Foreword

This book is aimed mainly at researchers and graduate students in the field of geometric mechanics, especially the theory of nonholonomic constraints. A wider audience may consist of pure mathematicians who are interested in applications of modern differential geometry to mechanics.

The first part of the book is devoted to the general theory of systems with linear nonholonomic constraints. In chapter 1 we review different approaches to the description of dynamics of nonholonomically constrained systems appearing in the literature. Our only addition to this literature is the systematic use of the Levi Civita connection of the kinetic energy metric. We discuss restrictions on motions of nonholonomically constrained systems provided by conserved quantities and by accessible sets of the constraint distribution.

In chapter 2 we treat the basic properties of the action of a Lie group on a smooth manifold, concentrating especially on the case when the action is proper. We also discuss the differential geometry of the space of orbits of a proper action using the concept of a differential space.

In chapter 3 we use the results presented in chapter 2 to discuss reduction of a proper action of the symmetry group of a nonholonomically constrained system. We begin with a discuss symmetry and reduction for a general dynamical system. Next, we define the notion of symmetry of a nonholonomically constrained system and then show how to reduce it, when the action of the symmetry group is proper. We specialize our results to the case when the action of the symmetry group is free and proper. We then compare the above results to different approaches to reduction of systems with nonholonomic constraints.

In chapter 4 we discuss reconstruction of solutions of the original equations of motion from solutions of the reduced equations. We also study relative equilibria and relative periodic orbits of nonholonomically constrained systems.

The second part of the book contains a comprehensive analysis of concrete systems with nonholonomic constraints. In chapter 5 we discuss the classical nonholonomically constrained system known as Carathéodory's sleigh. In order to illustrate the theory given in the preceding chapters, we derive its equations of motion in five different ways, construct the reduced system in three different ways, and carry out reconstruction explicitly.

In chapter 6 we treat the example of a smooth strongly convex rigid body rolling without slipping on a horizontal plane under the influence of a constant vertical gravitational force. We use traditional notation, which sometimes clashes with that of the preceding chapters.

In chapter 7 we give a comprehensive analysis of the motion of a rolling disk, which is a body of revolution whose edge rolls on a horizontal plane under the influence of a constant vertical gravitational field. The rim of the disk is a planar circle with its center at the center of mass. We assume that during the motion the lowest point of the rim remains in contact with the horizontal plane, which prevents the disk from taking off into space. The rolling disk has a symmetry group $E(2) \times S^1$, where $E(2)$ is the Euclidean motion group of the plane, and S^1 is the group of internal symmetries of the disk. We reduce these symmetries, solve the reduced equations of motion and then reconstruct the reduced motion. We give a complete qualitative analysis of the motion of the disk. We obtain a global gyroscopic stabilization principle, namely, relative equilibria are stable (= elliptic) if their energy is larger than a fixed number. For exceptional values of the parameters, the disk falls flat in finite time. We give an asymptotic analysis of the situation when the disk nearly falls flat and then rises up again. A surprising result of this analysis is the existence of a universal constant change in the angle of the point of contact.