Chapter I
Introduction

1.1. Nuclear Matter: A Brief Overview

In the early 20th century physics moved from a macroscopic level down to atoms and molecules, and with this the nature of the discipline changed dramatically. Deterministic classical physics was replaced by a more probabilistic and often unintuitive or even counterintuitive quantum approach. As physical investigation moved to yet smaller entities, from atoms to their nuclei, the question arose whether the quantum mechanical principles that had been developed for the study of atoms would be applicable to the new and fast-developing field of nuclear physics. It seemed that many general principles of non-relativistic quantum theory could be applied at the nuclear level, but most of the calculations of nuclear properties were of a phenomenological, or at least semi-phenomenological kind. Instead of an ab initio approach, starting from first principles, scientists used a mixture of theoretical considerations and empirical data to develop models in a quest for an understanding of the nuclear realm.

In the years following the discovery of the neutron by James Chadwick in 1932, Dmitri Ivanenko and Werner Heisenberg independently proposed that the atomic nucleus consisted of protons and neutrons, now commonly referred to as nucleons.[1] This idea was soon commonly accepted, and scientists turned to a closer investigation of nuclear structure.

A striking feature of the binding energies of nuclei is the way in which, after the removal of electromagnetic effects, they follow a simple formula

$$\alpha A + \beta A^{2/3} + \gamma (N - Z)^2 / A \,, \tag{1}$$

where A is the number of nucleons and the symmetry term at the end is small. From the absence of a term proportional to A^2, the number of pairs of nucleons, one can see that nuclear forces saturate. Various types of scattering experiments suggest that nuclei are roughly spherical and appear to have essentially the same density. The data are summarized in the expression called the Fermi model:

$$R = r_0 A^{1/3} \,, \tag{2}$$

[1]W. Heisenberg, 'Über den Bau der Atomkerne', *Z. Phys.* **77**, 1 (1932), *Z. Phys.* **78**, 156 (1932); *Z. Phys.* **80**, 587 (1933); D. Ivanenko, 'The neutron hypothesis', *Nature* **129**, 798 (1932); D. Ivanenko, 'Constitutive parts of atomic nuclei' *Comptes Rendus de l'Académie des Sciences de l'URSS* **1**, 50 (1933).

where R is the nuclear radius and r_0 is a constant fit to experiment. This indicates that nuclei have approximately constant density which is not dependent on their size and binding energies but dominated by volume and surface terms. Therefore it is possible to interpret α and r_o as constants relating to an infinite volume of nuclear matter.[2]

Among the first attempts to understand nuclear structure was the so-called liquid drop model. It was first conceived by George Gamow in 1929.[3] In the wake of Chadwick's discovery of the neutron, Heisenberg's seminal theory of the nucleus, and Ettore Majorana's work on nuclear exchange forces, Carl-Friedrich von Weizsäcker in his influential study of nuclear masses extended Gamow's model significantly in 1935.

The fact that the density and the binding energy per nucleon were approximately the same for all (stable) nuclei led to the comparison of the nucleus with a liquid drop, which similarly has a constant density, independent of the number of molecules. Using this analogy, Weizsäcker in 1935 developed his semi-empirical formula for the mass of a nucleus as a function of A (total number of nucleons) and Z (number of protons),[4] which became more widely used when Hans Bethe, in the first of his famous *Reviews of Modern Physics* articles reworked and simplified it.[5] The liquid drop model describes the nucleus as a classical fluid made up of neutrons and protons. It is envisaged to have a well-defined surface and the short-range forces, one attractive force holding the nucleons together and one repulsive force stopping the nucleons collapsing in. Systematic measurements of the binding energies of atomic nuclei, however, showed systematic deviations of observed values from those estimated using the liquid drop model.

Certain nuclei with special values for proton and neutron numbers (the so-called 'magic numbers') proved to be much more tightly bound than the liquid drop model predicted, and this suggested the existence of a shell structure within the nucleus. In 1949, Maria Goeppert Mayer[6] and Johannes Jensen[7] independently proposed an average potential which could reproduce the nuclear magic numbers. Thereby the shell structure in nuclei was theoretically established and it became a firm foundation of nuclear structure theory.[8]

The shell model treats the nucleons individually as opposed to treating the nucleus as a whole. The long-range repulsive Coulomb force and the strong short-range attractive nuclear force acting between nucleons are replaced by an average force. According to this model, the motion of each nucleon is governed by the average attractive force of all the other

[2]This matter is, however, immensely heavy. Hans Bethe, with Gerry Brown, estimated that a teaspoon full weighed as much as all the buildings in Manhattan.

[3]G. Gamow, 'Discussion on the structure of atomic nuclei', *Proc. Roy. Soc.* **A123**, 386 (1929), G. Gamow, 'Mass defect curve and nuclear constitution', *Proc. Roy. Soc.* **A126**, 632 (1929).

[4]C. F. von Weizsäcker, 'Zur Theorie der Kernmassen', *Z. Phys.* **96**, 431 (1935).

[5]H. A. Bethe and R. F. Bacher, 'Nuclear physics. A. Stationary states of nuclei', *Rev. Mod. Phys.* **8**, 165 (1937).

[6]M. Goeppert-Mayer and R. G. Sachs, 'On closed shells in nuclei', *Phys. Rev.* **235** (1948); M. Goeppert-Mayer, 'On closed shells in nuclei, II.', *Phys. Rev.* **75**, 1969 (1949).

[7]O. Haxel, J. H. D. Jensen and H. E. Suess, 'On the "Magic Numbers" in Nuclear Structure', *Phys. Rev.* **75**, 1766 (1949).

[8]See also M. Goeppert Mayer and J. H. D. Jensen, *Elementary Theory of Nuclear Shell Structure*, John Wiley & Sons, New York, 1955.

nucleons. The resulting discrete energy levels form 'shells', just as the orbits of electrons in atoms do. As nucleons are added to the nucleus, they drop into the lowest-energy shells permitted by the Pauli Principle, which requires each nucleon to have a unique set of quantum numbers to describe its motion.

A third nuclear model is the so-called optical model for nuclear reactions, used particularly for determining elastic scattering, total cross sections and transmission coefficients. The optical model is a quantum mechanical approach to the problem of scattering and absorption of particles impinging on a nucleus. Scattering of neutrons from nuclei is described by considering a plane wave in the potential of the nucleus, which comprises a real part and an imaginary part. This model is called the optical model since it resembles the theory of how light scatters when it enters a new medium.

Around the time of the development of the liquid drop model, Hideki Yukawa considered the question of the stability of the nucleus against break-up into its constituent nucleons, which could not be explained by the electromagnetic field of the protons or by gravitational forces which were negligible. Inspired by Heisenberg's 1932 nuclear theory and Enrico Fermi's 1934 theory of beta radioactivity, he attempted to find a unified picture of strong and weak interactions, and he suggested that a new field, a "meson field", could explain this phenomenon. In analogy with the photon in an electromagnetic field, he proposed a new kind of quantum, a field particle with finite mass that mediated the exchange forces within the nucleus.[9] The meson's finite mass was to ensure that the forces due to the meson field should have a finite range. From the size of the nucleus, which gave the range of the new interaction, Yukawa concluded that the mass of these conjectured particles (mesons) was about 200 electron masses. It was not until 1947, when Cecil Powell and his collaborators with the help of new experimental techniques, proved that there existed two distinct particles, both present in cosmic radiation, of which one was the previously unidentified, short-lived 'π-meson' (now called pion). It soon became clear that its properties were well-accounted for by Yukawa's theory.

It was hoped that Powell's discovery of the Yukawa meson would have been followed by the derivation of a potential governing the nucleon-nucleon interaction. The first attempt at calculating these interactions had been made by one of Heisenberg's students, Hans Euler, already in 1937. He calculated the properties of nuclear matter in second-order perturbation theory[10] working on the assumption that nucleons interacted via a two-body potential of Gaussian shape. He believed the force to be smooth at short distance and the comparative weakness of the force ensured that the contributions beyond the second order were small. But in order to achieve saturation, it was necessary for strong repulsion to occur in odd states, and this proved incompatible with later scattering experiments.

In Euler's calculation the second-order term from iterating the tensor force, although operating only in triplet states was very large and we will see later in this chapter that it basically gave the difference between the triplet attraction, which binds the deuteron

[9]H. Yukawa, 'On the Interaction of Elementary Particles', *Proc. Physico-Math. Soc. Japan* **17**, 48 (1935); See also H. Yukawa, 'Models and Methods in the Meson Theory', *Rev. Mod. Phys.* **21**, 474 (1949).

[10]H. Euler, 'Über die Art der Wechselwirkung in den schweren Atomkernen', *Z. Phys.* **105**, 553 (1937).

and the singlet attraction in which there is no tensor force and no bound state. Therefore investigators were worried that higher-order effects would also be large. However, the first-order term in the tensor force includes tensor-like correlations in the wave function, and the additional tensor interaction takes the wave function essentially to a (large) constant times the initial state. In other words, the coefficients pile up coherently for the second-order term, which acts as a triplet scalar interaction, and higher order terms are small. Gerry Brown, Gottfried Schappert and Chun Wa Wong[11] showed by Monte Carlo simulation that there are no coherent combinations in higher order which would contribute appreciably; thus, as far as the tensor force is concerned, the Euler second-order direct term is all that is left. It comes from the second-order pion and rho meson exchanges, as we shall develop, with opposite sign.

The properties of nuclear matter are known only for the saturation density $n_0 = 0.16$ nucleons/fm^3. However, the nuclear equation of state has been used in countless calculations of dense matter such as that in neutron stars, etc. None of these have, in a fundamental sense, described nuclear matter saturation at the one density n_0 in which we know its properties, although some calculations with three-body forces enforce essentially the same saturation as obtained by Brown–Rho scaling which we shall discuss in Chapter IV. We shall also discuss there that Brown–Rho scaling is essential in transforming the ^{14}C beta decay from a short half-life superallowed beta decay of some hours into an archeologically long $\tau_{1/2} \simeq 5370$ yr transition that makes carbon dating so effective.

The singular nature of the nuclear potential at short distances, i.e. the very strong repulsion at short distances, the so-called hard core, was described by Robert Jastrow in 1951.[12] The extreme case of an infinitely hard core would mean that all the potential matrix elements would be infinite in the uncorrelated wave functions. A possible solution of this problem was to build short-range correlations that would prevent the nucleons from being too close to each other.

A special many-body methodology needed to be developed to deal with the problem of the strongly repulsive core. Some of the features of the subsequent development of this many-body theory were already present in an earlier work by Eugene Feenberg on the structure of perturbation theory[13] and in the studies by Kenneth Watson of multiple scattering of a particle in a many-body medium.[14] Watson recognized the simplification resulting from the use of selective summations leading to vertex operators and modified particle propagators, although he never considered the full many-body problem.

Keith Brueckner used the algebra developed by Watson for studying the multi-scattering of a fast particle through an atomic nucleus. He modified it for his study of a particle in a

[11] G. E. Brown, G. T. Schappert, and C. W. Wong, 'Binding energy of nuclear matter', *Nucl. Phys.* **56**, 191 (1964).
[12] R. Jastrow, 'On the nucleon-nucleon interaction', *Phys. Rev.* **81**, 165 (1951).
[13] E. Feenberg, 'A note on perturbation theory', *Phys. Rev.* **74**, 206 (1948); E. Feenberg, 'Theory of scattering processes', *Phys. Rev.* **74**, 664 (1948).
[14] K. M. Watson, 'Multiple scattering and the many-body problem: applications to photomeson production in complex nuclei', *Phys. Rev.* **89**, 575 (1953); and N. C. Francis and K. M. Watson, 'The elastic scattering of particles by atomic nuclei', *Phys. Rev.* **92**, 291 (1953).

bound state and it was applied to the problem of nuclear saturation by Brueckner, Levinson and Mahmoud.[15] Their method depended on a treatment of the coherent particle motion which was exact in the limit of very many scatterers, and treated the incoherent motion as a perturbation. In this case the many-body potential energy could be expressed in terms of the low-energy scattering amplitudes. They applied the method to the two-body potentials given by pseudoscalar meson theory when the effects of nucleon pair formation were assumed to be small. In this approximation the many-body forces of the theory were negligible.

In a second paper, Brueckner extended the method developed for the treatment of the problem of nuclear saturation to the case of tensor forces.[16] The general result obtained expresses the many-body potential energy as a function of the triplet and singlet scattering phase shifts. One consequence was that the tensor force, which averaged to zero if the Born approximation was used to evaluate the scattering, gave a very sizable contribution to the potential energy.

In a third paper completing the series, Brueckner discussed the details of the structure of the nucleus.[17] He examined the characteristics of particle motion in the nuclear medium and he discussed the origin of the strong dependence of the potential energy on the nucleon momentum. Furthermore, he provided an equivalent formulation in which a uniform and constant potential was assumed but the nucleon moved with markedly reduced mass. The determination of the potential was shown to lead to a self-consistency equation which was to some extent similar to that appearing in the Hartree method of self-consistent fields.

Following these three papers on two-body forces and nuclear saturation, Brueckner and Levinson turned to the mathematical basis of their methods[18] and Richard Eden and Norman Francis considered how Brueckner's method related to the general theory of nuclear models. They described a framework for a unified theory of nuclear structure in which the wave functions for different nuclear models were obtained by transformations on the actual nuclear wave function. This formulation provided a basis for explaining the success of weak-coupling models of the nucleus and showed that these were not in conflict with the assumption that nucleons had very strong mutual interactions. Brueckner, Eden and Francis then moved on to relate their methods to correlations in the nucleus and their effect on high-energy nuclear reactions,[19] the nuclear shell model[20] and to the optical model.[21]

[15] K. A. Brueckner, C. A. Levinson and H. M. Mahmoud, 'Two-body forces and nuclear saturation. I. Central forces', *Phys. Rev.* **95**, 217 (1954).

[16] K. A. Brueckner, 'Nuclear saturation and two-body forces. II. Tensor forces', *Phys. Rev.* **96**, 508 (1954).

[17] K. A. Brueckner, 'Two-body forces and nuclear saturation. III. Details of the Structure of the Nucleus', *Phys. Rev.* **97**, 1352 (1955).

[18] K. A. Brueckner and C. A. Levinson, 'Approximate reduction of the many-body problem for strongly interacting particles to a problem of self-consistent fields', *Phys. Rev.* **97**, 1344 (1955).

[19] K. A. Brueckner, R. J. Eden, and N. C. Francis, 'High-energy reactions and the evidence for correlations in the nuclear ground-state wave function', *Phys. Rev.* **98**, 1445 (1955).

[20] K. A. Brueckner, R. J. Eden and N. C. Francis, 'Nuclear energy level fine structure and configuration mixing', *Phys. Rev.* **99**, 76 (1955).

[21] K. A. Brueckner, R. J. Eden and N. C. Francis, 'Theory of neutron reactions with nuclei at low energy', *Phys. Rev.* **100**, 891 (1955).

Keith Brueckner had recognized that the strong short-range interactions, such as the infinite hard core, would scatter nucleons to momenta well above the Fermi sea. Therefore the exclusion principle would have very little effect and could be treated as a small correction. In analogy to Watson's T-matrix

$$T = V + V\frac{1}{E - H_0}V + V\frac{1}{E - H_0}V\frac{1}{E - H_0}V + \cdots$$

with V the two-body potential, E the unperturbed energy and H_0 the unperturbed Hamiltonian, containing only the kinetic energy, he defined an operator G from the potential V. This G-Matrix obeys the well-known equation

$$G = V + V\frac{Q}{E - H_0}G\,,$$

where Q is the Pauli exclusion operator ensuring the intermediate states being above the Fermi surface. He showed that one could carry out a perturbation theory calculation in a Fermi gas treating G as the effective potential.

Brueckner's pioneering approach of solving the two-body scattering problem in the nuclear medium consisted of rearranging perturbation theory in such a way that the contribution to the total energy at each order was proportional to the number of particles. Energies per particle were manifestly finite. But its formulation was ambiguous, and it was not readily accepted by nuclear physicists, largely as a result of the very formal nature of the central proof of the theory. The theoretical structure needed a more concrete machinery to make it work. This was provided by Jeffrey Goldstone and Hans Bethe at Cambridge and Cornell.

Brueckner has rightly been attributed with 'taming' the short-range interactions between two nucleons; Hans Bethe, the master of organization and communication, set out 'to tame' the body of theory that Watson, Brueckner and their colleagues had created by recreating and reformulating Brueckner's work. As he stated in the introduction to his first important paper on the Nuclear Many-Body Problem,[22] while the success of the Brueckner model had been beyond question for many years, a theoretical basis for it had been lacking. He pointed out "it is well established that the forces between two nucleons are of short range, and of very great strength, and possess exchange character and probably repulsive cores. It has been very difficult to see how such forces could lead to any over-all potential and thus to well-defined states for the individual nucleons." Further explaining this point he emphasized that Brueckner had developed a powerful mathematical method for calculating the nuclear energy levels using a self-consistent field method, even though the forces are of short range. But the definitions on which the various concepts had been based had remained unsatisfactory.

Bethe's approach, based on a diagrammatic expansion of perturbation theory in a series ordered by the number of interacting particles, facilitated the understanding of Brueckner's work significantly. Bethe gave a self-contained and largely new description of Brueckner's method for studying the nucleus as a system of strongly interacting particles with the aim

[22] H. A. Bethe, 'Nuclear many-body problem', *Phys. Rev.* **103**, 1353 (1956).

of developing a method that was applicable to a nucleus of finite size while at the same time eliminating any ambiguities of interpretation and approximations required for computation. Thus Bethe, using the work of Brueckner and collaborators, produced an orderly formalism in which the evaluation of the two-body operator G would form the basis for calculating the shell model potential.

One of Bethe's students at Cambridge, Jeffrey Goldstone, by means of perturbation theoretical methods, established the so-called linked cluster expansion.[23] Using Feynman graphs to enumerate the terms of the perturbation series, and describing the states in a way that was equivalent to treating the independent-particle Fermi sea as a 'vacuum state', he proved the 'linked cluster' theorem for the non-degenerate case. About a decade later, Morita,[24] Brandow,[25] Johnson and Baranger,[26] Kuo, Lee and Ratcliff (KLR)[27] and others extended the linked cluster theorem to the degenerate case. The linked cluster theorems for the degenerate case are usually referred to as the folded-diagram methods. The KLR formalism has been widely applied to calculations of finite nuclei using realistic nucleon-nucleon interactions.[28]

As in other scientific contexts, analytic solutions to specific problems were a source of additional insight for Hans Bethe. Therefore, with Jeffrey Goldstone he went on to investigate the evaluation of G for the extreme infinite-height hard core potential. Bethe and Goldstone[29] defined a spatial wave function for two nucleons and derived the Bethe–Goldstone integro-differential equation for this function. The calculation of the Brueckner G-matrix is rather complicated. As we shall discuss in Chap. III, a simple and ingenious separation method for the calculation of G has been developed by Moszkowski and Scott.[30] In this method, the G-matrix in the $^1S_0$ and $^3S_1$ channels can be well approximated by a long-range potential $V_{\rm long}$ obtained by cancelling the repulsive core against part of the attractive well up to a separation distance $d \simeq 1.2$ fm. For $r < d$, $V_{\rm long} = 0$ and for $r > d$, $V_{\rm long} = V_{NN}$. We shall also discuss there that $V_{\rm long}$ is qualitatively similar to the low-momentum interaction V_{low-k}.

Starting from realistic NN potentials, the Brueckner–Bethe–Goldstone theory, which to first order is referred to as the Brueckner-Hartree-Fock (BHF) method, has been extensively applied to symmetric nuclear matter. However, the binding energy per particle BE/A and saturation density n_0 given by such calculations are all off the empirical value or $BE/A \simeq 16$

[23] J. Goldstone, 'Derivation of the Brueckner many-body theory', *Proc. Roy. Soc.* **A239**, 267 (1957).
[24] T. Morita, 'Perturbation theory for degenerate problems of many-fermion systems', *Prog. Theor. Phys.* **29**, 351 (1963).
[25] B. H. Brandow, 'Linked-cluster expansions for the nuclear many-body problem', *Rev. Mod. Phys.* **39** 771 (1967).
[26] M. B. Johnson and M. Baranger, 'Folded diagrams', Ann. Phys. (N.Y.) **62**, 172 (1971).
[27] T. T. S. Kuo, S. Y. Lee and K. F. Ratcliff, 'A folded-diagram expansion of the model-space effective Hamiltonian', *Nucl. Phys.* **A176**, 65 (1971).
[28] See Chap. III.
[29] H. A. Bethe and J. Goldstone, 'Effect of a repulsive core in the theory of complex nuclei', *Proc. Roy. Soc.* **A238**, 551 (1957).
[30] S. A. Moszkowski and B. L. Scott, 'Nuclear forces and the properties of nuclear matter', *Ann. Phys.* **11**, 65 (1960).

MeV and $n_0 = 0.16$ fm^{-3}. In fact they all lie on a so-called Coester band,[31] none reproducing the empirical BE/A and n_0 values simultaneously. Much effort has been devoted to improve the situation. In BHF, one includes only the lowest-order G-matrix diagram. As we shall discuss in Chap. III, a ring-diagram extension of the BHF method has been developed.[32] In this method the particle-particle hole-hole ring diagrams are summed to all orders. The ring-diagram results are an improvement over the BHF ones, but the obtained BE/A and n_0 are still significantly larger than the empirical values. There are indications that the inclusion of the Brown–Rho scaling, to be discussed in Chapter IV, may play an important role in reproducing the nuclear matter saturation properties.[33]

1.2. Development of Kuo–Brown Effective Interactions

"One of the authors, Gerry Brown, arrived at Princeton in early September, 1964. The next morning, as he came to the Palmer Physics Laboratory, Eugene Wigner, who just preceded him, opened the door for him. (It was a real contest to get ahead of Eugene and open the door for him which very few succeeded in doing.) Eugene asked Gerry, as we went into the building, what he planned to work on. "I plan to work out the nucleon–nucleon interaction in nuclei." Eugene said that it would take someone cleverer than him, to which Gerry replied that they probably disagreed what it meant to "work out". Gerry wanted to achieve a working knowledge, sufficiently good to be able to work out problems in nuclear physics. What follows in this book is the story of what Gerry and Tom Kuo, with considerable help from their students, collaborators and friends, put together."

Gerald E. Brown

Around the time of Gerry's arrival at Princeton in 1964, there were two main streams of activity. One was the Brueckner theory, which we have outlined in the preceeding section. Gerry called this a "realistic" approach because it dealt with the strongly repulsive hard core potential, which was later understood as having come from the strongly repulsive potential from ω-meson exchange, the ω being a massive photon. The other was the application of the Brueckner theory to finite nuclei. Hans Bethe had spent nearly two decades of his life organizing the Brueckner theory so that it would be amenable to calculations in finite systems such as nuclei. Brueckner theory had begun from many-body scattering theory where the effect of a vertical core was simply to keep the wave function excluded from the region occupied by the core.

The major work on the nucleus as a many body system, on the structure of the nucleus and on the spectra, nature of excited levels, etc. was led by Aage Bohr and Ben Mottelson

[31] F. Coester, S. Cohen, B. D. Day and C. M. Vincent, 'Variation in nuclear-matter binding energies with phase-shift-equivalent two-body potentials', *Phys. Rev. C* **1**, 769 (1970).
[32] H. Q. Song, S. D. Yang and T. T. S. Kuo, 'Infinite order summation of particle-particle ring diagrams in a model-space approach for nuclear matter', *Nucl. Phys.* **A462**, 491 (1987).
[33] L. W. Siu, J. W. Holt, T. T. S. Kuo and G. E. Brown, 'Low-momentum NN interactions and all-order summation of ring diagrams in symmetric nuclear matter', *Phys. Rev.* **C79**, 0540004 (2009).

in Copenhagen in the 1960's, the time of interest here.[34] We shall not treat their work in detail but will show how two-body interactions following from meson exchange give roughly equivalent descriptions. The various processes involving the mesonic interactions with each other and the way in which mesons fit into symmetry schemes are of great interest in themselves. Furthermore, the meson-focused approach helps to elucidate how nuclear physics relates to many modern developments in particle physics and other subdisciplines of physics.

Many years were spent in using the G-matrix to tame the infinitely repulsive interactions used in nuclear matter problems. It was natural to try it in finite systems, and some of this effort has been outlined above. A very important effort applied to finite nuclei was carried out by Dawson, Talmi and Walecka[35] who solved the Bethe–Goldstone equation not only for the ground state but also for excited states and then looked at the spectra of ^{18}O.

Igal Talmi[36] systematized the properties of subsystems of particles in partly filled shells of the shell model; e.g. the calcium isotopes where n particles are in the $f_{7/2}$ orbit. We would call these j^n where $j = 7/2$. He had an extremely simple formula for the energy of the partly filled shell

$$E(j^n, g.s.) = Cn + \frac{1}{2}n(n-1)\alpha + \left[\frac{1}{2}n\right]\beta\,, \tag{3}$$

where $[\frac{1}{2}n]$ is the step function which is equal to $\frac{1}{2}n$ if n is even and $\frac{1}{2}(n-1)$, if n is odd. The coefficients α and β are given by

$$\alpha = \frac{2(j+1)\overline{V}_2 - V_0}{2j+1}, \qquad \beta = \frac{2(j+1)}{2j+1}(V_0 - \overline{V}_2)\,, \tag{4}$$

$$\overline{V}_2 = \frac{\sum\limits_{J=\text{even}} (2J+1)V_J}{\sum\limits_{J=\text{even}} (2J+1)}\,, \tag{5}$$

$$V_J = (j^2 J|G|j^2 J)\,. \tag{6}$$

Talmi realized that any G-matrix model like that of Dawson, Talmi and Walecka mentioned earlier would be such that the force between the nucleons in the j^n configuration would be attractive such that α would be negative; i.e., that in the quadratic term there would be attraction between the nucleons. However, in the shell model nuclei he had looked at α was positive. This argument disabled essentially all of the models using the G-matrix as interaction to date, and spurred Kuo and Brown to go on to the next higher order.

[34]The work is summarized in Aage Bohr and Ben R. Mottelson, *Nuclear Structure*, (Vol. I Single Particle Motion; Vol. II Nuclear Deformations), W. A. Benjamin, New York, 1969.

[35]J. F. Dawson, I. Talmi and J. D. Walecka, 'Calculation of the level spectrum of O^{18} from the free two-nucleon potential', *Ann. Phys.* **18**, 339 (1962).

[36]I. Talmi, 'Effective interactions and coupling schemes in nuclei', *Rev. Mod. Phys.* **34**, 704 (1962).

We shall describe the Kuo–Brown calculations in Chapter II. Most of nuclear structure calculations in the 1960s had been carried out in Copenhagen under the guidance of Aage Bohr and Ben Mottelson[37] using only two types of effective interactions: (i) the pairing force and (ii) the quadrupole–quadrupole interactions

$$V_{ij} = -k \sum_{i.j} r_i^2 r_j^2 Y_2^m(\theta_i, \phi_i) Y_2^{-m}(\theta_j, \phi_j)(-1)^m$$

which were introduced by Elliott.[38]

Polarization in quantum mechanics is achieved in the nucleus by lifting a particle from a filled nuclear state to an unfilled one. Doing this with an operator having no angular dependence $P_0(\cos\theta)$ involves no change of shape, just an increase in size. With a $P_1(\cos\theta)$ it involves a small translation, which doesn't change shape, and comes out as a spurious mode. Thus a quadrupole excitation with $P_2(\cos\theta)$ gives the most favorable shape, and one expects two neighboring nucleons to interact importantly via this mode of core polarization. In Copenhagen a tremendous amount of work had been done treating nuclei between two closed shells with the quadrupole–quadrupole interaction.

George Bertsch[39] carried out the first microscopic calculation of the core polarization process and found that it is very important for the effective nucleon-nucleon interaction in nuclei. The calculation employed the Kallio–Kolltveit potential[40], which we viewed at the time as a realistic nucleon–nucleon interaction (in spite of it not having a spin-orbit or tensor force, it did give the correct scattering length and effective range). It is a hard-core, spin dependent (spin-singlet 's' and triplet 't') interaction of the form

$$V_i(r) = \begin{cases} \infty & \text{for } r \leq 0.4 \text{ fm} \\ -A_i e^{-\alpha_i (r - 0.4\text{fm})} & \text{for } r > 0.4 \text{ fm} \end{cases} \quad \text{for } i = s, t, \tag{7}$$

where

$$A_s = 330.8 \text{ MeV}, \quad \alpha_s = 2.4021 \text{ fm}^{-1}$$
$$A_t = 475.0 \text{ MeV}, \quad \alpha_t = 2.5214 \text{ fm}^{-1}.$$

The core-polarization contribution was evaluated with second-order perturbation theory, as given by the core-polarization diagram of Fig. 1, where each vertex is a G-matrix interaction derived from the above potential. The Moszkowski–Scott separation method, which we shall discuss in Chapter III, was employed for calculating the G matrix.

[37] A. Bohr and B. R. Mottelson, *Nuclear Structure*, (Vol. I Single Particle Motion; Vol. II Nuclear Deformations), New York: W. A. Benjamin, 1969.

[38] J. P. Elliott, 'Collective motion in the nuclear shell model. I. Classification schemes for states of mixed configurations', *Proc. Roy. Soc.* **A245**, 128 (1958); J. P. Elliott, 'Collective motion in the nuclear shell model. II. The introduction of intrinsic wave functions", *Proc. Roy. Soc.* **A245**, 562 (1958).

[39] G. F. Bertsch, 'Role of core polarization in two-body interaction', *Nucl. Phys.* **74**, 234 (1965).

[40] A. Kallio and K. Kolltveit, 'An application of the separation method in shell-model calculation', *Nucl. Phys.* **53**, 87 (1964).

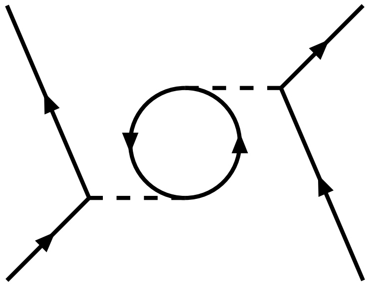

Fig. 1. The core polarization process calculated by Bertsch.

A more extensive calculation for the core polarization process was later performed by Kuo and Brown.[41] In their calculation, they employed a more realistic nucleon-nucleon interaction, the Hamada–Johnston potential[42] which was the 'best' nucleon-nucleon interactions at that time. The effects from the core-polarization process were found to be very important and desirable in both the Bertsch and Kuo–Brown calculations. Of particular interest is that the sign problem concerning the Talmi coefficient α, which was mentioned a little earlier, was largely resolved: its sign was negative when the effective interaction was given by the G interaction alone, but it became positive when G and core polarization were both included. In fact, as we shall discuss in Chapter III, the Kuo–Brown interactions turned out to be remarkably successful for a wide range of nuclei in both the (sd) and (pf) shells.[43] The rather successful results of the above initial calculations were 'very encouraging', and have led to many questions for further study.

The concept of Model Space is very useful in the theory of the nuclear shell-model. In treating nuclear many-body problems, one usually reduces the "full" many-body problem to a much smaller and more manageable one, often referred to as the model-space problem. Many-body problems are very difficult to solve, as we are all aware of, when A, the number of particles in the system, is large. In fact a many-body problem with A=3 is already a very hard problem, and in the real world, A is usually much larger. For example, A=18 when the nucleus ^{18}O is treated as a 18-nucleon problem. The wave function Ψ for the A=18 system is clearly very complicated, but to describe the low-energy properties of this nucleus, it may not be necessary to consider Ψ in its full glory; it may be sufficient to include just some low-energy parts of them. This is actually the approach commonly used in the shell-model description of ^{18}O, where this nucleus is treated as composed of an inert ^{16}O core with two

[41] G. E. Brown and T. T. S. Kuo, 'Structure of finite nuclei and the free nucleon-nucleon interaction–General discussion of the effective force', *Nucl. Phys.* **A92**, 481 (1967); T. T. S. Kuo and G. E. Brown, 'Structure of finite nuclei and the free nucleon-nucleon interaction–An application to ^{18}O and ^{18}F', *Nucl. Phys.* **85**, 40 (1966).

[42] T. Hamada and I. D. Johnston, 'A potential model representation of two-nucleon data below 315 MeV', *Nucl. Phys.* **34**, 382 (1962).

[43] T. T. S. Kuo and G. E. Brown, 'Reaction matrix elements for the 0f-1p shell nuclei', *Nucl. Phys.* **A114** (1968) 241.

valence neutrons in the 0d1s shell, corresponding to using a model space of

$$P = \sum_{2p'} |2p'0h\rangle\langle 2p'0h| \,, \tag{8}$$

where $2p'$ indicates the restriction that the two neutrons are confined in the $0d1s$ shell. The secular equation used in the shell model is of the form

$$PH_{\rm eff}P\Psi_m = E_m P\Psi_m; \quad m = 1, d \tag{9}$$

with $H_{\rm eff} = H_0 + V_{\rm eff}$, where $V_{\rm eff}$ is the effective interaction and H_0 denotes the single-particle Hamiltonian. The dimension of the model space is labeled d.

The above model-space many-body problem is much simpler than the original full space one. But 'there is no free lunch'; in this model space approach there is the difficult task of deriving the model space effective interaction $V_{\rm eff}$. Before the Kuo–Brown interactions, $V_{\rm eff}$ was generally determined empirically by fitting certain experimental data. That the Kuo–Brown interactions have worked well is an indication that $V_{\rm eff}$ can be microscopically derived from the free nucleon-nucleon interactions.

In Chap. II, we shall describe a folded-diagram theory for deriving the model-space effective interaction starting from the free NN interaction. The Kuo–Brown interactions have indicated that $V_{\rm eff}$ is mainly given by the sum of G and the second-order core polarization diagram often referred to as G_{3p1h}. But they are only a low-order approximation for the effective interaction $V_{\rm eff}$ between a pair of nucleons inside a nucleus. Using this folded diagram framework, we shall study the contribution to $V_{\rm eff}$ from higher-order diagrams. We shall discuss in Chap. III that the second-order results for $V_{\rm eff}$ and the corresponding all-order Kirson–Babu–Brown results are in fact quite similar. There are a number of different realistic nucleon-nucleon potential models, creating the uncertainty about which of them should be employed in nuclear structure calculations. This question will also be studied in Chapter III, and we shall show how one can derive a nearly universal low-momentum interaction V_{low-k} from the various NN potential models.

1.3. Towards a Unique Low-Momentum Nucleon–Nucleon Interaction

A fundamental problem in nuclear physics has long been the nucleon-nucleon (NN) potential V_{NN}. There have been a number of successful models for V_{NN}, such as the CD-Bonn,[44] Argonne,[45] Nijmegen[46] and Idaho[47] potentials. A common feature of these potentials is that they all can reproduce the empirical deuteron properties and low-energy phase shifts

[44] R. Machleidt, 'High-precision, charge-dependent Bonn nucleon-nucleon potential', *Phys. Rev. C* **63** (2001) 024001.

[45] R. B. Wiringa, V. G. J. Stoks and R. Schiavilla, 'Accurate nucleon-nucleon potential with charge-independence breaking', *Phys. Rev. C* **51**, 38 (1995).

[46] V. G. J. Stoks, R. A. M. Klomp, C. P. F. Terheggen and J. J. de Swart, 'Construction of high-quality NN potential models', *Phys. Rev. C* **49**, 2950 (1994).

[47] D. R. Entem, R. Machleidt, 'Accurate charge-dependent nucleon-nucleon potential at fourth order of chiral perturbation theory', *Phys. Rev. C* **68**, 041001 (2003).

very accurately. But they are, however, quite different from each other (see Chapter III). Certainly we would like to have a "unique" NN potential. (The gravitational potential $V(r) = -Gm_1m_2/r_{12}$ is unique as is the Coulomb potential $V(r) = kq_1q_2/r_{12}$.) In principle, the nonrelativistic NN potential should also be so. If there does exist one unique NN potential, then we will be confronted with the difficult question in deciding which, if any, of the existing potential models is the correct one. A partial answer to this question may be the low-momentum nucleon-nucleon potential V_{low-k} which was first developed, about 10 years ago, at Stony Brook (USA) and Napoli (Italy).[48–53]

An initial purpose for developing the V_{low-k} interaction was to have an energy-independent effective interaction which is more convenient for nuclear many-body calculations than the well-known Brueckner G-matrix. Because of its strong short-range repulsions, the bare interaction V_{NN} is not suitable for being directly used in perturbative calculations; it first needs to be 'tamed' into a smooth potential. The G-matrix is such a tamed interaction, and has been widely used for many many years. But G is not convenient to use, mainly because of its off-shell energy dependence. For example, to evaluate the matrix element $\langle k_1k_2|G(\omega)|k_3k_4\rangle$ for a certain vertex in a diagram we need to know the energy variable ω. But knowing the external indices (k_1, k_2, k_3, k_4) alone is in general not enough to determine ω; we need to have also information about what other particles in the diagram are doing. This makes the calculation using the G-matrix rather complicated, particularly for high-order diagrams. The V_{low-k} interaction to be described later does not have this off-shell energy dependence of the G-matrix.

The V_{low-k} project was started in about 1997 when Kuo, Coraggio and Bogner were visiting Arturo Polls and Angela Ramos in Barcelona. The Brueckner G-matrix had been the pillar in nuclear many body problems for 50 years or so. Maybe it was time to try something different! We thought about getting an energy-independent low-momentum interaction by way of a folded-diagram method, but were not at all sure how it would work. After much trial and error, we finally adopted the following renormalization (RG) method, namely deriving the V_{low-k} interaction by integrating out the high-momentum $(k > \Lambda)$ components of V_{NN}, Λ being a decimation scale. Certain low-energy physics, such as phase shifts, should be preserved by the integrating-out process. Thus, we have used the following

[48] S. K. Bogner, T. T. S. Kuo and L. Coraggio, 'Low momentum nucleon-nucleon potentials with half-on-shell T-matrix equivalence', *Nucl. Phys.* **A684**, 432c (2001).
[49] T. T. S. Kuo, S. K. Bogner, and L. Coraggio, 'A new theory of shell model effective interactions', *Nucl. Phys.* **A704**, 107c (2002).
[50] S. K. Bogner, T. T. S. Kuo, L. Coraggio, A. Covello and N. Itaco, 'Low-momentum nucleon-nucleon potential and shell model effective interaction', *Phys. Rev. C* **65**, 051301(R) (2002).
[51] T. T. S. Kuo, S. K. Bogner, L. Coraggio, A. Covello, and N. Itaco, 'Realistic low-momentum nucleon-nucleon potential', in *Challenges of Nuclear Structure* (Proceedings of the 7th International Seminar on Nuclear Structure; Maiori, Italy, May 27-31, 2001), ed. by A. Covello, p. 129, World Scientific Pub. Co. (2002).
[52] A. Schwenk, G. E. Brown, and B. Friman, 'Low-momentum nucleon-nucleon interaction and Fermi liquid theory', *Nucl. Phys.* **A703**, 191 (2002).
[53] S. K. Bogner, T. T. S. Kuo, and A. Schwenk, 'Model-independent low momentum nucleon interaction from phase shift equivalence', *Phys. Rep.* **386**, 1 (2003).

T-matrix equivalence formalism. We start from the half-on-shell T-matrix

$$T(k', k, k^2) = V_{NN}(k', k) + \mathcal{P}\int_0^\infty q^2 dq \frac{V_{NN}(k', q)T(q, k, k^2)}{k^2 - q^2}\,, \tag{10}$$

where $\mathcal{P}$ denotes the principal value integration. We then define an effective low-momentum T-matrix by

$$T_{low-k}(p', p, p^2) = V_{low-k}(p', p) + \mathcal{P}\int_0^\Lambda q^2 dq \frac{V_{low-k}(p', q)T_{low-k}(q, p, p^2)}{p^2 - q^2}\,, \tag{11}$$

noting that the integration limit is Λ. To preserve phase shifts, we require the following half-on-shell T-matrix equivalence:

$$T(p', p, p^2) = T_{low-k}(p', p, p^2);\; p', p \leq \Lambda\,, \tag{12}$$

which ensures that the low energy ($E_{lab} \leq 2\Lambda^2\hbar^2/m$) phase shifts of V_{NN} are preserved by V_{low-k}. The above equations define the effective low-momentum interaction V_{low-k}.

How to solve the above coupled equations was in fact a rather difficult task. It is a special inverse scattering problem, determining the V_{low-k} interaction backwards from the half-on-shell T-matrix. Also V_{low-k} has a special feature that it is nonvanishing only within the momentum model space $k < \Lambda$. As discussed in Chapter III, a folded-diagram solution for the above equations can be obtained. It is of the form

$$V_{low-k}(p', p) = \langle p'|\hat{Q} - \hat{Q}\int\hat{Q} + \hat{Q}\int\hat{Q}\int\hat{Q} - \cdots|p\rangle\,, \tag{13}$$

with the $\hat{Q}$-box given by

$$\hat{Q}(k', k) = V_{NN}(k'k) + \left\langle k'\left|V_{NN}\frac{P_\Lambda}{k^2 - H_0}V_{NN}\right|k\right\rangle\,, \tag{14}$$

where each intergral sign represents a 'fold', H_0 is the kinetic energy operator and P_Λ denotes that the intermediate states must have momenta greater than Λ.

The above solution for V_{low-k} can be rewritten as $V_{low-k} = V_{NN}(1+\Omega(0, -\infty))$ where Ω is the wave operator which satisfies $\Omega P\Psi = Q\Psi$ where Ψ is the full-space eigenstate and P is the model space projection operator ($Q = 1 - P$). The wave operator can be conveniently calculated using the Lee–Suzuki iteration method.[54] Clearly V_{low-k} as given above is not Hermitian. There are a number of ways to transform it into a Hermitian interaction, as discussed by Holt et al.[55]

A remarkable feature of V_{low-k} is its near uniqueness. As discussed in Chap. III, there are a number of high-precision models for the V_{NN} potential. Although they all fit low-energy NN phase shifts and deuteron properties very accurately, these potentials themselves are, however, quite different from each other. But after integrating out the high-momentum

[54] S. Y. Lee and K. Suzuki, 'The effective interaction of two nucleons in the s-d shell', *Phys. Lett.* **B91**, 173 (1980); K. Suzuki and S. Y. Lee, 'Convergent theory for effective interaction in nuclei', *Prog. Theor. Phys.* **64**, 2091 (1980).

[55] J. D. Holt, T. T. S. Kuo and G. E. Brown, 'Family of hermitian low-momentum nucleon-nucleon interactions', *Phys. Rev. C* **69**, 034329 (2004).

($k > \Lambda$, with $\Lambda \sim 2$ fm^{-1}) components of these potentials, the resulting low-momentum interactions are nearly identical to each other. The NN potentials are constrained by scattering data up to $E_{lab} \sim 350$ MeV, corrresponding to the above Λ value. Thus, we only know the low-momentum NN potentials up to this Λ. How to determine the NN potential at higher momenta is by and large still an open question and is model dependent.

1.4. Brown–Rho Scaling and Density-Dependent Nuclear Interactions

In the last few decades of the twentieth century, our understanding of hadron physics has undergone a profound transformation. This has resulted from the development of a fundamental relativistic quantum field theory of strong interactions, going by the name of quantum chromodynamics, or QCD. In this theory protons, neutrons, mesons, and all other hadrons are described as color-neutral bound-states of quarks held together through a force mediated by the exchange of gluons. Our confidence in QCD as *the* theory of strong interactions comes from the precise agreement between theory and experiment at high energies where the remarkable property of asymptotic freedom[56] tells us that QCD is weakly-coupled and therefore solvable with perturbative techniques. Moreover, QCD is in principle capable of describing essentially all of traditional nuclear physics, including the masses and other properties of protons and neutrons, as well as the force binding them together in nuclei. However, at the low energy scales characteristic of nuclear physics ($E < 1$ GeV), QCD is notoriously difficult to solve because the scale-dependent coupling constant has grown to the point that one can no longer employ perturbation theory. Merging traditional nuclear physics with the underlying fundamental theory of QCD is therefore a significant challenge and inspires many of the modern developments in nuclear theory.

As we've already discussed, for problems in low-energy nuclear physics, QCD cannot be solved with the tools usually employed for relativistic quantum field theories. An alternative method first suggested by Kenneth Wilson[57] is to discretize spacetime and let powerful supercomputers solve the resulting equations that describe how an interacting system of quarks and gluons evolves. This "lattice QCD" approach has been able to yield accurate hadron masses,[58] a qualitatively correct description of the nuclear force in relative S-waves,[59] and many other hadronic properties. However, it appears that much effort is still required before lattice QCD will be capable of producing realistic nuclear forces comparable in accuracy to models based on meson exchange.

An alternative to lattice gauge theory is to construct a low-energy effective theory of interacting hadrons by exploiting the known symmetry structure of QCD, an idea whose

[56]D. J. Gross and F. Wilczek, 'Asymptotically free gauge theories. I', *Phys. Rev. D* **8**, 3633 (1973); D. J. Gross and F. Wilczek, 'Ultraviolet behavior of non-abelian gauge theories', *Phys. Rev. Lett.* **30**, 1343 (1973); H. D. Politzer, 'Reliable perturbative results for strong interactions?', *Phys. Rev. Lett.* **30**, 1346 (1973).
[57]K. G. Wilson, 'Confinement of quarks', *Phys. Rev. D* **10**, 2445 (1974).
[58]S. Dürr et al., 'Ab initio determination of light hadron masses', *Science* **322**, 1224 (2008).
[59]N. Ishii, S. Aoki, and T. Hatsuda, 'Nuclear force from lattice QCD', *Phys. Rev. Lett.* **99**, 022001 (2007).

roots lie in the seminal work of Weinberg.[60] Although the QCD Lagrangian is invariant under chiral symmetry transformations in the limit of massless bare quark, the QCD vacuum breaks this symmetry. This "spontaneous breaking" of chiral symmetry gives rise to a set of light pseudo-Goldstone bosons (e.g., the pions) and constrains their dynamics. Together with the nucleons, the Goldstone bosons comprise the low-energy degrees of freedom of the effective theory. The nuclear force is then obtained by calculating pion-exchange processes in a well-defined power counting scheme that organizes them according to importance. Besides pion-exchange, a set of short-range contact interactions that model the exchange of heavier mesons are fit to experiment. The resulting theory is called chiral effective field theory and has been remarkably successful in describing properties of the Goldstone bosons as well as $\pi\pi$ and πN scattering. In systems with two or more nucleons, chiral effective field theory is considerably more difficult, owing to the slower convergence in the chiral expansion. Nevertheless, all terms contributing to the NN interaction at fourth-order in the chiral expansion (next-to-next-to-next to leading order or $\mathrm{N^3LO}$) have been calculated, and by fitting the 24 low-energy constants at this order one can very well reproduce NN scattering phase shifts and deuteron properties.[61]

Given the near universality of low-momentum interactions evolved to a scale of $\Lambda \simeq 2.0$ fm^{-1}, it may seem that there is little practical difference between traditional one-boson-exchange interactions and those derived from chiral effective field theory. However, it is well known that two-nucleon forces alone are insufficient to describe many properties of dense nuclear systems. Nowadays the nuclear many-body problem can be solved almost exactly for few-nucleon systems, and one finds that unevolved realistic two-nucleon forces systematically underbind ^{3}H, ^{3}He, and ^{4}He and significantly underpredict the nucleon-deuteron differential cross section at intermediate energies and backward angles. Moreover, as we've previously discussed, two-nucleon forces appear unable to predict simultaneously the saturation energy and density of symmetric nuclear matter. Given the small uncertainties associated with solving the above nuclear many-body problems, one can reasonably conclude that something is missing in the description of the nuclear force. Chiral effective field theory addresses this problem by introducing many-body forces with parameters fit to properties of light nuclei. Within a one-boson-exchange model, similar effects can be achieved through in-medium meson masses. Three-nucleon forces have the advantage that the uncertainties are currently better controlled, but density-dependent meson masses have the potential to directly connect with chiral symmetry breaking/restoration as we now discuss.

At low temperatures chiral symmetry is spontaneously broken by the QCD vacuum, but at large densities and/or temperatures the symmetry can be restored. The order parameter for the chiral phase transition is the chiral condensate $\langle \bar{q}q \rangle$, which gives the amplitude for finding a virtual quark-antiquark pair in the vacuum. A number of models suggest that this condensate is responsible for generating much of a hadron's total mass. Indeed, since the bare masses of the two lightest quarks are on the order of 5 MeV while the masses of

[60] S. Weinberg, 'Phenomenological Lagrangians', *Physica A* **96**, 327 (1979).

[61] D. R. Entem and R. Machleidt, 'Accurate charge-dependent nucleon-nucleon potential at fourth order of chiral perturbation theory', *Phys. Rev. C* **68**, 041001 (2003).

hadrons composed of them are typically 1 GeV, most hadronic mass (and therefore, most of the observable mass in the universe) is generated dynamically.

There are a number of models that connect the order parameter for chiral symmetry restoration with dynamical mass generation. In the Nambu-Jona-Lasinio (NJL) model,[62] which models QCD as a system of quarks interacting through zero-range contact interactions, bare quarks with small masses evolve into constituent quarks with masses on the order of ~ 300 MeV in vacuum. This constituent quark mass is directly proportional to the scalar quark condensate:

$$\frac{m^*}{m} \sim \frac{\langle \bar{q}q \rangle^*}{\langle \bar{q}q \rangle} \,. \tag{15}$$

The above scaling law is known as "Nambu scaling" and is characterized by hadron masses that scale linearly with the scalar quark condensate. This scaling law also comes out in QCD sum rule calculations.[63] Within the NJL model, the temperature and density dependence of the chiral condensate can be calculated, with the startling result that even at normal nuclear matter density found at the center of heavy nuclei, the chiral condensate can decrease by approximately 20–30%. This leads to the intriguing possibility that evidence for chiral symmetry restoration may be found even in normal nuclei.

The pion mass is generated by a different mechanism, since in the chiral limit (massless bare quarks) pions would be true Goldstone bosons and therefore massless. The pion in fact gets its mass from the explicit breaking of chiral symmetry due to the nonzero quark mass term in the QCD Lagrangian, and therefore ultimately by interacting with the Higgs particle which gives a bare quark its mass. One can therefore reasonably assume that the pion mass does not depend as sensitively on dynamical symmetry breaking as the non-Goldstone bosons.

In Brown–Rho scaling (BRS), which was derived in an attempt to address the question of scale invariance of chiral effective Lagrangians, one obtains that hadronic masses scale according to[64]

$$\sqrt{\frac{g_A}{g_A^*}}\frac{m_N^*}{m_N} = \frac{m_\sigma^*}{m_\sigma} = \frac{m_\rho^*}{m_\rho} = \frac{m_\omega^*}{m_\omega} = \frac{f_\pi^*}{f_\pi} = \Phi(n) \,, \tag{16}$$

where g_A is the axial coupling constant, f_π is the pion decay constant (an alternative order parameter for chiral symmetry breaking/restoration), Φ is a function of the nuclear density n, and all starred quantities represent in-medium (nonzero temperature and/or density) values as opposed to vacuum values. As pointed out by Lutz et al.,[65] the pion decay

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[62] Y. Nambu and G. Jona-Lasinio, 'Dynamical model of elementary particles based on an analogy with superconductivity. I', *Phys. Rev.* **122**, 345 (1961).
[63] T. Hatsuda and S. H. Lee, 'QCD sum rules for vector mesons in the nuclear medium', *Phys. Rev. C* **46**, R34 (1992).
[64] G. E. Brown and M. Rho, 'Scaling effective Lagrangians in a dense medium', *Phys. Rev. Lett.* **66**, 2720 (1991).
[65] M. Lutz, S. Klimt and W. Weise, 'Meson properties at finite temperature and baryon density', *Nucl. Phys.* **A542**, 521 (1992).

constant can be connected to the scalar quark condensate through the Gell-Mann, Oakes, Renner relation[66]

$$f_\pi^2 m_\pi^2 = -(m_u + m_d)\langle \bar{q}q \rangle \,, \tag{17}$$

where m_u and m_d are the bare up and down quark masses and $\langle \bar{q}q \rangle$ is the scalar quark condensate for the up quark. Assuming that the pion mass is protected by chiral invariance, this relation would produce

$$\left(\frac{f_\pi^*}{f_\pi}\right)^2 = \frac{\langle \bar{q}q \rangle^*}{\langle \bar{q}q \rangle} \,. \tag{18}$$

The resulting dependence of hadron masses on the square root of $\langle \bar{q}q \rangle$ in Brown–Rho scaling at low densities is different from the linear scaling obtained in the NJL model and QCD sum rules. At higher densities, Koch and Brown[67] showed that the entropy from reduced-mass hadrons fit the entropy from lattice gauge simulations if one had Nambu scaling (15). Thus, the exact connection between the scalar quark condensate and the in-medium pion decay constant remains an open question.

The density dependence of the quark condensate is given (at low densities) by[68]

$$\frac{\langle \bar{q}q \rangle^*}{\langle \bar{q}q \rangle} = 1 - \frac{\sigma_{\pi N}}{f_\pi^2 m_\pi^2} n + \cdots \,, \tag{19}$$

where $\sigma_{\pi N}$ is the pion-nucleon sigma term that describes how the nucleon mass depends on the masses of bare quarks. From πN scattering experiments one can infer $\sigma_{\pi N} \simeq 45$ MeV. Thus, at nuclear matter density one would obtain from equations (16), (18), and (19) that $\Phi(n_0) \simeq 0.8$. One therefore often takes a linear drop with density of the masses:

$$\frac{m^*}{m} \simeq 1 - 0.2 \frac{n}{n_0} \,. \tag{20}$$

Moreover, to the extent that the phenomenological form factors associated with nucleon-meson interaction vertices are governed by the nucleon radius, they too will decrease with density due to the increase in the nucleon size.[69]

In Chapter IV we shall explore the consequences of Brown–Rho scaling for nuclear structure. Since the masses of light mesons are expected to decrease as the nuclear density increases, the nuclear force in-medium would be different from that in free space. Such a density-dependent nuclear interaction could address the well known deficiencies in free-space NN interactions fit to NN scattering phase shifts. The exchange of the σ, ρ, and ω mesons are all important components of the nuclear force. The σ and ω act opposite to one

[66] M. Gell-Mann, R. J. Oakes and B. Renner, 'Behavior of current divergences under $SU_3 \times SU_3$', *Phys. Rev.* **175**, 2195 (1968).

[67] V. Koch and G. E. Brown, 'Model of the thermodynamics of the chiral restoration transition', *Nucl. Phys.* **A560**, 345 (1993).

[68] E. G. Drnkarev and E. M. Levin, 'The QCD sum rules and nuclear matter', *Nucl. Phys* **A511**, 679 (1990); **A516**, 715 (1990); T. D. Cohen, R. J. Furnstahl and D. K. Griegel, 'From QCD sum rules to relativistic nuclear physics', *Phys. Rev. Lett.* **67**, 961 (1991).

[69] M. Rho,'Axial currents in nuclei and the skyrmion size', *Phys. Rev. Lett.* **54**, 767 (1985).

another and to a large extent cancel in central forces, even as their masses decrease. The two most important contributions to the tensor force come from π and ρ-meson exchange, which act opposite to each other:

$$V_\rho^T(r) = -\frac{f_\rho^2}{4\pi} m_\rho \vec{\tau}_1 \cdot \vec{\tau}_2 \, S_{12} \, f_3(m_\rho r) \,, \tag{21}$$

$$V_\pi^T(r) = \frac{f_\pi^2}{4\pi} m_\pi \vec{\tau}_1 \cdot \vec{\tau}_2 \, S_{12} \, f_3(m_\pi r) \,, \tag{22}$$

where

$$f_3(mr) = \left(\frac{1}{(mr)^3} + \frac{1}{(mr)^2} + \frac{1}{3mr} \right) e^{-mr} \tag{23}$$

and S_{12} is the tensor operator $S_{12} = 3(\vec{\sigma}_1 \cdot \vec{r} \, \vec{\sigma}_2 \cdot \vec{r})/r^2 - (\vec{\sigma}_1 \cdot \vec{\sigma}_2)$. In Brown–Rho scaling the ρ meson is expected to decrease in mass at finite density while the pion mass remains nearly unchanged due to chiral invariance. Therefore, one unambiguous prediction of Brown–Rho scaling is the decreasing of the tensor force in a nuclear medium. As we shall discuss later, this decrease in the tensor force plays an important role for nuclear saturation as well as for the Gamow–Teller transition strength of ^{14}C, which is responsible for the archaeologically long lifetime of ^{14}C.