Preface
to the second enlarged edition

The appearence of the present second edition is motivated by a two-fold engagement: to improve parts of the original text and to add two chapters on recent developments.

The improvements include the clarification of obscurities and the correction of formal flaws. The additional chapters serve as useful supplements to the topics treated in the first edition. The discussion added to Section 5.4 concerns the embedding of infinitely divisible probability measures on an Abelian locally compact group, a problem deeply related to the structure of the underlying group.

The new Chapter 7 is devoted to extending the framework of an Abelian group to that of a commutative hypergroup. Chapter 8 written by Gyula Pap aims at the central limit problem of probability theory for Abelian groups. While this theme enriches the previous discussion of the asymptotic behavior of random sequences in groups, the approach to stochastic processes taken in Chapter 7 reaches beyond groups, i.e. to group-like structures defined via a generalized convolution. In both chapters limit theorems for random sequences are discussed with respect to the special structure of their state space, particularly for tori, p-adic groups and solenoids in Chapter 8, for polynomial and Sturm–Liouville structures in Chapter 7.

Although the technique employed in both chapters is based on the notion of the Fourier transform —as throughout the first edition—, hypergroups require special attention. Meeting this requirement within a reasonably sized text lead necessarily to an expository presentation in Chapter 7. The main idea for choosing to study infinitesimal arrays in Chapter 8 is the completion and extension of the only available reference to necessary and sufficient conditions in terms of moments of the convergence of such arrays towards weakly divisible distributions.

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In Chapter 7 the author prepares the necessary tools for the construction of hypergroup structures and their analysis before he enters the treatment of random walks and more general increment processes in these structures. Despite obvious similarities to the group case significant deviations justify the extended concept and open the route to further fruitful research.

The author is grateful to his publisher for having stimulated this second enlarged edition of his book.

Heartfelt thanks go to two of his colleagues and friends: to Michael Bingham, who proposed numerous ameliorations, and to Gyula Pap, who not only adapted the original typescript to the requirements of the publisher, but also added significant improvements and above all composed Chapter 8 on the basis of his recent research.

As usual and most remarkably again the team of World Scientific has cooperated optimally along the production of the present edition.

And, as not even multi-editions are free from dark spots, all future readers are invited to report on their concerns.

Tübingen, June 2009 Herbert Heyer

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Preface

The present book has been written for mathematically prepared readers who like to look beyond the boundary of a single topic in order to discover the interrelations with others. More concretely the author's idea is to direct the attention of probabilists to the applicability of the enlightening notion of a group to probability theory.

The interplay between probability theory and group theory is as old as the early investigations on translation invariant probability distributions and stochastic processes and has become an increasingly important field of research which meanwhile reached a certain state of maturity.

While the traditional approach to the basic theorems of probability theory often overshadows part of the structure of the problems, the awareness of group-theoretical concepts leads to a quick detection of common features of apparently unrelated situations. In other words, the perception of algebraic-topological structures in the state space of stochastic processes does not only yield interesting and applicable generalizations of known results but also sets a limit to such generalizations by describing their domains of validity within the general framework. In practice this approach helps to provide at least more transparent proofs of well-established theorems including Lévy's continuity theorem, the Lévy–Khinchin representation of infinitely divisible probability measures, transience criteria for convolution semigroups and characterizations of recurrent or transient random walks.

This primer in probabilities on Abelian topological groups with emphasis on separable Banach spaces and on locally compact Abelian groups is by its very conception an elementary introduction to the structural access to probability theory, no textbook in the habitual understanding and by now means a monograph. It should be studied by graduate students along with the course work and will make interesting accompanying reading for

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their lecturers. At the same time the book provides information beyond the particular topic and lays bare the possibility of incorporating certain problems of probability theory into a wider setting which may be chosen according to the actual aims of study.

Since the pioneering work of Grenander and Parthasarathy going back to the early 1960's structural aspects of probability theory have been stressed in various monographs. For probabilities on locally compact groups we mention the books by Berg and Forst and by Revuz, both of 1975, as well as the author's book of 1977. There is also an extensive literature on probabilities on linear spaces. We just cite the books by Araujo and Giné of 1980, by Linde of 1986 and by Vakhania, Tarieladze and Chobanyan of 1987. Our selection of topics from these sources has at least two motives: to stress the significance of the problems within the development of the theory, and to choose an approach to their solutions which at the same time is as direct and informative as possible. Clearly these aims can hardly be achieved without reference to some basic notions and facts from topological groups, topological vector spaces and commutative Banach algebras. Appendices at the end of the book are offered as desirable aids.

In the first part of the book (Chapters 1 to 3) we start by collecting the necessary measure theory on metric spaces including the Riesz and Prohorov theorems. It follows a detailed analysis of the Fourier transform for separable Banach spaces. The main focus of the subsequent discussion is the arithmetic of probability measures on such spaces, in particular the study of infinitely divisible probability measures. We establish the embedding of infinitely divisible probability measures into continuous convolution semigroups and then examine Gauss and Poisson measures. The Ito–Nisio theorem is applied to a construction of Brownian motion. The proof of the Lévy–Khinchin representation is prepared by a detailed discussion of Lévy measures and generalized Poisson measures. It is clear that the theory exposed for general separable Banach spaces covers the case of Euclidean space and also various cases of function spaces.

The second half of the book (Chapters 4 to 6) begins with the notion of convolution of Radon measures on a locally compact group. The exposition continues by developing the duality theory of locally compact Abelian groups including positive definite functions and measures. Then negative definite functions on such groups are studied, their duality with positive definite functions and their correspondence in the sense of Schoenberg with convolution semigroups. The construction of Lévy functions for any locally compact Abelian group is the basic step towards a Lévy–Khinchin repre-

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sentation of negative definite functions. The concluding chapter contains a discussion of transient convolution semigroups and random walks. A measure-theoretic proof of the Port–Stone transience criterion precedes the characterization of groups admitting recurrent random walks and the classification of transient random walks which solves the problem of renewal of random walks on a locally compact Abelian group. The theory developed in this part of the book can be easily specialized to the Euclidean case, but moreover to infinite dimensional lattices and tori.

Now the methodical framework of the book becomes visible. For separable Banach spaces as well as for locally compact Abelian groups dual objects and Fourier transforms of measures as functions on these dual objects are employed in order to determine the structure of infinitely divisible probability measures and convolution semigroups. For Banach spaces only restricted versions of the Lévy continuity theorem can be proved. In fact, by the lack of an appropriate Bochner theorem for positive definite functions harmonic analysis soon reaches its limits. In the case of locally compact Abelian groups, however, the Pontryagin duality provides a far more elaborate harmonic analysis which can be applied to obtain not only strong versions of the Lévy continuity theorem but also deep results on the potential theory of stochastic processes with stationary independent increments and random walks in the group.

To write a primer in probabilities on algebraic-topological structures became a matter of concern during the author's lecturing over about three decades, mostly at the University of Tübingen in Germany. Along with his research work at the interface between probability theory and harmonic analysis he taught on probability measures on Banach spaces, locally compact groups and homogeneous spaces. It turned out that graduate students majoring in probability theory or in analysis took those courses which led to seminars on "Stochastics and Analysis" in which central limit theorems for generalized random variables, stochastic processes in and random fields over general algebraic-topological structures were discussed. In recent years also analogs of these probabilistic objects for generalized convolution structures as Jacobi and Sturm–Liouville translation structures were considered. For the harmonic analysis of these structures the presentation of the case of a locally compact Abelian group provides the appropriate basis. Consequently, the present book may also be used as a preparatory text for the study of probability measures on hypergroups and hypercomplex systems.

In conceiving his book the author received encouragement from many colleagues and friends spread over the globe. Various scientific agencies

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like the German and the Japanese Research Societies made it possible to test preliminary versions of the manuscript in workshops and crash courses during research stays and sabbaticals at universities in Australia (Perth), Japan (Tokyo) and the US (San Diego). Acknowledgement of prime importance goes to Christian Berg and Gunnar Forst, to Werner Linde and to Daniel Revuz for their excellent monographs the contents of which reaches far beyond our exposition. Several people have read drafts of the text. Especially valuable was Gyula Pap's constructive criticism for which the author is most thankful. There were also capable secretaries who did a great job in preparing the typescript: Kerstin Behrends and Erika Gugl deserve praise for their skillful work. Last but not least I am grateful to M.M. Rao from the University of California at Riverside who invited the book into the series on Multivariate Analysis with World Scientific.

The author expresses his expectation that all obscurities contained in the text will be communicated to him and that despite such inevitable deficiencies the book may serve its modest purpose. There is no doubt that the following statement due to Pablo Picasso also applies to an author in mathematics

"Ce que je fais aujourd'hui est déja vieux pour demain."

Tübingen, March 2004 Herbert Heyer

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