

ANDERSON LOCALIZATION OF MATTER-WAVES IN A CONTROLLED DISORDER: A QUANTUM SIMULATOR?

ALAIN ASPECT

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We have observed exponentially localized wave function of ultracold atoms released into a one-dimensional waveguide in the presence of a controlled disorder created by laser speckle. We present this result, and elaborate on the significance of 1D Anderson localization, and on the prospects of extending that type of study to quantum gases in higher dimensions (2D and 3D) and with controlled interactions. We will also point out its relevance to the rapidly evolving domain of quantum simulators to study difficult problems of Condensed Matter.

1. Anderson localization: The naïve view of an AMO experimentalist

Fifty years ago, P W Anderson published a paper providing a revolutionary suggestion to interpret the sudden transition of certain materials from conductor to insulator, under a slight change of the amount of disorder in the material.¹ At that time, every physicist would understand (for instance based on the Drude model of a metal) that increasing the amount of disorder in a conducting material increases the hindrance to the displacement of electrons and thus decreases the conductivity, but no one had predicted a total cancellation of the conductivity, i.e. a transition from a conductor to an insulator, past a certain amount of disorder. The core of the prediction of this phase transition was a fundamental quantum phenomenon, interference between the many quantum amplitudes associated with various trajectories of the electron in a crystal with a certain disorder. Actually, the result was essentially based on a mathematical argument, and it took quite some time for a simple picture to emerge. It is interesting to quote what PW Anderson wrote when he received the Nobel Prize in physics in 1977, almost 20 years after his seminal paper. After noting that his first cited work (the theory of local magnetic moments) “had rapid and

permanent acceptance because of its timeliness and its relative simplicity”, he continued about what had become “Anderson Localization” (AL): “*Localization was a different matter: very few believed it at the time, and even fewer saw its importance; among those who failed to fully understand it at first was certainly its author. It has yet to receive adequate mathematical treatment, and one has to resort to the indignity of numerical simulations to settle even the simplest questions about it. Only now, and through primarily Sir Nevill Mott’s efforts, is it beginning to gain general acceptance.*” More than thirty years after this strong statement, and fifty years after the initial paper, it is surprising that there are still many open questions, even among the simplest. And it is remarkable that AMO physicists have been able to translate the Condensed Matter situation considered by Anderson, into AMO situations, which can be investigated experimentally.²⁻⁴

Anderson initial model starts from the so called “tight binding model” of an electron in a crystal⁵, where one considers a periodic lattice of trapping sites for the electron (Figure 1a). Actually, the electron can hop from one site to the nearest ones by quantum tunneling. For large enough tunneling amplitudes, the stationary solution of the quantum problem is a so called “extended” wave function, describing an electron that can freely propagate over the crystal. This free propagation of an electron in a perfect crystal can be alternatively described by a Bloch wave in a conduction band.⁵

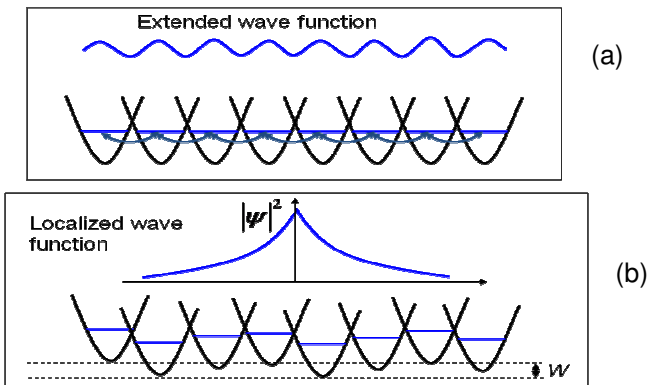


Figure 1. Anderson localization in the tight binding model. Starting from localized wave functions on each potential well of a regular lattice, and adding tunneling between neighbor sites, one obtain a solution of the Schrödinger equation which is an extended state, ie a freely propagating wave. If one adds disorder to the lattice by changing in a random way the depth of the various wells, the solution may become localized, ie a wave function with an exponentially decaying profile in all three dimensions of space (the figure is 1D for clarity). This happens when the amplitude W of the disorder is large enough.

If now one adds disorder to the tight binding model by assuming random values of the depths of the various trapping sites (Figure 1b), with a distribution of width W , propagation is hindered, and this corresponds to Ohm conduction with a conductivity decreasing when the amplitude W of the disorder increases. Anderson's conjecture, based on a general mathematical argument, is the existence a threshold in the level of disorder, beyond which the conductivity is more than reduced, it is totally cancelled, and the material becomes a perfect insulator. This breakdown of conduction is associated with a sudden change in the electronic wave function, which ceases to be extended and becomes spatially localized, with exponentially decreasing tails: there is no way for the electrons to get out from their localized states and propagate in the crystal. The surprise is that this happens even if the electron has still the possibility to tunnel between neighbor sites of the lattice: when one considers the various possible paths to go from one site to another one, the various quantum amplitudes associated with all these paths cancel when they are added. It must be emphasized that this is a quantum interference effect, involving a single quantum particle, for which one must add the various amplitudes associated to the various possibilities to go from one place to another.

After this initial conjecture based on the tight binding model of hopping electrons, an equivalent model based on wave physics emerged, to understand the existence of a conductor/insulator transition for a certain level of disorder.⁶ The starting point of this alternative approach is the equivalence⁷ between the running Bloch wave describing a particle freely propagating in a conduction band of a perfect crystal, and a matter wave freely propagating in a homogeneous non absorbing medium. Adding disorder to the latter model can be done by introducing impurities, on which the matter waves scatter (Figure 2). This leads to a diffusive propagation, an alternative model of Ohm conduction, based on the assumption that diffusion is incoherent and that one can add the intensities of the wavelets scattered on impurities. In contrast, when one adds the amplitudes of the scattered waves, as it should be done for coherent waves, one finds Anderson Localization, resulting from a destructive interference between wavelets multiply scattered onto impurities and pointing to the same final direction of propagation, in the forward direction (Fig. 2) or towards any other direction. For this to happen, an intuitive condition is that the memory of the initial direction is rapidly lost, i.e. that the mean free path ℓ between two scattering events is smaller than the wavelength λ of the wave, viz. that the dephasing between two scatterings remain small. Introducing the wave vector $k = 2\pi/\lambda$, this condition can be written as $k\ell < \text{constant} \sim 1$, which is known as the "Ioffe-Regel criterion". This condition, introduced in the context of Anderson localization by Mott,⁷ stresses the existence of a threshold – a so

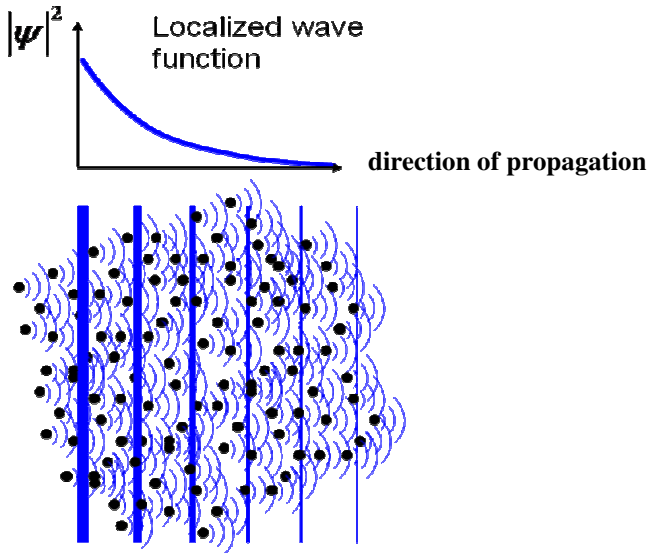


Figure 2. Exponential localization of waves in a medium with random scatterers. When a plane wave tries to propagate in a medium with many scatterers, the various rescattered wavelets interfere in the forward direction and add coherently to the “initial” wave. In 3D, if the mean free path ℓ between two scatterings is less than the wavelength, the forward interference is destructive, and the amplitude of the wave decreases exponentially along the propagation direction. This happens whatever the considered direction of propagation, and the stationary solution of the problem is an exponentially decaying profile in all three dimensions of space (the figure is drawn in 2D for clarity).

called “mobility edge” – at which the transition from “localized” to “extended” happens.

More than thirty years after Anderson Nobel lecture, and in spite of its wide acceptance, the exact status of the Ioffe-Regel criterion is still not fully clear. Firstly, rigorous derivations are based on the scaling theory developed by the famous “gang of four”,⁸ but they do not yield the value of the constant in the Ioffe-Regel condition. Secondly, the situations in dimensions lower than 3 have many intriguing features.⁸ For instance there is a general theorem that “in 1D all states are localized”, so that there seems to be a priori, no mobility edge to separate a regime of localized state from a regime of extended states, but as we will see below the situation may be more subtle. Dimension 2 is a marginal case, and as such very interesting from a theoretical point of view⁸, and with several open issues. In order to shed light on these issues, experimental observation of Anderson localization has been looked for in various domains of physics, in condensed matter but also in wave physics and in particular with light waves,⁹⁻¹² or microwaves.^{13,14}

2. Ultra-cold atoms in optical speckle: A good candidate to observe AL

Anderson was thinking of electrons in his original paper, but a *direct* observation of Anderson localization for electrons is very difficult. Firstly, it is hindered by a number of phenomena that can mask the single particle quantum effects genuinely induced by disorder: electrons interact with each other by Coulomb electric repulsion; they are affected by the vibrations of the lattice, which is a kind of a disorder that is not static (“quenched” in the usual jargon) and therefore cannot produce AL. Secondly, direct observation of electron wave functions in solids is very difficult, and most of the evidences are indirect and stem from conductivity measurements.

In contrast, ultra-cold atoms allow one to address the core of the phenomenon that Anderson had discovered, since they are genuine quantum particles described as matter waves, and interactions can be reduced at a negligible level so that one can study single particle behaviors. Moreover, with ultra-cold atoms, single atom matter waves can be directly visualized by absorption or fluorescence imaging of the atomic density of a dilute, non interacting, Bose Einstein Condensate. Indeed, such an ideal quantum gas is nothing else than many atoms in the same one atom wave function, which makes possible the direct observation of that wave function. It then becomes possible to realize experimentally and investigate an AL situation,¹⁵ provided that one can generate the adequate disordered potential where to place the ultracold atoms^{16,17}. This is possible by using a disordered light pattern, since atoms can be submitted to the dipole radiative force, deriving from a potential directly proportional to the light intensity. If the detuning between the light and the atomic resonance is large enough, fluorescence is totally negligible, and the atoms are submitted to a pure potential and experience no dissipation phenomenon. This is in contrast to experiments with light propagating in media with a high level of scattering, where the possibility of having ordinary absorption or diffusion must be considered, and sophisticated tests must be performed to ascertain that an observed exponential decay is due to AL.¹⁸

A disordered potential created by laser speckle has remarkable properties. The optical speckle field is produced by passing a laser beam through a diffusing plate, which imposes a phase depending randomly on the position. As a result, the complex electric field describing the light at any point in the far field can be considered a sum of many independent random variables, and it is thus a Gaussian random process¹⁹. Note that the light intensity, which is the squared modulus of the complex electric field, is not a Gaussian process. For instance, its probability distribution is a decaying exponential, i.e. a non symmetric distribution. Nevertheless it inherits many properties of the underlying Gaussian process, and in particular all the intensity-intensity

correlation functions, which are higher order correlation functions of the complex electric field, can be expressed as the second order correlation function of the electric field, which in turn is determined by diffraction from the diffusive plate onto the atom location. For instance, in the configuration of Figure 4, which we have used to study 1D AL, the scattering plate is elongated along the z -axis, and narrow along the x -axis, and this results in a speckle pattern, whose 3D spatial autocorrelation function is narrow along z and elongated along x and y . More precisely, in the experiment of,² the autocorrelation function of the disordered potential has a half-length of $97 \mu\text{m}$ along x , $10 \mu\text{m}$ along y , and $0.82 \mu\text{m}$ along z . For such a rectangular aperture, the autocorrelation along x is $V_R^2 [\sin(\Delta z / \sigma_R) / (\Delta z / \sigma_R)]^2$, where the correlation length of $\sigma_R = 0.26 \pm 0.03 \mu\text{m}$ corresponds to a half width $\pi\sigma_R = 0.82 \mu\text{m}$. Such a rapidly varying speckle is obtained with large numerical aperture optics. Because the speckle is produced by diffraction, its autocorrelation function has a Fourier transform, with a finite support the size of which is related to the numerical aperture of the optics. In other words, the random process describing the disordered potential has a power spectrum limited by a maximum spatial frequency. In the case of a rectangular aperture discussed here, the disordered potential has no component with a spatial frequency larger than $2/\sigma_R$.

The amplitude V_R of the disorder is directly proportional to the laser intensity, and the calibration factor is calculated knowing the geometry of the optical system and the constants of ^{87}Rb atom.

These properties of the disordered potential created by laser speckle have been carefully checked using the atoms themselves as a local probe of the potential.¹⁶

3. One dimensional AL?

In the experiment sketched on Figure 4, the atoms are guided along z by a strong and narrow laser beam, acting as a matter wave guide with a typical diameter of a few microns only. They are transversely confined, and cold enough to be in the lowest state of the transverse trapping potential. On the other hand, they can freely move along z . When the anisotropic laser speckle described above is applied, the guided atoms thus experience a transversely invariant but longitudinally (along z) disordered potential that will affect their motion along z . We are then in a situation allowing us to study Anderson localization in one dimension (1D AL).

But what is the point of studying 1D AL? According to the scaling theory of localization,⁸ there is always localization in 1D. This means that whatever the (non trivial) potential, for each energy E of a particle of mass M (parameterized by a quantity with dimension of a wave vector, $k = (2ME)^{1/2}/\hbar$) there is a

solution of the Schrödinger equation which is a localized state. At first sight, there is thus no point in addressing the question of 1D Anderson localization. However, when we considered the question of ultra cold atoms placed in a 1D optical speckle disorder, we found a yet unexplored approach to it, which allowed us to point out several fundamental features of AL in such a situation²⁰, and to explore them experimentally, as we explain now.

Many theoretical studies of AL of waves are based on a model of scattering impurities described as randomly positioned Dirac peaks, i.e. infinitely high and infinitely narrow potential peaks, a so-called uncorrelated disorder. In contrast, a laser speckle potential, as drawn on Figure 3, is made of randomly positioned peaks of *finite height* and *finite width*, a very different situation (correlated disorder). Firstly the peak heights distribution decays exponentially, which means that it is exponentially improbable to find peaks with a height many times the average value \bar{V} of the potential, and in a finite size sample there is a maximum value V_{\max} of the potential. Secondly, the random potential has no spatial variation more rapid than the typical size σ_R of a speckle grain (or more precisely no Fourier component with a spatial frequency larger than $2/\sigma_R$). These two properties entail dramatic consequences.

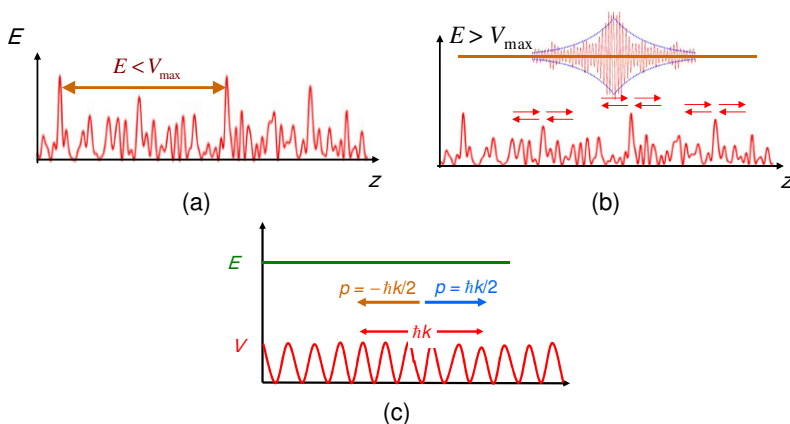


Figure 3. Localization of a particle in a 1D laser speckle disordered potential. a) For an energy smaller than the highest peaks of the potential, localization results from bouncing between two peaks, a classical effect. b) If the energy is larger than the highest peak, one can find as a solution of the Schrödinger equation an exponentially localized wave function, resulting from the addition of many wavelets due to scattering on the peaks. This is a fully quantum localization phenomenon, equivalent to Anderson localization. c) If one considers a periodic potential of spatial frequency k , a particle with momentum $p = \hbar k/2$ cannot propagate and is exponentially localized because it is Bragg reflected, as a result of the many wavelets scattered from the periodic structure. Localization of Figure b can then be interpreted as a Bragg diffraction of each momentum component on the corresponding periodic component of the disordered potential.

Let us consider the first property that in a finite size sample L there will be no peak with a height larger than a maximum value V_{\max} of the potential. We can then distinguish two regimes. If we take a particle with an energy E significantly below V_{\max} , it is very likely to find two peaks much larger than the particle energy in a finite size sample, and (because of the negligible tunneling through these peaks), there is a trivially localized state, corresponding to the classical trapping of a particle bouncing between two potential barriers (Figure 3a).²¹ But what happens if the particle has an energy E larger than V_{\max} , so that a classical particle would propagate from one end of the sample to the other, without any blocking? A numerical solving of the Schrödinger equation in such a *weak disorder* shows that one can have an exponentially localized wave function (Figure 3b). One can interpret this localization as resulting from the interference between the many wavelets scattered at the various peaks of the speckle potential, as sketched on Figure 3b. In contrast to the trivial case of a particle trapped between two peaks, it is a pure quantum effect of the kind considered by Anderson. To understand it intuitively, we can use a Born approximation approach, and think about the disordered potential as composed of many harmonic Fourier components, each characterized by a spatial frequency k . A particle with momentum $p = \pm \hbar k/2$ cannot propagate in such a periodic potential, because it would be backscattered due to Bragg reflection (Figure 3c). Again this is a fully quantum effect, resulting from the interference of the many matterwave wavelets scattered from the periodic structure, whose phases differ by 2π , for the wavelets scattered by two points separated by one period $2\pi/k$. But there is more. When we take into account the second property of a speckle disordered potential (no Fourier component with a spatial frequency larger than $2/\sigma_R$), we conclude that there is a cut off value $k_{co} = 1/\sigma_R$ such that a matter wave with a momentum larger than $\hbar k_{co}$ will not localize. We have thus an *effective mobility edge*, separating the regime of Anderson localization from the regime of free propagation.

To conclude this section, in a weak 1D disordered potential produced by a laser speckle, a first order perturbative approach allowed us to predict a localization phenomenon which has the characteristic features of AL: (i) it happens in a regime where there is no classical localization, and it is due to interference between the many wavelets scattered on the disordered potential; (ii) there is an effective mobility edge separating a regime of localization (localized states) from a regime where the atom can propagate freely over the whole sample of finite size (extended state).²⁰ It was then tempting to investigate experimentally such a situation.

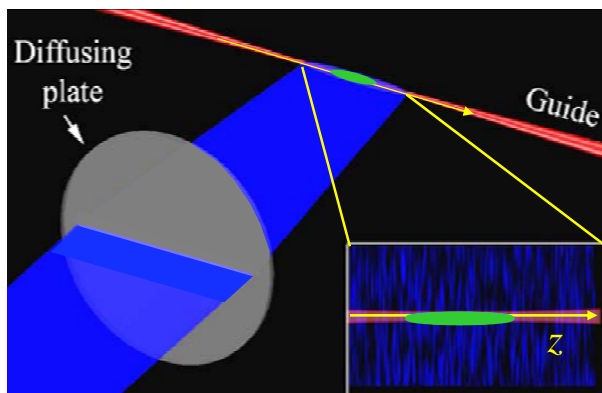


Figure 4. Ultra-cold atoms in a one dimensional speckle potential. The atoms are held by a matter wave guide that confines them transversely to the axis z , but let them travel freely along z . A laser beam passing through an aperture in a diffusing plate elongated along z , creates a laser speckle, ie a disordered intensity pattern, varying fast along z , and smoothly perpendicular to z . This realizes a situation where particles are submitted to a 1D disordered potential along z .

4. Direct observation of Anderson localized 1D wave functions²

Our experiment (sketched in Figure 4), starts with a small elongated BEC (1.7×10^4 atoms of ^{87}Rb), trapped in an anisotropic opto-magnetic hybrid trap,²² with trapping frequencies $\omega_z/2\pi = 5.4$ Hz and $\omega_{x,y}/2\pi = 70$ Hz. The transverse and longitudinal Thomas Fermi radii are $3 \mu\text{m}$ and $35 \mu\text{m}$ respectively, and the chemical potential is $\mu_{\text{ini}}/h = 219$ Hz, where h is the Planck constant). The transverse trapping is provided by a far detuned laser beam (wavelength $1.06 \mu\text{m}$, to be compared to the resonant wavelength of Rb, $0.78 \mu\text{m}$), which creates an optical waveguide along the horizontal z -axis. A shallow magnetic trap confines the BEC in the longitudinal direction. It is suddenly switched off at $t = 0$, and the BEC starts expanding along z in the waveguide, under the effect of the initial repulsive interaction energy associated with the chemical potential μ_{ini} . A weakly expelling magnetic field compensates the residual longitudinal trapping of the optical waveguide, so that the atoms can freely expand along z over several millimeters. The expanding BEC can be imaged at any chosen time t after release by suddenly switching off the optical guide and irradiating the atoms with a resonant probe of duration $50 \mu\text{s}$. An ultra sensitive EMCCD camera allows us to make an image of the fluorescing atoms with a resolution of $15 \mu\text{m}$ and a 1D atomic density sensitivity close to $1 \text{ atom} / \mu\text{m}$. We can then follow the ballistic expansion of the atoms, and check by looking at the propagation of the fore front of the successive density profiles,

that the largest k vector in the expanding atoms corresponds to the initial chemical potential, i.e. $k_{\max} = (4m\mu_{\text{ini}})^{1/2}/\hbar$, with m the atomic mass.

When we switch off the longitudinal trapping in the presence of weak disorder, the BEC starts expanding, but the expansion rapidly stops, in stark contrast with the free expansion case. A plot of the density profile, in linear and semi logarithmic coordinates (Fig. 5c-d), then shows exponential wings, a clear signature of Anderson localization. This observation was done in a regime allowing AL. Firstly, the disorder is weak enough ($V_R/\mu_{\text{in}} = 0.12$) that the initial interaction energy per atom is rapidly converted into a kinetic energy of the

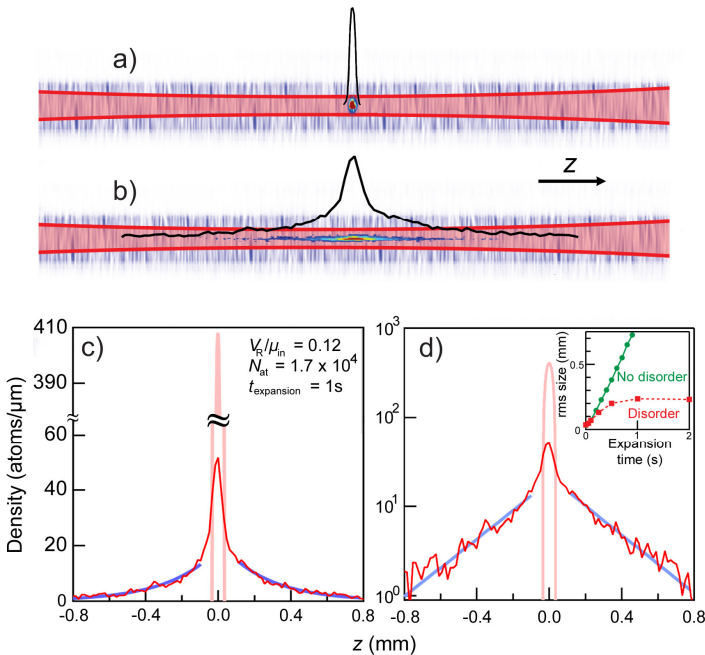


Figure 5. Observation of exponential localization. (a) A small BEC (1.7×10^4 atoms) is formed in a hybrid trap, which is the combination of a horizontal optical waveguide ensuring a strong transverse confinement, and a loose magnetic longitudinal trap. A weak disordered optical potential, transversely invariant over the atomic cloud, is superimposed (disorder amplitude V_R small compared to the chemical potential μ_{in} of the atoms in the initial BEC). (b) When the longitudinal trap is switched off, the BEC starts expanding and then localises, as observed by direct imaging of the fluorescence of the atoms irradiated by a resonant probe. On a and b, images and sketched profiles are for illustration purpose, they are not on scale. (c-d) Observed density profile of the localised BEC, 1s after release, in linear or semi-logarithmic coordinates. The inset of Fig (d) (rms width of the profile vs time, with or without disordered potential) shows that the stationary regime is reached after 0.5 s. Solid lines in Fig (c) are exponential fits to the wings, corresponding to the straight lines of Fig (d). The narrow profile drawn at the centre represents the trapped condensate before release.

order of μ_{in} for atoms in the wings, a value much larger than the amplitude of the disordered potential so that there is no possibility of a classical reflection on a potential barrier, as in figure 3a. Secondly, the atomic density in the wings of the profiles is small enough (two orders of magnitude less than in the initial BEC) that the interaction energy is negligible compared to the atom kinetic energy, and we can consider that we have many non-interacting atoms described by the same wavefunction. Lastly, the atomic matter-wave k vector distribution is bounded, with a maximum value k_{max} smaller than half the cut-off in the spectrum of the speckle disordered potential used here, i.e. $k_{\text{max}} \sigma_{\text{R}} < 1$. Indeed, for the observations of Figures 5 and 6, we have $k_{\text{max}} \sigma_{\text{R}} = 0.65 \pm 0.09$, and all the k vector components are below the effective mobility edge.

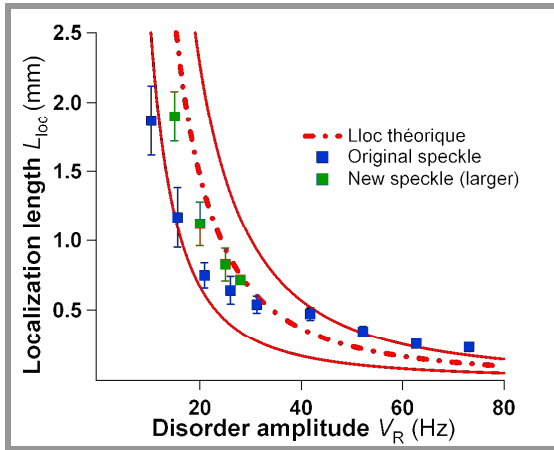


Figure 6. Localization length vs amplitude of the disordered potential. The measured localization length L_{loc} is obtained by an exponential fit to the wings of the stationary localized density profiles, as shown in Fig. 5c-d. The error bars correspond to a confidence level of 95% of the fit. The number of atoms is $N_{\text{at}} = 1.7 \times 10^4$ ($\mu_{\text{in}} / h = 219$ Hz). The dash-dotted line represents formula (1), where k_{max} is determined from the observed free expansion of the condensate. The two solid lines are the limit of the uncertainty in applying equation (1), associated with the evaluation of k_{max} and the evaluation of σ_{R} . The two sets of data correspond to two different diffusive plates that produce speckle pattern with different homogeneities. When the homogeneous region is broader, the measured values get closer to the dash-dotted line in the case of a large localization length.

An exponential fit to the wings of the density profiles yields the localization length L_{loc} , which we can compare to the theoretical value²⁰

$$L_{\text{loc}} = \frac{2\hbar^4 k_{\text{max}}^2}{\pi m^2 V_{\text{R}}^2 \sigma_{\text{R}} (1 - k_{\text{max}} \sigma_{\text{R}})} \quad (1)$$

valid only for $k_{\max} \sigma_R < 1$. In Fig. 6, we plot the variation of L_{loc} with the amplitude of the disorder, V_R , for the same number of atoms, *i.e.* the same k_{\max} . The dash-dotted line is a plot of Equation (1) for the values of k_{\max} and σ_R determined as explained above. It shows a good agreement between our measurements and the theoretical predictions. The shaded area reflects the variations of the dash-dotted line when we take into account the uncertainties on σ_R and k_{\max} . The uncertainty in the calibration of V_R does not appear in Fig. 3. We estimate it to be not larger than 30%, which does not affect the agreement between theory and experiment.

An intriguing result of ref [20] is the prediction of density profiles with algebraic wings when $k_{\max} \sigma_R > 1$, *i.e.* when the initial interaction energy is large enough that a fraction of the atoms have a k -vector larger than the effective mobility edge $1/\sigma_R$. This predicted behaviour results from the sum of the many exponentially localized wave functions with different localization lengths, up to the effective mobility edge where the localization length is infinite, while atoms associated with a k -vector beyond $1/\sigma_R$ eventually escape. We have investigated that regime by repeating the experiment with a BEC containing a larger number of atoms (1.7×10^5 atoms), *i.e.* a larger initial interaction energy ($\mu_{\text{in}}/\hbar = 519$ Hz), so that a fraction of the atoms have a momentum larger than the effective mobility edge ($k_{\max} \sigma_R = 1.16 \pm 0.14$). Note however that we are still in the weak disorder regime ($V_R/\mu_{\text{in}} = 0.15$). Figure 7a shows a log-log plot of the observed density profile in such a situation, suggesting a power law decrease in the wings, with an exponent of 1.95 ± 0.10 , in agreement with the

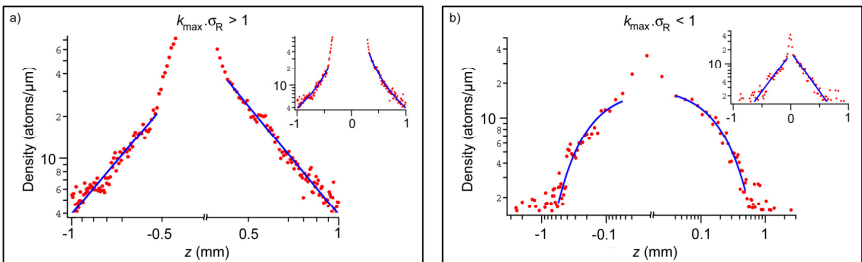


Figure 7. Algebraic vs exponential regimes in a 1D speckle potential. Log-log and semi-log plots of the stationary atom density profiles showing the difference between the algebraic ($k_{\max} \sigma_R > 1$) and the exponential ($k_{\max} \sigma_R < 1$) regimes. a) Density profile for $V_R/\mu_{\text{in}} = 0.15$ and $k_{\max} \sigma_R = 1.16 \pm 0.14$. The momentum distribution of the released BEC has components beyond the effective mobility edge $1/\sigma_R$. The fit to the wings with a power law decay $1/z^\beta$ yields $\beta = 1.92 \pm 0.06$ for the left wing and $\beta = 2.01 \pm 0.03$ for the right wing. The inset shows the same data in semi-log plot, and confirms the non-exponential decay. b) For comparison, similar set of plots (log-log and semi-log) in the exponential regime with the same $V_R/\mu_{\text{in}} = 0.15$ and $k_{\max} \sigma_R = 0.65 \pm 0.09$.

theoretical prediction of wings decreasing as $1/z^2$. The semi-log plot in inset confirms that an exponential would not work as well. To allow comparison, we present in Figure 7b a log-log plot and a semi-log plot for the case $k_{\max} \sigma_R = 0.65$ with the same $V_R/\mu_{\text{in}} = 0.15$, where we unambiguously conclude in favor of exponential rather than algebraic tails. These data support the existence of a cross-over from exponential to algebraic regime for a BEC in a speckle potential.

5. Beyond the effective mobility edge

The analysis leading to the existence of an effective mobility edge is based on the (lowest order) Born approximation, and one may wonder what a more exact approach predicts beyond the effective mobility edge. In the case of a speckle disorder, a perturbative expansion to higher orders is possible since high order correlators of the potential can easily be calculated. Indeed the momentum theorem, which is valid for the Gaussian random process describing the light electric field whose squared modulus yields the potential, allows one to express any higher order moment as a function of the second order moment, which is easily calculated by the diffraction theory. A perturbative calculation up to order 4, confirmed by a diagrammatic approach and numerical calculations, yields interesting results,²³ shown on figure 8 where one plots the inverse localization length vs. the disorder amplitude V_R and the k -vector associated to the energy of the solution of the Schrödinger equation (see section 3). Firstly, one sees that beyond the effective mobility edge at $1/\sigma_R$, the inverse localization length is not strictly zero, in agreement with the general result of the scaling theory for a one dimensional problem. Note however that there is a crossover at $k\sigma_R = 1$, where the localization length suddenly increases by several orders of magnitudes, so that in a system with a finite size smaller than L_{loc} there is no localization observable. The sharpness of that crossover increases when the amplitude of the disorder decreases, as expected for an expansion where ϵ_R is the small parameter. The figure also shows that there is a second effective mobility edge at $k\sigma_R = 2$. It can readily be interpreted by invoking second order Bragg diffractions, corresponding to a phase difference of 4π between successive backscattering events on a periodic component of the disorder.

Another interesting result of the calculation at the fourth order (in the potential) is that there is a difference between the calculated localization lengths calculated for a repulsive or attractive speckle potential. This difference is due to the third order term, which is odd in V_R .

The result can be generalized to higher orders, and one predicts the existence of successive effective mobility edges at $k\sigma_R = 3, 4, \text{etc.}$

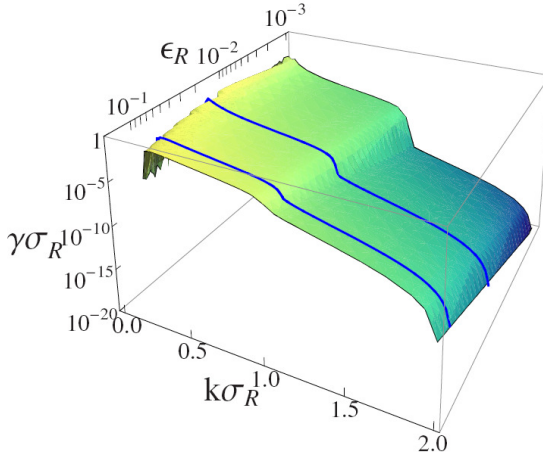


Figure 8. Inverse localization length L_{loc} (Lyapunov exponent) calculated two orders beyond the Born approximation for particles in 1D speckle potentials. The speckle (with a correlation length σ_R) is created with a square diffusive plate. The particle momentum $\hbar k$ and the strength of disorder are indicated in reduced units ($\epsilon_R = 2m\sigma_R^2 V_R / \hbar^2$) where V_R is the amplitude of the disorder). The solid lines correspond to $\epsilon_R = 0.1$ and $\epsilon_R = 0.02$.

6. Future prospects: AL and beyond

Direct imaging of atomic quantum gases in controlled optical disordered potentials is a promising technique to investigate a variety of open questions on disordered quantum systems. As in other problems of condensed matter simulated with ultra-cold atoms,^{24,25} it offers unprecedented possibilities to measure important properties, such as localization lengths.

The good quantitative agreement between our measurements and the theory of 1D Anderson localization in a speckle potential demonstrates the high degree of control in our set-up. We thus anticipate that it can be used as a quantum simulator for investigating Anderson localization in higher dimensions,^{26,27} first to look for the mobility edge of the Anderson transition, and then to measure important features of the Anderson transition that are not known theoretically, such as critical exponents. It will also become possible to investigate the effect of controlled interactions on Anderson localization, as well as other predicted effects such as localization of quasi-particles,^{28,29} Bose glass³⁰ and Lifshits glass³¹, and to study Fermi gases and Bose-Fermi mixtures where rich phase diagrams have been predicted.³²

Acknowledgments

The authors are indebted to Pierre Chavel, Thierry Giamarchi, Maciej Lewenstein and Gora Shlyapnikov for many fruitful discussions, to P. Georges and G. Roger for assistance with the laser and to Frédéric Moron, André Villing and Gilles Colas for invaluable technical assistance on the experimental apparatus. This research was supported by the French Centre National de la Recherche Scientifique (CNRS), Délégation Générale de l'Armement (DGA), Ministère de l'Education Nationale, de la Recherche et de la Technologie (MENRT), IXSEA, Agence Nationale de la Recherche (ANR), and Institut Francilien de Recherche sur les Atomes Froids (IFRAF); by the programme FINAQS of the European Union and by the programme QUEDDIS of the European Science Foundation (ESF).

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