PREFACE

The present volume is the proceedings of the 5th Japan-China Seminar on Number Theory "Dreaming in dreams" held during August 27-31, 2008 at Kinki University, Higashi-osaka, Japan, the organizers being Shigeru Kanemitsu and Jianya Liu with Professor Takashi Aoki as the local organizer.

The title sounded somewhat romantic or exotic and one of the participants, Professor Tim. D. Browning, a relative of the world famous poet R. Browning, expressed a poetic view that the title suggested that one could dream of proving the RH or whatsoever of the hardest nuts to crack in dreams. But we chose this title in view of the following due reason. Osaka is most well-known for its world famous Osaka Castle. The builder of Osaka Castle, Hideyoshi Toyotomi, a hero in the 16th century made a poem at his deathbed. "Like a dew drop was I born and into a dew drop am I fading, all that prevails in Naniwa (the present world) is like dreams in a dream." Here Naniwa sounds the same as the old name of Osaka. Also many of us went to the center of the city (for empty orchestra, perhaps), Namba, which is the modern name of Naniwa. This gives a good reason to entitle the seminar. This may not sound poetic but associatively logical. Indeed, at the end we have a poem composed by Professor Chaohua Jia. Thus we now have at least four poets among participants, including Professors Tianxin Cai (a professional), Chaohua Jia, Jianya Liu (who composed a poem in the proceedings of the 4th China-Japan Seminar), and Tim D. Browning.

The atmosphere was enjoyable as usual and we believe everyone enjoyed the 5 rich days. We organized various social activities including reception party at Sheraton-Miyako Hotel to which we thank for their great hospitality and generosity of providing us with champagne bottles. We made a tour to Kyoto (the bus was arranged through Sheraton-Miyako) and visited a few musts there. Some of the foreign participants paid multiple visits to Kinkakuji Temple and Kiyomizu Temple. Evening events were also entertaining which included empty orchestra activities at various places. We found that not only Chinese participants who were known to be good and enthusiastic singers, but most of the Western participants shared the same spirit. We found Professors Trevor Wooley, Winfried Kohnen, Katsuya Miyake and Yumiko Hironaka most entertaining. Professor Jörg Brüdern, though didn't sing, promised to play the guitar at the next occasion. S. Kanemitsu, to his regret, missed the chance of attending the ever-night show including Professor Tim. D. Browning and Professor Koichi Kawada.

Now about the contents of the seminar and the present proceedings. The talks ranged over a wide spectrum of contemporary number theory. As can be seen from the papers themselves as well as in the following brief descriptions in this volume, we succeeded in assembling topics from Analytic Number Theory (Classical and Modern with emphasis on additive number theory), Theory of Modular Forms, Algebraic Groups and Algebraic Number Theory.

In the proceedings we collected not only papers from the participants but from those invitees who could not attend the seminar, including Professors Andrzej Schinzel (who was about to come), Igor Shparlinski and Ken Yamamura.

In [Browning] a new direction of research in analytic number theory is exhibited, i. e. a quantitative study on the distribution of rational points of some variety—specifically, a del Pezzo surface of degree 4, $V \subset \mathbb{P}^4$ which is defined over the rationals and is assumed to have a conic bundle structure. The main result is the asymptotic formula for the counting function

$$N_{U_0,H}(B) = \#\{x \in U_0(\mathbb{Q}) :\ H(x) \leq B\},$$

where U_0 is a certain Zariski open subset and $H(x)$ is a certain norm function. The formula reads

$$N_{U_0,H}(B) = c_{V_0,H} B (\log B)^4 + \text{error term},$$

establishing the Manin conjecture in this case, where $c_{V_0,H}$ is the constant conjectured by Peyre.

The proof involves various ingredients, the geometric Picard group $\mathrm{Pic}(V_0) \simeq \mathbb{Z}^5$, analysis of conic sections and classical techniques including lattice points counting and divisor problems for binary forms.

[Brüdern-Kawada-Wooley] is the 8th of their series of papers "Additive representation in thin sequences" I-VII and is a timely summary which looks over their recent results in an enlightening way.

Their main concern is the diagonal form

$$\lambda_1 x_1^k + \cdots + \lambda_s x_s^k,$$

as $x_1, \cdots, x_s$ ranges over $\mathbb{Z}$ or a subset thereof, where $s \geq 2$, $k \geq 1$ and $\lambda_1, \cdots, \lambda_s$ are non-zero real numbers.

The purpose of the paper is two-fold; on one hand, it centers around Diophantine inequalities and on the other on the potential of the methods developed in the series, including the Davenport-Heilbronn Fourier transform method, a counterpart of the Hardy-Littlewood circle method for Diophantine inequalities. Starting from the case of additive cubic forms, the authors give a very clear survey on their hitherto contributions, giving proofs of some of the important theorems, which makes the paper more instructive and readable.

In the paper [Hoshi-Miyake] the authors are concerned with the FIP (Field Isomorphism Problem) on k-generic polynomial $f_{\mathbf{t}}^G(X) \in k(\mathbf{t})[X]$, where k is a field of arbitrary characteristic, G is one of finite groups D_3, D_4, D_5 (dihedral groups) and C_3, C_4 (cyclic groups), and $k(\mathbf{t})$ is the rational function field over k with n indeterminates $\mathbf{t} = (t_1, t_2, \cdots, t_n)$. A monic separable polynomial $f_{\mathbf{t}}^G(X)$ is called a k-generic polynomial for G if

- (G_1) the Galois group of $f_{\mathbf{t}}^G(X)$ over $k(\mathbf{t})$ is isomorphic to G and

- (G_2) every G-extension $L/K, K \supset k$ may be obtained as $L = \mathrm{Spl}_K f_{\mathbf{a}}^G$, the splitting field of $f_{\mathbf{a}}^G$ over K, for some $\mathbf{a} = (a_1, a_2, \cdots, a_n) \in K^n$.

The FIP reads: For a field $K \supset k$ and $\mathbf{a}, \mathbf{b} \in K^n$, determine whether $L_{\mathbf{a}} = \mathrm{Spl}_K f_{\mathbf{a}}^G$ and $L_{\mathbf{b}} = \mathrm{Spl}_K f_{\mathbf{b}}^G$ are isomorphic over K or not. Numerical examples are given in the cases $G = C_4$ and $G = D_5$.

Two more related problems are stated without details: Subfield Problem and Field Intersection Problem for generic polynomials.

In his paper [Jia], C. -H. Jia gives some developments over the results on dynamics of the w-function introduced by W. -S. Goldring in 2006, where dynamics refers to the orbit of successive iterates of the function. For $n = p_1 p_2 p_3 \in S := C_3 \cup B_3$ (p_i's are prime, not all equal), the w-function is defined by

$$w(n) = P\left(p_1 + p_2\right) P\left(p_2 + p_3\right) P\left(p_3 + p_1\right),$$

where $P(n)$ signifies the largest prime factor of n. The objective is to classify the elements of S and there are many results obtained by Goldring, Y.-G. Chen et al.

The inverse problem of finding the inverse image of the w-function is also of interest. One of the conjectures of Goldring states that every element of C_3, which is the set of all $n = p_1p_2p_3$ with p_i all distinct primes, has infinitely many C_3-parents, i.e. there are infinitely many $m \in C_3$ such that $w(m) = n$. Jia proves, by making a novel use of the large sieve method, some quantitative results including Theorem 9 saying that there is an element of $r_2r_2q \in C_3$, where $r_1, r_2 \sim \sqrt{x}\log x$, $q \leq 4x$ which has many distinct C_3-parents (where the data is quantitative).

In the paper [Kohnen], Kohnen surveys on the recent results his paper jointly with Mason on the generalized modular functions (GMF) (of weight zero) on Γ, where $\Gamma \subset SL_2(\mathbb{Z})$ is a subgroup of finite index. A GMF is a holomorphic function $f : \mathcal{H} \to \mathbf{C}$ satisfying the following two conditions.

i) $f(\gamma \circ z) = \chi(\gamma)f(z)$ $(\forall\gamma \in \Gamma)$, where $\chi : \Gamma \to \mathbf{C}^*$ is a (not necessarily unitary) character of Γ,

ii) f is meromorphic at the cusps of Γ.

In the paper [Komori-Matsumoto-Tsumura], the authors report on some recent developments on the second author's
Problem: Is it possible to find some functional relations for multiple zeta-functions which include some value-distribution for MZV's (multiple zeta values)? Here the multiple zeta-function of complex variables $\mathbf{s} = (s_1, s_2, \cdots .s_r)$ is defined by

$$\zeta(s_1, s_2, \ldots, s_r) = \sum_{m_1>m_2>\cdots>m_r\geq 1} \frac{1}{m_1^{s_1} m_2^{s_2} \cdots m_r^{s_r}}.$$

and the MZV of depth r is $\zeta(k_1, k_2, \ldots, k_r)$ with $k_1, k_2, \ldots, k_r \in \mathbb{N}, k_1 > 1$.

The authors are concerned with the multi-variable (version of the) Witten zeta-function defined by

$$\zeta_r(\mathbf{s}; \mathfrak{g}) = \sum_{m_1=1}^{\infty} \cdots \sum_{m_r=1}^{\infty} \prod_{\alpha\in\Delta_+} \langle \alpha^\vee, m_1\lambda_1 + \cdots + m_r\lambda_r \rangle^{-s_\alpha},$$

where $\mathfrak{g}$ is a complex semi-simple Lie algebra with rank r, $\mathbf{s} = (s_\alpha)_{\alpha\in\Delta_+} \in \mathbb{C}^n$ and the data appearing on the right-hand side are certain quantities associated with $\mathfrak{g}$.

Theorem 5.1 seems to be the culmination of the results which gives a general form of the functional relations for the multiple zeta-function $\zeta_r(\mathbf{s}, \mathbf{y}; \Delta)$ with an additive character of a root system, i.e. for

$$\sum_{w\in W^I} \Big(\prod_{\alpha\in\Delta_{w^{-1}}} (-1)^{-s_\alpha} \Big) \zeta_r(w^{-1}\mathbf{s}, w^{-1}\mathbf{y}; \Delta).$$

However, to deduce explicit functional relations for specific root systems from Theorem 5.1 seems rather cumbersome and the authors give some handy formulas in §6 which they apply in later sections to deduce explicit relations for $\mathfrak{g} = A_3$, B_2, B_3, C_3. There are given many concrete examples which make the paper readable.

[Liu] is a short introduction to Maass wave forms, which originated from his lectures at Postech in 2007. Restricting to the simplest setting of the Maass forms of weight 0 on the full modular group, he manages to give a quick introduction to its formidable theory and renders the situation more familiar to analytic number-theorists. From the contents one can see the flowchart of the paper. He starts from Fourier expansion of Maass forms, proving thereby the Chowla-Selberg formula for the Eisenstein series, goes on to the spectral decomposition of the Hilbert space of square-integrable automorphic functions with respect to the non-Euclidean Laplacian, establishing the facts about the Laplace eigenvalues. After introducing Hecke theory, he introduces the automorphic L-functions and develop analytic methods to study them. Towards the end of the paper, a Linnik-type problem for Maass forms is studied, exhibiting how analytic number theory can develop on such exotic stages.

There are two nice collections of problems by Andrzej and Igor'.

In [Schinzel] there is a collection of problems concerning the number $N(f)$ of non-zero coefficients of a polynomial $f \in K$, K being a field. f is called an $N(f)$-nomial, e. g. $x^n - a$ is a binomial while $4x^{20} + 7x^{18} + 64$ is a trinomial. Problems are about the estimate from above or below on $N(f)$. E. g. Problem 2 asks about the existence (and boundedness if it exists) of a constant $C(K)$ such that every trinomial over K has an irreducible factor f with $N(f) < C(K)$.

In [Shparlinski] many open problems are given on the estimate of exponential and character sums according to the author's taste and interest, with enlightening annotation. Problems fall into two categories; in the first category new improvements of the estimates are asked for, while the second is concerned with applications of these sums. We shall talk about the second category.

Problem 3.6 has a natural interpretation in the study of polynomially growing sequences on orbits of the dynamical system generated by the map $u \to gu$ in $\mathbb{Z}/g\mathbb{Z}$. Problem 3.11 gives the estimate for complexity of factorization algorithm for polynomials over $\mathbb{F}_p$. Problem 3.13 has applications to the study of distribution of Selmer groups of a certain family of elliptic curves. Problem 3.13 is concerned with the estimate of the exponential sum of a sparse polynomial and has applications to number theory, computer science and cryptography. Problem 3.43 is about the exponential sum of a non-linear recurrence sequence and has applications to pseudo-random number generation.

We invited Professor Yamamura to contribute his list of determinantal expressions for the class number of Abelian number fields. The theory started from the paper of Carlitz and Olson in 1955 and has been pursued constantly. The list is in the spirit of Dilcher-Skula-Slavutskii volume on Bernoulli numbers, is complete and will be useful for researchers in the relevant problems.

And we also need to add one sentence, Professor Haruo Tsukada added corrigendum to his paper published in the last proceedings.

Finally, vote of thanks is due. We would like to thank Kinki University for its generous permission of using its excellent facilities. The conference room was equipped with modern conveniences and was very useful in conducting the seminar. We would like to thank Professor Kohji Chinen for his help in preparing posters of the seminar as well as constant support in keeping the working conditions in the conference room pleasant. We would like to thank Dr. Hiromitsu Tanaka for technical help in manipulating modern devices and for his thoughtful arrangement of things. We would like to thank Sheraton-Miyako Hotel for its excellent service and hospitality; especially thanks are due to Messers H. Fujihara and K. Morimoto for their thoughtful support throughout.

As in the case of the last proceedings, Professor Jing Ma from Jilin University made a devoted help in editing and we record here our hearty thanks to her for her excellent and beautiful preparation of the manuscript of the proceedings. It was a pity that she could not attend the seminar, but she came to Japan in July, 2009 to complete the editing work.

Finally, we would like to express our hearty thanks to S. Kanemitsu's students, Mr. N. -L. Wang and Ms. X. -H. Wang for their devoted support in making the stay of foreign participants more comfortable and pleasant.

As usual we complete the preface by a poem. This time Professor Chaohua Jia composed it.