherung an die Konvergenzgrenz* supervised by Frobenius and Schottky.

Knopp went to Japan and taught at the Nagasaki College of Commerce during 1908–09. In 1910 he returned to Germany and married the painter Gertrud Kressner. They had one son and one daughter. The couple moved to Tsingtao (Qingdao) and taught at the German–Chinese Academy during 1910–11.

In 1911 they returned to Germany and he taught at the Military Technical Academy and Military Academy and also received his habilitation at the University of Berlin. Knopp was an officer in the German Army during World War I. He was wounded at the beginning of the war, resulting in his discharge from the German Army.

He was appointed extraordinary professor at the University of Königsberg in 1915 and became an ordinary professor there in 1920.

In 1926 he accepted a chair (Erster Lehrstuhl) at the University of Tübingen succeeding Ludwig Maurer, Knopp retired from there in 1950.

Knopp was the cofounder of *Mathematische Zeitschrift* in 1918, and was the editor from 1934 to 1952. He wrote excellent books on complex functions. *Theorie und Anwendung der unendlichen Reihen* was published in 1922 and *Elemente der Funktionentheorie* was published in 1936.[3]

He also produced the sixth edition of Hans von Mangoldt's (1854–1925) famous three volume book *Höhere Mathematik: eine Einführung für studierende und zum Selbststudium*. The book has since then appeared as jointly authored by von Mangoldt and Knopp and has been popular with generations of German students in mathematics, physics and engineering.

Knopp died on 20 April 1957 in Annecy, France.

Figure 60. Konrad H. T. Knopp

George G. Lorentz

George Lorentz was born on 25 February 1910 in St. Petersburg, Russia. Lorentz attended a Russian secondary school and later a German school, thereby becoming fluent in German. In 1926 he entered Tiflis Institute of Technology in Gruzia (Georgia). Two years later he transferred to Leningrad State University, where he received his

diplom in mathematics in 1931. In 1936 Lorentz became a dozent at Leningrad State University. The Leningrad Mathematical Society was disbanded because the communist government believed that mathematics was too theoretical. His father was arrested on false charges in 1937 and died in a concentration camp in 1938. This tragedy interrupted his studies for about five years.

During World War II Lorentz and his newly wedded wife Tanny escaped Leningrad and arrived at a refugee camp at Torun, Poland. The Lorentzes were registered as ethnic Germans.

In 1943 Lorentz submitted two mathematical papers to Professor Knopp for possible publication in *Mathematische Zeitschrift*. With the help of Wilhelm Süss, Knopp was able to relocate the Lorentz family, including their new-born son Rudolph to Tübingen.

Lorentz received his doctorate at Tübingen in 1944 with his thesis *Eine Fragen der Limitierungs theorie* under supervision of Knopp.

He then moved to the University of Toronto in 1949 where he was able to supervise doctoral students even though his title was instructor. In 1953 he moved to Wayne State University and in 1958 to Syracuse University.

In 1968 he was appointed professor at the University of Texas Austin from where he retired in 1980.

He became a United States citizen in 1959. His son Rudolph became a professor of mathematics at the University of Duisburg, Germany.

Lorentz wrote 130 papers and five books. His hobbies include chess and stamp collecting.

He died on 1 January 2006 in Chico, California.

Figure 61. Paul L. Butzer

Figure 62. Paul I. Bernays

Paul L. Butzer

Paul Butzer was born on 15 April 1928 in Muelheim, Ruhr, Germany. Because of his father's anti-Nazi stand, his family left Germany in 1937, settling first in England. From 1941 to 1955, Paul Butzer lived in Montreal, Canada. He studied at Loyola College (now Concordia University) and the University of Toronto. He was naturalized as a Canadian citizen in 1948. Butzer received his Ph.D. in 1951 at the University of Toronto for his thesis *On Bernstein Polynomials* under supervision of George G. Lorentz.

After teaching at McGill University, he returned to Germany in 1955 where he worked at the Universities of Mainz, Freiburg and Würzburg. He was appointed at the Rheinisch-Westfälische Technische Hochschcele (Aachen University of Technology) in 1958, becoming full professor in 1962 and Emeritus Professor in 1993. He is the author of some 270 research papers and six books or monographs, the editor of fifteen conference proceedings, as well as associate editor of a dozen mathematical journals located in six countries. He is a Corresponding/Associate Fellow/Honorary Member of the Societe Royale des Sciences de Liege, the Academie Royale de Belgique, the Mathematische Gesellschaft in Hamburg, and Honorary Professor at the University of Nanjing, China. He has received honorary doctorates from the Universities of Liege, York (United Kingdom) and Timisoara (Romania). He has had 35 doctoral students and 32 of his former students became professors at universities in Germany, Austria and Greece.

Paul Isaac Bernays

Paul Bernays was born on 17 October 1888 in London, England to Swiss parents. He studied at TH Charlottenburg (now TU Berlin), University of Berlin and Göttingen where he received his doctorate in 1912. His dissertation was *Über die Darstellung von positiven, ganzen Zahlen durch die primitiven, binären quadratischen Formen einer nicht-quadratischen Diskriminante* supervised by Landau.

Bernays was at the University of Zürich as Zermelo's assistant until 1917 when Hilbert offered him a position as his assistant. Bernays received two habilitations at Zürich and Göttingen with his thesis on the completeness of propositional logic in 1919. He was then appointed *Nicht-Beamte Ausserordentlichen Professor* (non-civil service associate professor) in 1922 on Hilbert's recommendation.

After Hitler came to power in 1933 Bernays lost his position and moved to Switzerland because he was a Swiss citizen. He is best known for his two volume work *Grundlagen der Mathematik* (1934–1939) with Hilbert. He also revised Hilbert's *Grundlagen der Geometrie* many times. The tenth edition was published in Zürich. Bernays also published Axiomatic Set Theory in 1958.

Bernays died on 18 September 1977 in Zürich, Switzerland.

Harald August Bohr

Harald Bohr was a younger brother of Niels Bohr (1885–1962). Harald was born on 22 April 1887 in Copenhagen, Denmark. Niels and Harald's mother came from a wealthy Jewish family. Harald had been a member of Denmark's soccer team which placed second in the 1908 Olympic games in London.

He obtained his Ph.D. in 1910 at Kobenhavns Universitet with his thesis *Bidrag til de Dirichletske Rækkers Theori* supervised by Edmund Landau. In 1914 they proved the Bohr–Landau theorem on the distribution of zeros of the Zeta function. They proved that all but an infinitesmal proportion of the zeros of the Zeta function lie in a small neighborhood of the line $S = \frac{1}{2}$.

Bohr became professor of mathematics at the Polytechnic Institute in Copenhagen in 1915 and in 1930 he moved to the University of Copenhagen.

Figure 63. Harald A. Bohr

Figure 64. Carl L. Siegel

For most of his life Bohr was a sick man who suffered from terrible headaches. He was a man of refined intellect and a very humane person, always eager to help his pupils, colleagues and friends.

He died on 22 January 1951 in Copenhagen, Denmark.

Carl Ludwig Siegel

Carl Siegel was born on 31 December 1896 in Berlin. He studied at Berlin attending lectures by Frobenius and Planck. During World War I he was called to military service. After the war he studied at Göttingen where he received his Ph.D. in 1920 with his thesis *Approximation algebraischer Zahlen* supervised by Landau.

Siegel was a privatdozent at Göttingen from 1921 to 1922. When Arthur Schonflies (1853-1928) retired from Frankfurt in 1922 Siegel succeeded him at the age of 26. Siegel's colleagues at Frankfurt were Max Dehn, Ernst Hellinger, Otto Szasz and Paul Epstein. So Frankfurt became one of the most productive mathematics research centers in Germany together with Göttingen and Berlin. A mathematics seminar initiated by Max Dehn in 1922 continued until 1935. They made a rule that they would study all the mathematical works in their original languages.

The number of Ph.Ds awarded in the Faculty of Natural Sciences and Mathematics at Frankfurt are as follows[1]:

	Natural Sciences	Mathematics
1915-1920	60	4
1921-1925	303	7
1926-1930	267	9
1931-1935	240	8
1936-1940	248	6
1941-1945	115	1
1946-1950	80	3
1951-1955	259	9
1956-1960	342	4

In the autumn of 1935, Epstein, Hellinger and Dehn were dismissed from their chairs and Szasz had been removed earlier. Although Siegel was Aryan, he despised the Nazis regime.

Siegel moved to Göttingen in 1938 succeeding Rolf Nevanlinna. In 1940 Siegel emigrated to the United States. He was at the Institute for Advanced Study at Princeton from 1940 until 1951, being appointed to a permanent professorship there in 1946.[3]

In 1951 he returned to Göttingen for the rest of his career. He never married, and devoted his life to research. Siegel criticized Serge Lang's (1927–2005) book *Diophantine geometry* published in 1962, because Siegel's own contribution to the subject had altered beyond recognition. Louis Mordell (1888–1972) also wrote a critical review of the book in 1964. Siegel received the Wolf Prize in 1978 for his contributions to the theory of number, the theory of several complex variables, and celestial mechanics.

Siegel died on 4 April 1981 in Göttingen and was buried in the Göttingen Stadtfriedhof where Hilbert, Klein, Schwarzschild were also buried.

Felix Hausdorff

Felix Hausdorff was born into a wealthy Jewish family on 8 November 1868 in Breslau, Germany (now Wroclaw, Poland). The family moved to Leipzig when Felix was still young. Hausdorff received his Ph.D. in 1891 at the University of Leipzig with his thesis *Zur Theorie der astronomischen Strahlenbrechung* (On The Theory of Astronomical Refraction) supervised by Ernest Burns (1848–1919).

He submitted his habilitation thesis *On The Absorption Of Light In The Atmosphere* to Leipzig in 1895 and served as a privatdozent until 1901. He married Charlotte Sara Goldschmidt in Leipzig in 1899.[3] In 1901 he was appointed extraordinary professor at Leipzig. He left for Bonn in 1910 to be an extraordinary professor, working with Eduard Study (1862–1930). From 1913 to 1921 he was ordinary professor at Greifswald. He then moved back to Bonn in 1921 where he stayed until he was forced to give up his chair in 1935.

Hausdorff was also interested in literature and philosophy. He published a play *"The Doctor's Honour"* and a collection of poems entitled *Ecstasies* as well as a book of aphorisms.[4] He was an excellent pianist and occasionally composed songs. In fact he had once considered becoming a composer but his father discouraged him.

In 1914 he published his famous text *Grundzuge der Mengenlehre* (Basic Features Of Set Theory) which won him world wide recognition.

Hausdorff is given credit for laying the foundations of set-theoretic topology and the Hausdorff separation axiom. He introduced and investigated a class of measures and a type of dimension that may assume fractional values. These important concepts are known as Hausdorff measure and Hausdorff dimension.[4]

One day after his seventieth birthday on 9 November 1938 a mob came to his house shouting "We are going to send you to Madagascar where you can teach mathematics to the apes." He was deeply shocked.[4]

In January 1942 Hausdorff's family was ordered to move to a monastery in the suburb of Endenich. Felix, his wife Charlotte and his wife's sister, Edith all took barbiturates on 25 January 1942. Felix and Charlotte died on 26 January and Edith, died after a few days. The family had lived on Hindenburgstrasse which is now called Hausdorffstrasse in Bonn.[4]

Wilhelm Karl Joseph Killing

Wilhelm Killing was born on 10 May 1847 in Burbach (near Siegen), Westphalia. He studied at Münster and Berlin where he received his Ph.D. in 1872 with his dissertation *Der Flächenbüschel zweiter Ordnung* (Bundles of surfaces of the second degree) supervised by Weierstrass.[3]

In 1875 he married Anna Conmer. He taught at several gymnasiums in Berlin and Brilon. In 1882 he moved to the Catholic Academy in Braunsberg which is now called Braniewo, Poland located between Danzig and Königsberg. He taught there for ten years. During this period he produced some of the most original mathematics ever produced. He published a book on non-euclidean geometry in Leipzig in 1885. Killing introduced *the Lie algebra in Programmschrift* in 1884 in Braunsberg without knowing Lie's work.

Figure 65. Felix Hausdorff

Figure 66. Wilhelm K. J. Killing

Killing eventually met Sophus Lie (1842–1899) and Friedrich Engel (1861–1941) in Leipzig in the summer of 1886 to discuss the simple Lie algebra. Killing also introduced the term "characteristic equation" of a matrix.[3]

In 1892 he was appointed to a chair at the University of Münster from where he retired in 1919. He was Rector of the University of Münster from 1897 until 1898. Killing was honored with the Lobachevsky Prize in 1900.

He is best remembered by the Killing vector and the Killing equation in tensor analysis.

He died on 11 February 1923 in Münster, Germany.

Sofia (Sonja) Vasilyevna Kovalevskaya

Sofia Kovalevskaya was born on 15 January 1850 in Moscow, Russia. Her parents were Vasily Korvin-Krukovsky, an artillery general, and Velizaveta Schubert.[3] Russian middle names show father's first name. She married Vladimir Onufrievich Kovalevski when she was 18 years old because she wanted to go abroad but her father would not allow it without a husband.

When Sofia was eleven years old, her room was covered with pages of Ostrogradski's calculus lecture notes. In 1869 Sofia traveled to Heidelberg where two professors of mathematics taught. Leo Königsberger was ordinary professor just appointed in 1869 and Paul DuBois-Reymond was extraordinary professor appointed one year earlier.

Physicist Kirchhoff, chemist Bunsen and physiologist Helmholtz were also at Heidelberg at that time.

In 1871 she moved to Berlin where Weierstrass, Kummer and Kronecker lectured. She was not allowed to attend lectures, because she was a woman so Weierstrass tutored her privately.

By the spring of 1874, she had completed three papers on partial differential equations, Abelian integrals and Saturn's Rings. Weierstrass considered each of these papers worthy of a doctorate. In 1874 she was awarded her Ph.D. in absentia at the University of Göttingen. This demonstrates that Göttingen was more liberal than Berlin with regard to female academics.

Despite a strong recommendation from Weierstrass, Kovalevskaya was unable to obtain an academic position. She moved back to Russia with her husband and had a daughter Sophia Vladimirovna, nicknamed "Foufie" in 1878.[5]

In the spring of 1883, Sofia's husband, from whom she had been separated for two years, committed suicide.[3] His business partners were swindlers who had put Vladimir deeply into debt.

Gösta Mittag-Leffler (1846-1927) was able to appoint Kovalevskaya privatdozent at the new Hogskola in Stockholm[4] in early 1884. She was appointed extraordinary professor in June 1884. She and Mittag-Leffler's sister Anne-Charlotte wrote a play together, *The Struggle for Happiness*, which was favorably received when it was performed in Moscow.[4]

She became an editor of the new journal *Acta Mathematica*. In 1888 she won the prestigious Bordin prize awarded by the French Academy of Sciences for her paper on the motion of a rigid body. This brought her international fame and as a result she was promoted to full professor in 1889.

In the same year, on the initiative of Pafnuty Chebyshev (1821-1894), she was elected a corresponding member of the Russian Imperial Academy of Sciences. At the height of her career, Kovalevskaya died of influenza complicated by pneumonia on 10 February 1891 in Stockholm, Sweden.

The existence and uniqueness of quasi-linear partial differential equations with appropriately specified boundary conditions is known as the Cauchy-Kovalevski theorem.

Figure 67. Carl D. T. Runge

Figure 68. Sofia V. Kovalevskaya

Carl David Tolmé Runge

Carl Runge was born on 30 August 1856 in Bremen, Germany. He was awarded a doctorate in 1880 from Berlin with his thesis *Über die Krümmung, Torsion und*

geodätische Krümmung der auf einer Fläche gezogenen Curven supervised by Weierstrass.[10] Runge was a privatdozent at Berlin from 1883 to 1886.

In 1886 he was appointed ordinary professor at TH Hannover where he stayed eighteen years. In 1904 he was appointed to a newly created chair (Vierter Lehrstuhl) at Göttingen from where he retired in 1924. His colleagues at Göttingen were Klein, Minkowski, Landau and Hilbert.

Runge also studied the wavelengths of the spectral lines of elements other than hydrogen. After he moved to Göttingen one of his students were Max Born.

Runge is well known to students due to the Runge-Kutta method for solving ordinary differential equation by numerical analysis. Runge and Kutta never worked together. Wilhelm Kutta (1867-1944) introduced the method in his doctoral dissertation *Beiträge Zur näherungsweisen Integration totaler Differentialgleichungen* in 1900 at the University of Munich (München). His advisers were Ferdinand Lindemann and Gustav Bauer. He taught at TH München, Jena, Aachen and Stuttgart. Runge's daughter Nerina (Nina) married Richard Courant on 22 January 1919. Runge died of a heart attack on 3 January 1927 in Göttingen. His house was on Wilhelm Weber Strasse in Göttingen between houses of Felix Klein and David Hilbert.

Karl Hermann Amandus Schwarz

Hermann Schwarz was born on 25 January 1843 in Riesengebirge, Germany. After studying at a gymnasium in Dortmund he enrolled at the University of Berlin. He received his Ph.D. there in 1864 for his thesis *De superficiebus in planum explicabilibus primorum septem Ordinum.* After completing his habilitation thesis in 1867 he was appointed extraordinary professor at Halle-Wittenberg where he stayed for two years. From 1869 to 1875 he was at the Eidgenosische Technische Hochschule in Zürich. Then in 1875 he succeeded Lazarus Fuchs at Göttingen. Schwarz was appointed to the chair of Weierstrass in 1892. In the same year he was elected to the Berlin Academy of Sciences.

Schwarz's colleagues at Berlin were Frobenius, Fuchs and Schottky. Schwarz failed to produce significant mathematical research after his move to Berlin, and Göttingen became the most eminent German university for mathematics. Schwarz retired in 1917 and his successor was Erhard Schmidt.

Schwarz married Kummer's daughter. He was the captain of the local voluntary fire brigade and he assisted at the railroad station by closing train doors.[3]

Schwarz made an important contribution in 1865 when he discovered what is now known as the Schwarz minimal surface.[3] He also worked on conformal mapping. The Cauchy–Schwarz inequality is well known to students in mathematics. The integral form has the form

$$\left| \int_a^b f^*(x)\ g(x)dx \right|^2 \leq \int_a^b f^*(x) f(x)dx \int_a^b g^*(x)g(x)dx$$

In linear vector space

$$\|(a \cdot b)\| \leq \|a\|\|b\|$$

Schwarz died on 30 November 1921 in Berlin, Germany.

Gustav Herglotz

Gustav Herglotz was born on 2 February 1881 in Wallern, Bohemia (now Volary, Czech Republic), but spent most of his childhood in Vienna. There he formed a close friendship with three other students Heinrich Tietze (1880–1964), Hans Hahn (1879–1934) and Paul Ehrenfest (1880–1933). They became known as the "inseparable four". Herglotz studied astronomy at München where he received his Ph.D. in 1902 with his thesis *Über die Scheinbaren Helligkeitsverhältinisse eines planetarischen Körpers mit drei ungleichen Hauptträgheitsachsen.*[3] He was advised by Hugo von Seeliger (1849–1924) and Ludwig Boltzmann (1844–1906).

Herglotz submitted his habilitation thesis at Göttingen in 1904 to become a privatdozent until 1907. After working one year as an extraordinary professor at TH Wien he was appointed ordinary professor at Leipzig in 1909 where he stayed for sixteen years. At Leipzig, Herglotz supervised at least 25 doctoral candidates including Emil Artin.

In 1925 Herglotz succeeded Carl Runge at Gottingen from where he retired in 1948. Saunders Mac Lane described his experience with Herglotz.[86]

He had a vast knowledge of classical applied mathematics and classical analysis. He delivered stunningly beautiful lectures on mechanics, mathematical optics, functions with positive real parts, and Lie groups.

Physicist Max Born came to Göttingen as professor of physics in 1921 with his wife Hedwig (Hedi), a poet. Around 1925 Max Born became too involved in quantum

mechanics and neglected his wife. She had an affair with Herglotz or Gusti as Hedi Born referred to him. Gossip about Hedi and Herglotz was swirling around Göttingen. Hedi wanted to leave Max Born and marry Herglotz.

She complained that Max reduced her to a formula and did not understand her. The affair lasted eight years until the Borns were forced by the Nazis to leave Göttingen for England.[15]

Herglotz suffered a stroke in 1946 and retired in 1948. One week before he died Hedi Born went to see him but he did not recognize her.

Herglotz died on 22 March 1953 in Göttingen, Germany.

Figure 69. Hermann A. Schwarz

Figure 70. Gustav Herglotz

Emil Artin

Emil Artin was born on 3 March 1898 in Vienna, Austria. His father was an art dealer and his mother was an opera singer. Artin was an accomplished musician playing the flute and harpsichord.[3]

His study at the University of Vienna was interrupted by the World War I and he served in the Austrian Army until the end of the War. He entered the University of Leipzig in 1919 and received his doctorate in 1921 under the supervision of Gustav Herglotz. His dissertation was *Quadratische Körper im Gebiete der höheren Kongruenzen.*[10] He had his habilitation in 1923 at the University of Hamburg where he became an ordinary professor in 1926 at the age of 28.

Artin married Natalie Jasny, one of his students, in 1929. He had to leave Germany for the United States in 1937 because his wife was half-Jewish.

Artin made contributions to solve the Hilbert problems No. 9, 11, and 12 and he solved the seventeenth problem.[12] He was most productive in the ten year period 1921-1931. Artin taught eight years at Indiana University at Bloomington from 1938 to 1946, twelve years at Princeton University from 1946 to 1958 and then moved back to the University of Hamburg in 1958.

His main fields of contribution were class field theory, the theory of braids and Artin rings. Artin supervised 34 Ph.D. candidates (13 in Hamburg, 2 in Indiana, 1 in Columbia and 18 in Princeton). David Gilbarg, John Tate, Jr., Max Zorn, Serge Lang and Hans Zassenhaus are some of his students.

Artin wore an black leather jacket or sometimes a long winter coat belted at the waist. Some students emulated him in dress and manner at Princeton.[12]

His son, Michael Artin became a distinguished mathematician serving as President of the American Mathematical Society in 1991-1992.

Artin died on 20 December 1962 in Hamburg, Germany.

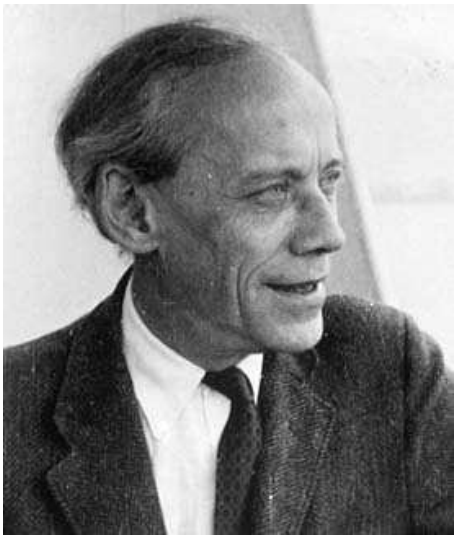


Figure 71. Emil Artin

Figure 72. Theodor F. E. Kaluza

Theodor Franz Eduard Kaluza

Theodor Kaluza was born on 9 November 1885 in Opeln, Germany (now Opole, Poland). He received his Ph.D. at Königsberg in 1907 supervised by Wilhelm Franz

Meyer (1856–1934). Two years later he submitted his habilitation thesis on Tschirnhaus transformations and worked as a privatdozent until 1922.

Normally after one or two years as privatdozent one is appointed extraordinary or ordinary professor. In Kaluza's case, however, it took 13 years for him to be promoted to extraordinary professor at Königsberg. In April 1921, encouraged by Einstein he published an original paper to unify Einstein's theory of gravity and Maxwell's electromagnetic theory. Kaluza's ideas involved the introduction of a fifth dimension.[3] This theory is now known as Kaluza–Klein field theory named after Oskar Klein (1894–1977).

Kaluza was appointed ordinary professor at Kiel succeeding Ernst Steinitz in 1929. In 1935 he moved to Göttingen succeeding Richard Courant and he retired from there in 1954. Kaluza and G. Joos published the textbook *Höhere Mathematik für die Praktiker* in 1938.

He died on 19 January 1954 in Göttingen, Germany.