

Contents

Preface	ix
Notation and conventions	xiii
I Introduction	1
1 The risk process	1
2 Claim size distributions	6
3 The arrival process	11
4 A summary of main results and methods	13
II Martingales and simple ruin calculations	21
1 Wald martingales	21
2 Gambler's ruin. Two-sided ruin. Brownian motion	23
3 Further simple martingale calculations	29
4 More advanced martingales	30
III Further general tools and results	39
1 Likelihood ratios and change of measure	39
2 Duality with other applied probability models	45
3 Random walks in discrete or continuous time	48
4 Markov additive processes	54
5 The ladder height distribution	62
IV The compound Poisson model	71
1 Introduction	72
2 The Pollaczek-Khinchine formula	75
3 Special cases of the Pollaczek-Khinchine formula	77
4 Change of measure via exponential families	82
5 Lundberg conjugation	84
6 Further topics related to the adjustment coefficient	91

7	Various approximations for the ruin probability	95
8	Comparing the risks of different claim size distributions	100
9	Sensitivity estimates	103
10	Estimation of the adjustment coefficient	110
V	The probability of ruin within finite time	115
1	Exponential claims	116
2	The ruin probability with no initial reserve	121
3	Laplace transforms	126
4	When does ruin occur?	128
5	Diffusion approximations	136
6	Corrected diffusion approximations	139
7	How does ruin occur?	146
VI	Renewal arrivals	151
1	Introduction	151
2	Exponential claims. The compound Poisson model with neg- ative claims	154
3	Change of measure via exponential families	157
4	The duality with queueing theory	161
VII	Risk theory in a Markovian environment	165
1	Model and examples	165
2	The ladder height distribution	172
3	Change of measure via exponential families	180
4	Comparisons with the compound Poisson model	188
5	The Markovian arrival process	194
6	Risk theory in a periodic environment	196
7	Dual queueing models	205
VIII	Level-dependent risk processes	209
1	Introduction	209
2	The model with constant interest	222
3	The local adjustment coefficient. Logarithmic asymptotics . .	227
4	The model with tax	239
5	Discrete-time ruin problems with stochastic investment	242
6	Continuous-time ruin problems with stochastic investment . .	248

IX	Matrix-analytic methods	253
1	Definition and basic properties of phase-type distributions . . .	253
2	Renewal theory	260
3	The compound Poisson model	264
4	The renewal model	266
5	Markov-modulated input	271
6	Matrix-exponential distributions	277
7	Reserve-dependent premiums	281
8	Erlangization for the finite horizon case	287
X	Ruin probabilities in the presence of heavy tails	293
1	Subexponential distributions	293
2	The compound Poisson model	302
3	The renewal model	305
4	Finite-horizon ruin probabilities	309
5	Reserve-dependent premiums	318
6	Tail estimation	320
XI	Ruin probabilities for Lévy processes	329
1	Preliminaries	329
2	One-sided ruin theory	336
3	The scale function and two-sided ruin problems	340
4	Further topics	345
5	The scale function for two-sided phase-type jumps	353
XII	Gerber-Shiu functions	357
1	Introduction	357
2	The compound Poisson model	360
3	The renewal model	374
4	Lévy risk models	384
XIII	Further models with dependence	397
1	Large deviations	398
2	Heavy-tailed risk models with dependent input	410
3	Linear models	417
4	Risk processes with shot-noise Cox intensities	419
5	Causal dependency models	424
6	Dependent Sparre Andersen models	427
7	Gaussian models. Fractional Brownian motion	428
8	Ordering of ruin probabilities	433
9	Multi-dimensional risk processes	435

XIV Stochastic control	445
1 Introduction	445
2 Stochastic dynamic programming	447
3 The Hamilton-Jacobi-Bellman equation	448
XV Simulation methodology	461
1 Generalities	461
2 Simulation via the Pollaczeck-Khinchine formula	465
3 Static importance sampling via Lundberg conjugation	470
4 Static importance sampling for the finite horizon case	474
5 Dynamic importance sampling	475
6 Regenerative simulation	482
7 Sensitivity analysis	484
XVI Miscellaneous topics	487
1 More on discrete-time risk models	487
2 The distribution of the aggregate claims	493
3 Principles for premium calculation	510
4 Reinsurance	513
Appendix	517
A1 Renewal theory	517
A2 Wiener-Hopf factorization	522
A3 Matrix-exponentials	526
A4 Some linear algebra	530
A5 Complements on phase-type distributions	536
A6 Tauberian theorems	548
Bibliography	549
Index	597