Preface

In applied mechanics, and in other physical and engineering sciences, the first task for examining a problem theoretically is to formulate the governing equations accurate enough to describe the dominant physical processes, and the second is to find effective means of solution. For both tasks, understanding the scales of physical relevance and utilizing them for expediting the mathematics are essential. In many physical problems several scales can be present either in space or in time, caused by either inhomogeneity of the medium, or complexity of the mechanical process. Depending on the objectives, one may focus on the process in a particular range of scales. For example, in the study of gas dynamics, if we are primarily concerned with the physics over a length scale much greater than the distances separating the particles or the molecules, detailed interactions of the particles are often overlooked and the medium is regarded as a continuum. Conservation laws of mass, energy, and momentum are derived by extending the basic laws of particle mechanics. Constitutive relations among macroscale variables are added on the basis of some basic experiments. Examples of these relations are those between heat flux rate and the temperature gradient, between stresses and strains in a solid, and between stresses and strain rates in a fluid, etc. However, a more fundamental approach is to first construct microscale models, and then to deduce the macroscale laws and the constitutive relations by properly averaging over the microscale. Thus in the kinetic theory of gases, models of colliding molecules are constructed to yield the hydrodynamical equations for the continuum as well as theoretical formulas of viscosity and thermal diffusivity, etc.

For many multiphase media such as fluid-saturated porous solids (soil, bones, or tissues), or of laminated or fiber-reinforced elastic solids, the primary interest is often restricted to the large-scale behavior of the composite. Theoretical derivation of the macroscale equations based on microscale considerations and prediction of the constitutive coefficients have been a challenge in both computational mathematics and theoretical mechanics. Under certain conditions, analytical theories have been advanced for physical problems governed by differential equations. These latter theories have the advantage of enhancing physical understanding and reducing the computational labor. In particular, for materials or processes with a periodic microstructure, the perturbation method of multiple scales can be used to derive averaged equations for a much larger scale from considerations of the small scales. In the mechanics of multiscaled media, the analytical scheme of upscaling is known as the *Theory of Homogenization*. Closely parallel methods have also been developed in the vast literature of wave propagation, where averages over the nearly periodic short waves are taken and the main attention is shifted to the long-scale evolution of wave envelopes. In wave theories, the technique is variably known as the envelope theory, or coupled-mode theory, etc.

Started by mathematicians in the late 1970s, the literature of homogenization theory for inhomogeneous media has mushroomed in recent decades. Attempts to derive or rederive the macroscale equations have been applied to numerous fields of applications. Several monographs have also appeared. While the key tool of the theory is the perturbation method of multiple scales now widely used in a variety of problems in applied analysis, existing treatises are often presented in abstract mathematical language which is not easily accessible to many students and researchers.

The present authors share the view that the general methods of homogenization deserve to be more widely understood and practiced by applied scientists and engineers. Hence this book is aimed at providing a less abstract treatment of the basic theory and its applications to a diverse range of problems involving inhomogeneity originated from either material structures, motion, or boundary geometry. Each chapter deals with a different class of physical problems. We hope the content can be useful not only to newcomers wishing to learn the essence of the method, but also to specialists of one field who may wish to extend their expertise to other fields. To tackle a new problem, we adopt the approach of first discussing the physically relevant scales, then identifying the small parameters and their roles in the normalized governing equations. The details of asymptotic analysis are explained only afterward. Whenever possible we shall include known quantitative results of the constitutive coefficients, which can be obtained analytically only in a few cases, and must in general be obtained by solving numerically the so-called cell problems. Since these numerical tasks are often not trivial, we discuss in Chapter 2 the mathematical alternative of variational bounds for the relatively simple problem of heat conduction, and illustrate how to estimate the range of possible values of the constitutive coefficients. Applications are then extended to seepage in porous media in Chapter 3, where the empirical law of Darcy is derived theoretically. Extensions to three-scale seepage flow are discussed and the nonlinear effects of convection are sketched. Relevant to environmental applications is the topic of shear-enhanced diffusion, i.e., dispersion. Two examples are analyzed in Chapter 4: one for a spatially periodic porous medium and the other for wave motion periodic in both space and time. Multiscale problems have a long history in elastic composites which are of great importance to construction and manufacturing industries. Comprehensive treatises describing various approximations and computational schemes are already available. In Chapter 5 we discuss the use of homogenization theory to two examples in elastic composites. A fairly detailed account is again given to the prediction of variational bounds of the constitutive coefficients defined by homogenization theory. The mechanics of poro-elasticity is of basic interest in soil mechanics, geophysics, and biomechanics. The coupling between pore fluid and the solid matrix is often treated as two interacting continua. We discuss in Chapter 6 the derivation of Terzaghi–Biot theory from micromechanical considerations. In the final chapter we illustrate how the method of homogenization can be employed in wave dynamics in an inhomogeneous environment. In particular, we shall also show that the asymptotic theory can be straightforwardly extended to random media where the random fluctuations are weaker than the stochastic average. A few examples involving linear or weakly nonlinear properties will be treated to predict the mean field. On the whole, our primary emphasis is on analytical and approximate derivations of macroscale equations. Less effort is devoted to their solutions and the implied physical significance. Details of numerical computations are left to the cited references.

In most of this book the mathematical prerequisite is kept at the level comfortable for graduate students in theoretical engineering sciences. Some prior exposure to perturbation methods is helpful but not necessary, since the details are explained from ground zero. The parts on variational bounds in Chapters 2 and 5 are likely to be more demanding to many engineering students. To introduce to them one of the theoretical topics which has been enriched by both engineers and mathematicians alike, derivations have been described in considerable detail in order to make the material more accessible. We hope that the scope of coverage here is sufficiently broad to appeal to theoretical researchers who are interested in either the analytical tools or the variety of applications.