

Huijgens' Synchronization: A Challenge

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Oscillations are common almost everywhere, be it in biology, in economics, in physics and many other fields. Everyone is familiar with the day-night rhythm, or the regular or less regular heart-beat of a human, the pig cycle in economy or the flashing of fire-flies and so on. All the above examples have in common that the oscillations seem to happen naturally, but there are also other more or less forced type of oscillations like for instance in chemistry, electrical circuits and acoustics. Probably the most basic example of an oscillator is a pendulum clock that runs at a fixed frequency and such that the exact time is given by the clock. Design and construction of a fully accurate mechanical clock is – even today – a very challenging task; the reader is referred to [Rawlings, 1994; Penman, 1998] for some background on this.

Sofar we haven't defined what exactly an oscillation is, and for the time being it suffices to understand it as a dynamic feature that “repeats” itself in time, usually with a fixed time period. As mentioned, such oscillations are abundant in every day life, but even more remarkable, in biological systems the “biological clock” is – or becomes – identical. The flashing of a flock of fire-flies may serve as a good example of this. In fact, there is a large literature on this “synchronization”, i.e. “conformity in time” in biological systems (and in some cases this conformity in time agrees with the

earth time clock of 24 hours a day). A nice account on synchronization in – amongst others – biology can be found in [Strogatz, 2003]. Synchronization, sometimes also referred to as coordination or cooperation, also happens in other than biological systems. Again many examples exist, but an illuminating example is dancing at a music festival – as should be clear the music made by orchestra already forms an illustration of synchronization – where the dancers’ motion synchronizes with the rhythm of the music.

There are numerous examples of synchronization reported in the literature, but here we will in particular only focus on some of the earlier reported cases. For a more complete account on synchronization *per se* we refer to [Pikovsky *et. al.*, 2001]. Probably the earliest writing on synchronization is due to Huygens, see [Huygens, 1660] and describes the (anti-phase) synchronization of a pair of pendulum clocks. Christiaan Huygens (1629-1695) is a famous Dutch scientist who worked on subjects from astronomy, physics and mathematics. He was famous for his work in optics and the construction of (pendulum) clocks and telescopes. Huygens describes his observation that the two pendulum clocks as described in Figure 1.1 exhibit (mirrored) synchronized motion, even when they are initiated at different initial conditions, see [Huygens, 1660], or for an English translation the reader may consult [Pikovsky *et. al.*, 2001]. This is by Huygens called “sympathy of two clocks”, who also ingeniously linked this to the beam to which the two clocks are attached.

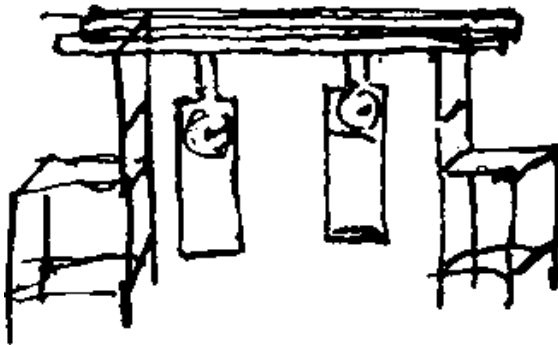


Fig. 1.1 Clocks attached to a beam on two chairs.

Indeed, the unprecedented contribution at this point is the correct explanation by Christiaan Huygens for the (anti-) phase synchronization of the two pendulum clocks. The reader should realize that Huygens lived

in an era that differential calculus did not exist yet – or rather Huijgens himself was one of the earliest initiators of it, whereas Newton and Leibnitz only at the end of the 17th century, beginning 18th century, fully developed the concepts from differential calculus. Therefore, as in the preceding centuries, see e.g. the work of another Dutch (or rather Flemish) scientist, Simon Stevin, the best one can do expect in studying new phenomena, is the combination of “Spiegheling ende Daet”, i.e. reflection and experiment, [Devreese and van den Berghe, 2003]. This is exactly what Huijgens did regarding his two pendulum clock experiment as described in Figure 1.1, namely he repeated the study about phase synchronization of the two pendula under systematically changing the set-up, and most notably, by changing the distance between the two clocks hanging on the beam. He then also reached the conclusion that once the clocks are positioned close to each other, in phase synchronization will occur. Huijgens realized the importance of his discovery as this would allow ships sailing at the ocean to obtain a second “position” measurement, besides the relative position of the sun, so that the ship can determine its position at the ocean.

Today, several experiments mimicking the Huijgens' experiment have been reported, see e.g. [Panteleone, 2002; Benett *et. al.*, 2002], all aiming at demonstrating that in-phase or anti-phase synchronization of a number of pendulum-like oscillators can be achieved. In-phase synchronization in a Huygens-type setup was explained in [Blekhman, 1998] by means of a model of coupled Van der Pol equations.

At this point it is worth noting that many of the reported toy-experiments of Huijgens' synchronization are a simplification of the work done by Huijgens! Mathematically, the experiment as reported by Huijgens can be cast into a set of differential equations. Namely, for each of the pendulum clocks a simple oscillator equation suffices – at least under the assumption that the so-called escapement mechanism that keeps the clocks running, can be modeled in a simple manner – and in addition the beam to which the clocks are attached needs to be modeled by a partial differential equation with suitable boundary conditions. Thus, the set-up consisting of two pendula linked to a free hanging beam is automatically an infinite dimensional system, for which a rigorous study of the in-phase or anti-phase synchronization of the two pendula is, as far as is known, still never addressed in the literature. Such a study, albeit dealing with the (partial) stability of an infinite dimensional system, typically is expected to involve a study of the energy of the overall system, and thus some kind of Lyapunov (or infinite dimensional extensions like the Lyapunov-Krasovskii functional

approach) seems in order. So far, most of the studies dealing with the Huijgens' synchronization problem treat a finite dimensional approximation, e.g. in an earlier attempt [Oud *et.al.*, 2006] we proposed a 3 degrees of freedom approximation of the Huijgens' set-up, see Figure 1.2, which to some extent does capture the effect that energy from one pendulum can be transferred to the other via the swinging platform.

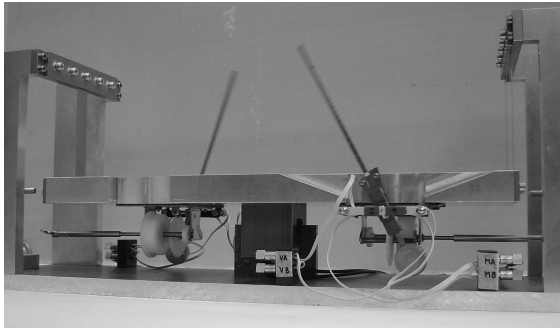


Fig. 1.2 A setup with two metronomes.

In the recent paper [Dilao, 2009] a slightly different approach is given to arrive at a finite dimensional (4 dof) model mimicking partly the Huijgens' experiment. Again, in this case a fairly complete analysis on in phase and anti-phase synchronization of the oscillators is possible.

Numerical results related to the 'true' infinite dimensional Huijgens' problem illustrating the in phase and anti-phase synchronization may form a next step, compare this to [Czolczynski *et. al.*, 2007]. Thus, besides several experimental illustrations and simulation studies, a rigorous analysis regarding the synchronization of pendula is still pending. For a thorough understanding of Huijgens' synchronization one can essentially distinguish two challenges. On the one hand, this is the issue of determining all the possible "stationary" solutions (like the in-phase and anti-phase pendulum clocks) and secondly, a stability-study regarding these "stationary" solutions. However, the old Huijgens' synchronization problem, and in particular its analysis is even today a very timely research theme. It centers about advanced nonlinear dynamics, hybrid dynamics – given the impulsive nature of the escapement mechanism – and in particular regarding the synchronization of multiple clocks, energy-based control techniques come into play. The contributions in this book largely fit into this broad arena of exciting dynamical systems. On one hand, there are several contributions

that aim at a complete stability analysis for the error dynamics between various kinds of oscillators, coupled through a relatively simple proportional error term. In many cases, energy and Lyapunov-like arguments are developed towards a successful stability theory.

On the other hand, several authors have contributed towards the study of all potential limiting or stationary solutions. Despite the developed results in this book, a complete and rigorous proof of the in-phase respectively anti-phase synchronization such as observed and described by Huygens more than three centuries ago is still missing. However, it is strongly felt that the contributions given here may help in a further understanding of synchronization of pendulum clocks and more generally other oscillators. It is clear that for a successful analysis for such situations besides the particular type of oscillators also the selected type of coupling is of key importance. It is our belief that the results given here provide important and illuminating insight in the old and intriguing synchronization problem. In addition, it may look artificial in the beginning, but the actual "sympathy" between pendulum clocks inherently contains both in experiments as well in analysis typical hybrid aspects that tremendously complicate the problem. In that sense, Dynamics and Control of Hybrid Mechanical Systems, remains a very exciting and challenging field of research.

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