Chapter 1

Introduction

> If an individual is to gain a thorough understanding of a hard-won concept he must personally experience some of the painful intellectual struggle that led to it. A lecture or a text can do no more than point the way.
>
> [Lin and Segel (1988)] p. 504

> "No one studies willingly, the hard, slow lesson of Sophocles and Shakespeare – that one grows by suffering."
>
> Clara Park, in [Manin (2007)]

Aside 1.0.1. When I was in college I wrote a term paper for a philosophy course and received a grade of B. The professor wrote, "The trouble with your paper is apparently that you exhausted yourself in devising a style that combines the worst elements of [Rudolf] Carnap and Earl Wilson and have not as yet come up with something for which this style might serve as a vehicle." Then he added, "I retract this characterization of your style which I do like although I think it gets somewhat out of hand in places."

I cannot resist the impulse to employ two different styles, one to somehow reflect my everyday man-on-the-street personality, the other to exhibit my aspiration to think abstractly, clearly, and impersonally.

Some readers may not care for the mathematical abstractions and will prefer the copiously captioned illustrations flowing with the ideas in this book. These asides are written for that sort of reader. Only here do I permit myself use of the first person form of address, so that my personality can appear – as if by magic – in the otherwise impersonal landscape of my abstract so-called *magnum opus*.

Then again, there may be a reader who would prefer to avoid distractions in the midst of abstractions. Fine, such a one may merrily skip across the asides.

1.1 Why Would I Have Valued This Book in High School?

Aside 1.1.1. Here is a personal story that antedates my high school years, but actually sets the stage for all my subsequent interests.

When I was kid growing up in Flatbush, that's in Brooklyn, New York City, I had a little group of friends interested in science. "Dickie" Matthews was a bit older, and I remember his huge workroom, strung with wires and ham radio equipment and glass tubes for chemistry experiments. "Bobbie" Reasenberg was more my age, about 13, and his father was an engineer, and they had a really nice house. One day in 1955 he showed me a book with a yellow paper jacket called "Giant Brains," by Edmund C. Berkeley[Berkeley (1949)]. It fired my imagination big time. When my mother saw this flame she did something amazing. She found out that Berkeley had an office in Greenwich Village on West 11 Street, which was about a 45 minute subway ride from our house. One Saturday we went to visit. At a very nice brownstone on West 11 Street we went up to the second floor and there were these men surrounding a terrific looking machine on a tabletop, densely packed with relays and wire bundles. They were excited about adding a magnetic memory drum to it. That machine was "Simon," considered by some historians to be the world's first personal computer. Mr. Berkeley taught me Boolean algebra and the habit of putting the date on my notes. So began my interests in the brain, mind, machines and consciousness.

In a sense I am writing this book to myself when I was in high school. I would have enjoyed the illustrations and diagrams even if I could not follow all the mathematics. But even the mathematics would have been gripping, because I would have seen that the author is not holding back details, nor condescending to tell me that something patently opaque to my untrained young eyes is "obvious." I would have found value in references to articles and books where I could search further for understanding. I would have enjoyed the quotations from principal researchers, not only for the pleasure of their distinctive writing styles, but also the specific insights only they could convey. And I would have really appreciated how the book brought between two covers a very interesting range of scientific subjects but all to one goal, the understanding of one important topic. The stories and asides would have served my desire to know the author and make with him a personal connection. It would have been fine if they are sometimes entertaining. More importantly, it would have been encouraging to know it is possible to gain understanding if one sets a high standard and relentlessly persists.

1.2 Who Else Would Value This Book?

Aside 1.2.1. I think this book would be valuable for someone who has already studied thermodynamics, but has always felt unsatisfied – even if they can remember the equations and solve the problems. That dissatisfaction seethes beneath the sense of disconnect between the intuitive physics and the formal mathematics, between the calculus as practiced by authors of thermodynamics textbooks, and the calculus as taught in mathematics courses.

If you are a high school student of mathematics or physics you will like the streamlined, no frills, brisk presentation of basic calculus and linear algebra – with all the easy proofs.

High school science teachers should definitely be up to speed on modern mathematical technology in general, and know more about their subject than they are responsible for teaching. Hence, this book provides auxiliary material for teachers of mathematics, physics, chemistry, computer programming and biology.

Likewise, researchers might find new material here, if only from a different viewpoint.

The book also provides a new model-building technology called "timing machinery" implemented in MATLAB that is potentially useful for simulating alternative muscle contractions models. Hence, this book is also for scientists interested in how global behavior emerges from local rules, for that is what happens when myosin molecules – coupled by feedback from their effects – *cooperate* to generate the relatively huge forces that are responsible for animal behavior [Jülicher *et al.* (1997)][Duke (2000)][Lan and Sun (2005)].

1.3 Physics & Biology

> Since antiquity, motion has been looked upon as the index of life. The organ of motion is muscle.
>
> [Szent-Györgyi (2004)]

Aside 1.3.1. The discovery of DNA and the subsequent explosion in molecular biology research was instigated by a handful of people, not the least of whom was Francis Crick – a physicist. Obviously, the scientists studying muscle contraction must know their physics well, since muscle contraction is the primary cause of all motion in animals and physics is the scientific study of motion.

The range of physics tools used for studying muscle contraction is prodigious. All the way from basic classical components like polarizing filters, dichroic mirrors, centrifuges and viscometers, to classical instruments like microscopes, to electronic instruments such as the oscilloscope, the photomultiplier tube and three-dimensional electron tomography, to machinery that depends on X-rays such as X-ray crystallography and X-ray diffraction probes and sub-millisecond time-sliced synchrotron X-ray sources, to modern equipment that could not be understood and deployed without quantum mechanics, such as photodiodes, lasers, and optical tweezers for manipulation of individual muscle contraction molecules. (The story of "X-ray diffraction work on muscle structure and the contraction mechanism" is reviewed in [Huxley (2004)].)

When I was in high school and strongly interested in physics I had an overwhelming feeling that there was no way I could truly understand it without personally repeating all of the basic experiments performed throughout its history [Strong (1936)][Shamos (1959)]. I got over that, of course, by realizing that it is necessary to think of modern tools and instruments as "black boxes" which have a history and a structure, sure, but for the experimental purpose at hand must be considered as opaque relations between inputs and outputs. Unless something weird happens, in which case a jump into the box is necessary.

1.4 Motivation

Aside 1.4.1. Masters of scientific research are finally investigating human consciousness. There have been some amazing insights [Blackmore (2004)] [Metzinger (2009)] but still, there is no scientific consensus on how to define consciousness, let alone a technology based on a science of consciousness for engineering useful consciousness structures.

But if I had a scientific theory of consciousness I would certainly want everyone to know about it. How would I do that? I would speak and write about it. These communication activities – indeed all intentional human activities – are implemented by conscious control of muscles, whether in the throat or at the fingertips, and so on. So, even though there is no scientific consensus about consciousness, it is at least reasonable to ask, is there a scientific consensus about muscle contraction? Fortunately, there is on this topic a great wealth of scientific understanding – if no consensus. I write this book to provide in one place a story of muscle contraction starting from first principles in mathematics and physics.

> **Since antiquity, motion has been looked upon as the index of life. The organ of motion is muscle. Our present understanding of the mechanism of contraction is based on three fundamental discoveries, all arising from studies on striated muscle. The modern era began with the demonstration that contraction is the result of the interaction of two proteins, actin and myosin with ATP, and that contraction can be reproduced in vitro with purified proteins. The second fundamental advance was the sliding filament theory, which established that shortening and power production are the result of interactions between actin and myosin filaments, each containing several hundreds of molecules and that this interaction proceeds by sliding without any change in filament lengths. Third, the atomic structures arising from the crystallization of actin and myosin now allow one to search for the changes in molecular structure that account for force production** [Szent-Györgyi (1974)].

The fun of it is that such a theory, expressed in conformity with modern standards of streamlined mathematical rigor, must refer to basic concepts in several branches of science normally taught in separate courses of separate departments of educational institutions. A standard textbook in mathematics, or particle mechanics, or thermodynamics, or chemistry, or physiology of muscle, in general cannot – and perhaps should not – be responsible for joining into a seamless whole some fundamentals of the science of muscle contraction. Even though an outsider, I set myself the challenge of obtaining a clear sense of that multidisciplinary science, and this book is the result.

The other half of the story, how conscious human beings choose – or sometimes forced – to perform particular activities with their muscles, remains to be told.

Sharply distinguishing thought from action divides the labor of understanding. Understanding their relationship is a goal for a different book. Indeed,

> **[O]n the sensorimotor theory the primary function of the central representational system is the planning and control of voluntary action. Hence all representations should be viewed as available for playing the functional role of action plans, which can lead to the development of motor programs that, when activated, trigger motor behavior. This means that representations would normally involve higher-level action planning in the frontal cortex. It also means that language production, a species of motor output, shares the action-planning representational system with nonverbal behavior systems** [Newton (1996)].

The word "arm" may be associated with armed force, army, armada, armistice, and armor. Not to mention Armageddon. Focus on the human arm, which has many muscles. For example, the biceps brachii muscle. This is the muscle in the front of the upper arm, the one that bulges in arm-wrestling, or when someone demands, "Let's see your muscle."

> **Human muscles provide the mechanical energy necessary to set the body in motion. Some muscles, such as those in the lower limbs, provide large**

> **forces required to walk or run while others, like those around the wrist, have the dexterity needed to perform complex tasks. Understanding how these muscles function is an integral component of comprehending skeletal motion. Models and simulations of muscles are used not only to analyze human locomotion, but also to design robotic devices or to treat orthopaedic abnormalities** [Aigner and Heegaard (1999)].

An arm muscle consists of muscle fiber cells within which are many parts that together produce force. How does that work? You could dismiss the question by saying that a supernatural force directs everything in the natural world, and it is not the job of human beings to question the supernatural. That line of thought is not scientifically interesting, so move on to speculate on the *mechanism* of muscle contraction.

Perhaps along with gravitational force, electrical force, and magnetic force, there is a new kind, muscle force. Perhaps there is a rack and pinion inside a muscle, in other words, a gear that has teeth rotating while meshed with a straight series of matching teeth. Or maybe there is a system of rubber-like bands or springs that are controlled by the brain. Or maybe there is a system of balloons that expand but are linked in a way to produce contraction. Or, assuming there is a more fundamental level at which force is generated, maybe there is a kind of molecule that shrinks when it receives a special signal.

> **It came as a shock when the electron microscope revealed muscle to consist of "thick" myosin and "thin" actin filaments, which did not shorten on contraction, but only slid alongside one another. Initially no connection was seen between the two** [Szent-Györgyi (2004)].

A great nineteenth century self-taught English educator, researcher, and public intellectual named Thomas Henry Huxley – known as "Darwin's bulldog" for his support of Charles Darwin's theory of evolution – among other accomplishments coined the word "agnostic" to denote his personal view on theology, and originated the idea that birds evolved from dinosaurs. His grandchildren included Aldous Huxley, author of "Brave New World," and Andrew Huxley.

Andrew F. Huxley was awarded with Alan Lloyd Hodgkin the *1963 Nobel Prize in Physiology or Medicine* for work in mathematical biology on nerve conduction. They provided a system of differential equations that model the action potentials – a.k.a. "spikes" – of electro-chemical activity that zoom among neurons in the brain and along nerves in the body. In particular, spikes trigger muscles to contract.

Andrew Huxley's answer to the question, what is the mechanism for muscle contraction, is *not* that there is a kind of shrinking molecule, but that there are two kinds of molecular filaments bridged by tiny little molecular "arms" between them which – like rowers in a racing boat – collaborate to propel one filament relative to the other. In other words, the answer to how an arm works is that inside of it there are many, many tiny, tiny arms.

Aside 1.4.2. This answer reminds me of the idea that "causality is circular," that "all explanations deriving events from something completely other than themselves become explanations because somewhere along the way they introduce the outcome itself and thus turn the account into one in which the outcome is already contained in the ground." [Rosch (1994)]

1.5 The Principle of Least Thought

Aside 1.5.1. The name of this principle is supposed to be reminiscent of the "Principle of Least Action." Whereas that principle belongs to particle mechanics – indeed, to quantum mechanics via Feynman's path integral approach – my Principle of Least Thought belongs to the psychology of teaching and learning.

I crave understanding through the smallest number of steps, where the steps are no larger than a certain size. That size is, for me, perhaps smaller than it is for more intuitive people, the people who can make big leaps in thought without much exertion. By "smallest number" I mean there should be no extraneous fluff, no extra symbols, no irrelevant or misleading ideas. For me, understanding is not so much a demand for mathematical rigor; rather, it is an anxiety to grasp intuitive plausibility. Then again, the very effort to achieve rigor has been for me a terrific boost to intuition. Rigor cleans the window through which intuition shines.

Even the slightest increment of understanding requires a leap of intuition, that is, an un-reportable thought process somehow "behind the scenes" of reportable conscious thought. For different individuals and for different topics, there is a largest possible leap, beyond which one may feel not only lack of understanding, but even frustration and discouragement. An exposition readily apprehended by one class of individuals might be impenetrable to another. This exposition is tuned to my own capacity for intuitive leaps, hence may be too pedestrian for some, perhaps over the heads of others.

The mean of two numbers is the point half-way between them. If an increment of understanding is called a step, then one step is a means towards a goal. Think of clearing a new path through a jungle towards a treasure you know is there. *Meaning* is the process of finding means between what is understood and what is not understood. The Principle of Least Thought says, "Find the smallest number of steps, none of which exceeds your stride." Once the path is found there is an urge to return to earlier steps and explore side paths. This enlarges understanding.

1.6 Measurement

To make a measurement in the everyday world of objective procedures – algorithms for doing things with muscles – requires counting. Time measurements end up as counting marks as a physical body passes by (think of seconds-hand sweeping past numerals around circumference of a circular clock). Space measurements also end up as counting marks as a physical body passes by (think of fingertip of hand moving past markers on a ruler set against a rectangular solid body).

A physical body "passing by" implies a physical body (pointer) moving relative to another physical body (mark), even if the pointer is the orientation of the eyeball when attention is on the mark. For measurement, ultimately all that matters is the "counting of proximities" between pointers and marks.

1.7 Conceptual Blending

Aside 1.7.1. There is a large computer science/mathematics research industry that centered around the concept of "finite state machine." This is a mathematical abstraction of the idea that a system consists of states – usually represented by circles or dots in a diagram – connected by arrows representing the possibilities for transitions between states. There are many, many variations on this idea including triggered transitions, nested transitions, conditional transitions, and probabilistic transitions. I developed timing machinery as a conceptual blend [Fauconnier and Turner (2002)] of *time* with *finite state machine*, but I had not familiarized myself with the published literature on that idea.

At first timing machinery was called "symbol train processing." I programmed a simulator in *Mathematica* in 1990 and submitted an article to *The Mathematica Journal* under the title, "An Object-Oriented Interpreter for Symbol Train Processing." The referees were negative. One reviewer said I was unfamiliar with the literature on neural networks, and that that was obvious from my lack of references. Another reviewer wrote, "From the abstract alone, I can tell you that the article is flaky." A reviewer who wrote more lengthy comments said there wasn't much originality present and that there are "controversial and unsupported statements that would concern experts in the field." He said he suspected that the system would be very slow. He went on to emphasize my apparent lack of familiarity with literature on finite state machines and artificial intelligence. The reviewer closed by suggesting changes including a reference to "message passing parallel systems" and wrote that nevertheless he was "giving a tepid recommendation to publish."

The MATLAB timing machine simulator in this book is most definitely a message passing parallel system, and is expected to be very fast.

In this book I also advance a Theory of Substances as a foundation for macroscopic thermodynamics in terms of a conceptual blend of prior research [Fermi (1956)][Tisza (1961)][Falk *et al.* (1983)][Schmid (1984)][Herrmann and Schmid (1984)][Callen (1985)][Fuchs (1996)][Fuchs (1997)][Haddad *et al.* (2005)][Herrmann and Schmid (Year not available)].

Aside 1.7.2. Very early in my life I was gripped by the desire to understand what seemed to me then, and what still seems to me now, the absolutely interesting theories collected under the general heading of "mathematical physics." Specifically, I wanted to understand special relativity and quantum mechanics. I have made some progress on the former, essentially because its intuitive foundations in thoughts about electricity, magnetism, and light are accessible to the visual imagination. All that is not relevant to this book. But quantum mechanics is a different order of difficulty, because my understanding is that historically it was a deep problem in thermodynamics – the problem of black body radiation - that Max Planck solved by inventing quantization [Kuhn (1978)].

Thermodynamics is a very, very hard subject to understand, and not until twenty years ago did I feel sufficiently competent to renew my effort.

Since then I encountered a breakthrough in the little book [Fermi (1956)]. I became convinced – by his proofs unlike I had seen elsewhere – that there is an essentially *algebraic* root at the bottom of thermodynamics.

1.8 Mental Model of Muscle Contraction

There are many trillions of cells in a human body. Cells are thousands of times smaller than bodies, and the molecules out of which cells are made are thousands of times still smaller. Each cell contains over a billion non-water molecules [Goodsell (1998)] and also envelops over a thousand times as many water molecules ([Tester and Modell (2004)] p. 433).

At body temperature liquid water molecules are in-between and ceaselessly banging into all the other molecules at random. Therefore movement of the much larger molecules is impeded – the net effect is like a scuba diver trying to swim in molasses. The faster a molecule moves, the more it is impeded. But even without moving on its own, say, due to changing position or shape because of a chemical reaction , the bombardment by water molecules makes it jiggle around. In mathematical physics the scenario of a collection of large moving spheres being banged around at random by a great many smaller spheres is modeled by a "stochastic differential equation" called the Langevin Equation. This equation has a rich history in physics and – it might seem strangely – in mathematical finance [Lemons and Gythiel (1997)][Reimann (2001)] [Perrin (2005)][Davis and Etheridge (2006)][Lindén (2008)].

The smallest complete unit of muscle contraction is the sarcomere, which is most certainly not a collection of spheres. There are tens of thousands of sarcomeres in a single muscle cell. Actually there is a hierarchy of distinguishable structures from arm to muscle to muscle fiber to myofibril to sarcomere [Nelson and Cox (2005)].

A sarcomere is a complex but spectacularly symmetrical, periodic arrangement of several kinds of molecules along filaments. The crystalline regularity of this intricate nano-structure is what makes it possible by X-ray crystallography to measure precisely the relative positions and angles of the molecules [Reconditi *et al.* (2002)] [Nelson and Cox (2005)].

The sarcomere is not a static structure, of course, since it is the ultimate source of animal motion. Over many decades in the twentieth century, hundreds of scientists from all over the world contributed to views on how chemical reactions are linked to mechanical actions of molecules [Szent-

Györgyi (2004)]. The result is a set of simple and very beautiful ideas with a substantial mathematical and experimental foundation. For example,

> **The ultra-structure of a sarcomere. (a) The thick and thin filaments between the Z-disk and the M-line. (b) Side-view of the sarcomere. myosin motors are arranged all along the thick filament. The thick filament is connected to the Z-disk via the elastic titin molecules. Titin molecules restrain the movement of the Z-disk away from the thick filament, thus they are passive force generators. (c) Geometry of the actin-myosin interaction. (d) The myosin motor consists of three domains. The angle, u, between the motor domain and the light-chain domain (LCD) changes during the power-stroke. The stalk, which consists of a coiled-coil motif, actually continues into the thick filament. A bend is thought to occur in the light-chain, angling it upward to actin** [Lan and Sun (2005)].

myosin is the molecule that is responsible for muscle contraction. There are different ways to graphically represent the structures of molecules. Generally the idea is to capture the essence of shapes sometimes involving thousands of atoms without actually drawing each atom. It turns out that representations in terms of just two main kinds of overall shapes – springs called "$\alpha-$helices" and ribbons called $\beta-$sheets – composed of many linked atoms are extremely useful. In particular the distinctive shape of the myosin molecule has been determined and represented in this way [Reedy (2000)].

The mechanical operation of the myosin molecule that produces force is called the "powerstroke," and is like a tiny arm that bends at its tiny elbow.

One theory of the transduction of energy stored in a chemical called ATP (adenosine triphosphate) into mechanical motion of the tiny arm is often presented in textbooks [Voet and Voet (1979)] and the cooperation of many tiny arms to produce large forces can be simulated on a computer:

> **Basic cycle of the swinging cross-bridge model. The myosin molecule makes stochastic transitions between a detached state D, and two attached states, $A1$ and $A2$, which are structurally distinct.**

> **In general, the transition rates f, α, g and the corresponding reverse rates depend on the strain of the elastic element. Owing to the free-energy change associated with ATP hydrolysis, the forward rates are predominately faster than the reverse rates and the molecule is driven one way around the cycle: $D \to A1 \to A2 \to D$. One ATP molecule is split during each cycle** [Duke (2000)].

This books is directed towards answering the question, what is the mathematical science education necessary for understanding such mechanisms?

1.9 Organization

The parts of this book are as follows:

Chapters 2–4 **Mathematics** is shot through with diagrams of dots connected by arrows, both of which are usually decorated with labels. Such diagrams are used to represent structures (think of airline flight paths), relationships (think family trees), and operations (think flowcharts). These kinds of diagrams also represent interconnected algebraic operations, and are themselves collectively amenable to algebraic operations. Experience with diagrams has condensed into a mathematical industry called **Category Theory**. This book provides just enough category theory to modernize the presentation of thermodynamics: there is even a proposal in modern theoretical physics that physical theories are best understood as diagrams in a certain category. This book implicitly adopts that framework [Baez and Lauda (2009)][Döring and Isham (2008)].

Calculus is the mathematical theory of change. The book adopts a modern formulation of calculus in terms of rigorously defined infinitesimals, and many proofs involving calculus are explicitly justified by algebraic rules of calculation with numbers or infinitesimals, or both.

Chapters 5–10 **Physics defined is the scientific study of motion**. The deterministic theory of particles is simple and beautiful. Newton's Laws of mechanics, Lagrange's Reformulation, Hamilton's Principle, and Hamilton's Equations are presented in detail. The treatment of Legendre Transform is important in this story and even more so in the thermodynamics to follow, and so is explained very carefully to lay bare its algebraic essence and boiled down to an algorithm [Alberty (2001)].

Aristotle discussed "substance" in his *Metaphysics*. That is a rather profound philosophical work. In this book I offer a mathematical **Theory of Substances**.

Physics is shot through with analogies [Muldoon (2006)]. Many seemingly different physical processes are quite analogous. Energy transduction links the energy in processes of different kinds, including thermal, motion, deformation, chemical, and electrical processes Fig. 1.1. In other words, thermodynamics links the dif-

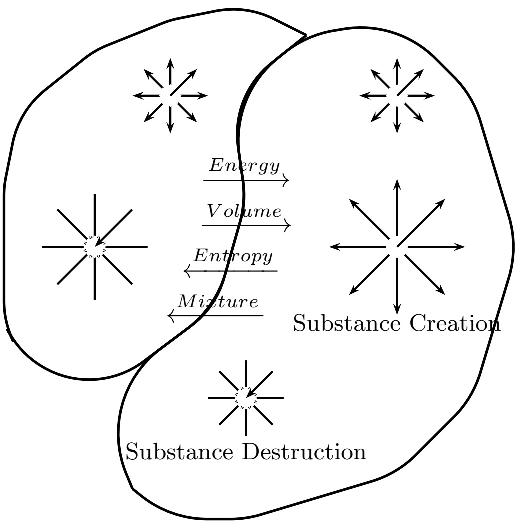

Fig. 1.1 Bodies in contact that are sustaining internal creation and destruction of substances as well as external flows of substances.

ferent ways that energy may be carried around, and hence, in a sense, is the center of physics. There are material substances, such as chemical species, and immaterial substances, such as energy, volume, momentum, probability, and entropy. All substances are tied together by the conserved substance, energy, which in thermal processes is carried by the indestructible immaterial substance, entropy Fig. 1.2.

$$\overset{\bullet}{E}_{\mathcal{A}} = \mathrm{Th}_{\mathcal{A}} + \mathrm{Vo}_{\mathcal{A}} + \mathrm{Ch}_{\mathcal{A}} + \mathrm{Me}_{\mathcal{A}} + \mathrm{El}_{\mathcal{A}}$$

Fig. 1.2 The Power Balance Equation. $\overset{\bullet}{E}_{\mathcal{A}}$ is the rate of change of energy – the power – of system $\mathcal{A}$. Successive terms on the right denote the rate of change of energy in the system due to thermal, volume, chemical, mechanical, and electrical processes, respectively.

If $\mathcal{A}$ is an isolated system then its total energy is constant, so $\overset{\bullet}{E}_{\mathcal{A}} = 0$. In that case, by the Power Balance Equation, changes in system energy by one type of process must be compensated by changes in energy due to one or more other processes. This is the meaning of *energy transduction* and in particular the transduction of energy in chemical processes to mechanical processes underlies muscle contraction.

Chapters 6–7 I invented "timing machinery" in 1989 while working as a computer programmer at The Rockefeller University in a laboratory where the neurophysiology of vision was studied. I wanted to abstract the essence of inter-communicating neurons. The result was the idea of a multitude of states timing out and consequently emitting signals that triggered other states (or even themselves) into commencement of timing out. This idea blossomed in my mind into a personal industry leading over the years to a general, parallel, graphic programming language.

These chapters offer **timing machinery** for computer simulation of systems such as the molecular machinery of muscle contraction. The timing machine interpreter is written in **MATLAB** to take advantage of its inherent parallelism.

Chapter 11 My fascination with **muscle contraction** and the reason for writing this book stems from the facts that

(1) it is crucial for nearly all vertebrate animal function behavior; (2) it is of significant interest to health professionals and sports physiologists; (3) its mechanism has been intensely studied by top-notch scientists for many decades and there are some very interesting alternative – perhaps even controversial – theories of how it works; (4) some basic mathematics and physics for understanding it can be but never have been published in a single book.

The closing chapter is a decade-by-decade chronology of muscle contraction research from the 19th century up through 2010. My idiosyncratic selections from the literature are biased by my interests in thermodynamics and simulation.

1.10 What is Missing?

Aside 1.10.1. It is a quirk of mine that, upon getting the gist of some theory or general explanation in the literature, I ask myself, "What is missing?" This is partly to counter my natural tendency to become wildly enthusiastic about the theory. But attempting to answer the question helps me put the story into better perspective. So, for example, if I read an article about a new theory of consciousness, I want to know how it relates, say, "attention" to "qualia," or, what consciousness experiments it cites.

This same habit is certainly applicable to my own work. There are numerous primary omissions:

Proofs The book has many proofs of theorems, some rather severely pedantic calculations, some more detailed than those in the literature, and some intrinsically very nice. But there are also theorems whose proofs are omitted because – dare I say it – the proofs are immediately obvious from the antecedent definitions, or, the proofs are exceedingly routine calculations. These omissions may seem to fly in the face of my Principle of Least Thought, but actually they just reveal my own – albeit small – intuitive leaps.

Adjoint Functors Arguably the most beautiful, universal, and natural concept introduced by category theory into mathematics is that of *adjoint functor*. The book does not define this abstract concept. Rather, it suggests with several leading examples that such a definition is inevitable.

Infinitely Slow Processes Classical thermodynamics is shot through with the intuition of an "infinitely slow process." In the Theory of Substances in Chapters 7–10 all of that is absorbed into a single term that may be set equal to zero in the Power Balance Equation. This amounts to ignoring the generation of entropy in the process under study. Indeed, it ignores the Second Law of Thermodynamics which says in part that *all* natural processes generate some entropy. The virtue of this unnatural assumption is that a large swath of classical thermodynamics is presented under the rubric of "equilibrium thermodynamics." This is not necessary for the intuitive and rigorous understanding of thermodynamics.

Heat Engines & Absolute Temperature Although the practice and theory of heat engines have been absolutely crucial in the history of thermodynamics, it turns out they are not essential in this book's

Theory of Substances. The same goes for the concept of "absolute temperature." This is not to say, of course, that heat engines and absolute temperature cannot be introduced within the framework of this book. It is just that those concepts may be left to self-imposed challenges for the reader. Likewise for the "Zeroth Law of Thermodynamics."

Numerical Calculations Not only are there no numerical calculations in this book, there are no standard units of measurement such as "Newton" or "Joule" or for that matter, "meter." Instead, generic placeholders for units of measurement such as [**FRC**]or [**NRG**]or [**DST**]are employed. Any convention may substitute standard units for these placeholders. As for calculations, there are a great many standard textbooks to consult, and some of the best are referenced in this book.

Phase Transformation The important theory of phase transformations is omitted because so far as I know phase transformations – gas to liquid to solid, and so on – play no role in muscle contraction.

Statistical Thermodynamics One of the challenges I set to myself in writing this book was to become convinced that it is possible to offer a rigorous mathematical theory of classical thermodynamics and chemical thermodynamics without mention of the molecular theory and accompanying statistical considerations. Therefore, Particle Mechanics in Chapter 6 and Stochastic Timing Machinery in Chapters 11–12 are entirely separate from the Theory of Substances in Chapters 7–10.

Muscle Contraction Simulation Code This book is not a report on original scientific research on muscle contraction that culminates in a new model for simulation. It is focused on the parts of mathematical science education that this outsider considers relevant to any such simulation, and so stops short of adopting a particular viewpoint among those cited in the last chapter.

Small Systems Biological motors such as the muscle contraction system are not classical chemical-mechanical thermodynamic systems. Discussion of cutting-edge mathematical physics of non-equilibrium thermodynamics of small systems as in, for example [Jarzynski (1997)] and [Astumian and Hänggi (2002)], is beyond the scope of this book.

1.11 What is Original?

Aside 1.11.1. I claim three original contributions to mathematical science education. First, baby steps towards a *Ground* in which to situate a new Foundation of Mathematics. All of the mathematics in this book is built on that foundation. Second, a new graphical programming language – *Stochastic Timing Machinery* – with source code for an interpreter to calculate simulations of concurrent processes such as clouds of particles moving in response to force fields. This is partly motivated by the literature on muscle contraction which includes models based on elaborations of such processes.

Third, my *Theory of Substances* is a new algebraic thermodynamics including an improved version of the "Second Law of Thermodynamics" called the *Entropy Axiom*.