

Contents

<i>Preface</i>	vii
<i>List of Symbols</i>	xvii
1. Preliminaries	1
1.1 Background Sketch	1
1.2 Semiperfect Rings and Perfect Rings	17
1.3 Frobenius Algebras, and Nakayama Permutations and Nakayama Automorphisms of QF -Rings	27
1.4 Notation in Matrix Representations of Rings	31
2. A Theorem of Fuller	35
2.1 Improved Versions of Fuller's Theorem	36
2.2 M -Simple-Injective and Quasi-Simple-Injective Modules .	49
2.3 Simple-Injectivity and the Condition $\alpha_r[e, g, f]$	52
2.4 ACC on Right Annihilator Ideals and the Condition $\alpha_r[e, g, f]$	60
2.5 Injectivity and Composition Length	63
3. Harada Rings	69
3.1 Definition of Harada Rings	69
3.2 A Dual Property of Harada Rings	83
3.3 The Relationships between Harada Rings and Co-Harada Rings	100
4. The Structure Theory of Left Harada Rings	107
4.1 Left Harada Rings of Types $(\#)$ and $(*)$	107

4.2	A Construction of Left Harada Rings as Upper Staircase Factor Rings of Block Extensions of QF -Rings	108
4.3	The Representation of Left Harada Rings as Upper Staircase Factor Rings of Block Extensions of QF -Rings	115
5.	Self-Duality of Left Harada Rings	139
5.1	Nakayama Isomorphisms, Weakly Symmetric Left H -Rings and Almost Self-Duality	140
5.2	Self-Duality and Almost Self-Duality of Left Harada Rings	142
5.3	Koike's Example of a QF -Ring without a Nakayama Automorphism	150
5.4	Factor Rings of QF -Rings with a Nakayama Automorphism	153
6.	Skew Matrix Rings	157
6.1	Definition of a Skew Matrix Ring	157
6.2	Nakayama Permutations vs Given Permutations	163
6.3	QF -Rings with a Cyclic Nakayama Permutation	171
6.4	Strongly QF -Rings	183
6.5	Block Extensions of Skew Matrix Rings	187
7.	The Structure of Nakayama Rings	191
7.1	Kupisch Series and Kupisch Well-Indexed Set via Left H -Rings	192
7.2	Nakayama QF -Rings	201
7.3	A Classification of Nakayama Rings	203
7.4	An Example of a Nakayama QF -Ring of $KNP(1 \rightarrow 1)$ -Type	229
7.5	The Self-Duality of Nakayama Rings	232
8.	Modules over Nakayama Rings	235
8.1	Characterizations of Nakayama Rings by Lifting and Extending Properties	235
9.	Nakayama Algebras	243
9.1	Nakayama Algebras over Algebraically Closed Fields	243
9.2	Nakayama Group Algebras	250
10.	Local QF -rings	261
10.1	Local QF -rings	261

10.2	Examples of Local QF -Rings with Radical Cubed Zero . .	270
	<i>Open Questions</i>	275
	<i>Bibliography</i>	277
	<i>Index</i>	287