Preface

> Il ne suffit pas d'observer, il faut se servir de ses observations, et pour cela il faut généraliser. [...] Le savant doit ordonner; on fait la science avec des faits comme une maison avec des pierres; mais une accumulation de faits n'est pas plus une science qu'un tas de pierres n'est une maison.[1]
>
> *Henri Poincaré (1902) La science et l'hypothèse*

Theoretical formulations of applied seismology are substantiated by observable phenomena. Reciprocally, our perception and understanding of these phenomena necessitate rigorous descriptions of physical behaviours. As stated by Bunge in his book on "Philosophy of Science, Vol. I: From problem to theory",

> A nice illustration of the intertwining of empirical and theoretical events in the actual practice of science is offered by seismology, the study of elastic disturbances of Terra. [...] In conclusion, in order to "read" a seismogram so that it may become a set of data regarding an event (e.g.,

[1] It is not enough to observe. One must use these observations, and for this purpose one must generalize. [...] The scientist must organize [knowledge]; science is composed of facts as a house is composed of bricks; but an accumulation of facts is no more a science than a pile of bricks is a house.

To emphasize this statement of Poincaré, let us also consider the following quotation.

> As the biggest library if it is in disorder is not as useful as a small but well-arranged one, so you may accumulate a vast amount of knowledge but it will be of far less value to you than a much smaller amount if you have not thought it over for yourself; because only through ordering what you know by comparing every truth with every other truth can you take complete possession of your knowledge and get it into your power.
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> *Arthur Schopenhauer (1851) Parerga and Paralipomena, Volume 2*

an earthquake) or an evidence relevant to a theory (e.g., about the inner structure of our planet), the seismologist employs elasticity theory and all the theories that may enter the design and interpretation of the seismograph.

The present book emphasizes the interdependence of mathematical formulation and physical meaning in the description of seismic phenomena. Herein, we use aspects of continuum mechanics, wave theory and ray theory to explain phenomena resulting from the propagation of seismic waves.

The book is divided into three main parts: *Elastic continua*, *Waves and rays* and *Variational formulation of rays*. There is also a fourth part, which consists of *Appendices*. In *Part 1*, we use continuum mechanics to describe the material through which seismic waves propagate, and to formulate a system of equations to study the behaviour of such a material. In *Part 2*, we use these equations to identify the types of body waves propagating in elastic continua as well as to express their velocities and displacements in terms of the properties of these continua. To solve the equations of motion in anisotropic inhomogeneous continua, we use the high-frequency approximation and, hence, establish the concept of a ray. In *Part 3*, we show that, in elastic continua, a ray is tantamount to a trajectory along which a seismic signal propagates in accordance with the variational principle of stationary traveltime. Consequently, many seismic problems in elastic continua can be conveniently formulated and solved using the calculus of variations. In *Part 4*, we describe two mathematical concepts that are used in the book; namely, homogeneity of a function and Legendre's transformation. This part also contains a *List of symbols*.

The book contains an *Index* that focuses on technical terms. The purpose of this index is to contribute to the coherence of the book and to facilitate its use as a study manual and a reference text. Numerous terms are grouped to indicate the relations among their meanings and nomenclatures. Some references to selected pages are marked in bold font. These pages contain a defining statement of a given term.

This book is intended for senior undergraduate and graduate students as well as scientists interested in quantitative seismology. We assume that the reader is familiar with linear algebra, differential and integral calculus, vector calculus, tensor analysis, as well as ordinary and partial differential

equations. The chapters of this book are intended to be studied in sequence. In that manner, the entire book can be used as a manual for a single course. If the variational formulation of ray theory is not to be included in such a course, the entire *Part 3* can be omitted.

Each part begins with an *Introduction*, which situates the topics discussed therein in the overall context of the book as well as in a broader scientific context. Each chapter begins with *Preliminary remarks*, which state the motivation for the specific concepts discussed therein, outline the structure of the chapter and provide links to other chapters in the book. Each chapter ends with *Closing remarks*, which specify the limitations of the concepts discussed and direct the reader to related chapters. Each chapter is followed by *Exercises* and their solutions. While some exercises extend the topics covered, others are referred to in the main text. Reciprocally, the footnotes attached to these latter exercises refer the reader to the sections in the main text, where a given exercise is mentioned. Often, the exercises referred to in the main text supply steps that are omitted from the exposition in the text. Also, throughout the book, footnotes refer the reader to specific sources included in the *Bibliography*.

"Seismic waves and rays in elastic media" strives to respect the scientific spirit of Rudzki, described in the following statement[2] of Marian Smoluchowski, Rudzki's colleague and friend.

> Tematyka geofizyczna musiała nęcić Rudzkiego, tak wielkiego, fantastycznego miłośnika przyrody, z drugiej zaś strony ta właśnie nauka odpowiadała najwybitniejszej właściwości umysłu Rudzkiego, jego dążeniu do matematycznej ścisłości w rozumowaniu.[3]

[2] Smoluchowski, M., (1916) Maurycy Rudzki jako geofizyk / Maurycy Rudzki as a geophysicist: Kosmos, **41**, 105 – 119

[3] The subject of geophysics must have attracted Rudzki, a great lover of nature. Also, this very science accommodated the most outstanding quality of Rudzki's mind, his striving for mathematical rigour in reasoning.