Chapter 1

Introduction

The goal of this book is to provide an extension of the previous book *Introduction to Modern Physics: Theoretical Foundations*, refered to as Vol. I. That volume develops the underlying concepts in twentieth-century physics: quantum mechanics, special relativity, and general relativity. Included in it are applications in atomic, nuclear, particle, and condensed matter physics. It is assumed in Vol. I that readers have had a good calculus-based introductory physics course together with a good course in calculus. Several appendices then provide sufficient background so that, with very few exceptions, the presentation is self-contained. Many of the topics covered in that work are more advanced than in the usual introductory modern physics books. It was the author's intention to provide the best students with an overview of the subject, so that they are aware of the overall picture and can see how things fit together as they progress.

As projected in Vol. I, it is now assumed that mathematical skills have continued to develop. In this volume, readers are expected to be familiar with multi-variable calculus, in particular, with multiple integrals. It is also assumed that readers have some familiarity with the essentials of linear algebra. An appendix is included here on functions of a complex variable, since complex integration plays a key role in the analysis. The ground rules now are that anything covered in the text and appendices in Vol. I is assumed to be mastered, while anything covered in the problems in Vol. I will be re-summarized. Within this framework, readers should again find Vol. II to be self-contained.

There are over 175 problems in this book, some after each chapter and appendix. The problems are not meant to baffle the reader, but rather to enhance the coverage and to provide exercises on working skills. The problems for the most part are not difficult, and in most cases the steps are clearly laid out. Those problems that may involve somewhat more algebra are so noted. The reader is urged to attempt as many problems as possible in order to obtain some confidence in his or her understanding of the framework of modern theoretical physics.

In chapter 2 we revisit quantum mechanics and reformulate the theory in terms of linear hermitian operators acting in an abstract Hilbert space. Once we know how to compute inner products, and have the completeness relation, we understand the essentials of operating in this space. The basic elements of measurement theory are also covered. We are then able to present quantum mechanics in terms of a set of postulates within this framework. The quantum fields of Vol. I are operators acting in the abstract many-particle Hilbert space.

Chapter 3 is devoted to the quantum theory of angular momentum, and this subject is covered in some depth. There are a variety of motivations here: this theory governs the behavior of any isolated, finite quantum mechanical system and lies at the heart of most of the applications in Vol. I;[1] it provides a detailed illustration of the consequences of a continuous symmetry in quantum mechanics, in this case the very deep symmetry of the isotropy of space; furthermore, it provides an extensive introduction to the theory of Lie groups, here the special unitary group in two dimensions SU(2), which finds wide applicability in internal symmetries. An appendix explores the use of angular momentum theory in the multipole analysis of the radiation field, which is applicable to transitions in any finite quantum mechanical system.

Chapter 4 is devoted to scattering theory. The Schrödinger equation is solved in terms of a time-development operator in the abstract Hilbert space, and the scattering operator is identified. The interaction is turned on and off "adiabatically", which allows a simple construction of initial and final states, and the S-matrix elements then follow immediately. Although inappropriate for developing a covariant scattering analysis, the time integrations in the scattering operator can be explicitly performed and contact made with time-independent scattering theory. It is shown how adiabatic damping puts the correct boundary conditions into the propagators. A general expression is derived for the quantum mechanical transition rate. Non-relativistic scattering from a static potential provides a nice example of the time-independent analysis. If the time is left in the scattering operator, one has a basis for the subsequent analysis in terms of Feynman

[1] For example, here we validate the "vector model" used there.

diagrams and Feynman rules. The tools developed in this chapter allow one to analyze any scattering or reaction process in quantum mechanics.

Lagrangian field theory provides the dynamical framework for a consistent, covariant, quantum mechanical description of many interacting particles, and this is the topic in chapter 4. We first review classical lagrangian particle mechanics, and then classical lagrangian continuum mechanics, using our paradigm of the transverse planar oscillations of a string. The string mechanics can be expressed in terms of "two-vectors" (x, ict) where c is the sound velocity in the string. We then discuss the quantization of these classical mechanical systems obtained by imposing canonical quantization relations on the operators in the abstract Hilbert space.

The appending of two additional spatial dimensions to obtain four-vectors $(\mathbf{x}, ict)$, where c is now the speed of light, leads immediately to a covariant, continuum lagrangian mechanics for a scalar field in Minkowski space, which is then quantized with the same procedure used for the string. We develop a covariant, continuum lagrangian mechanics for the Dirac field, and discuss how anticommutation relations must be imposed when quantizing in this case. A general expression is derived for the energy-momentum tensor, and Noether's theorem is proven, which states that for every continuous symmetry of the lagrangian density there is an associated conserved current. A full appendix is dedicated to the lagrangian field theory of the electromagnetic field.

Symmetries play a central role in developing covariant lagrangian densities for various interacting systems, and chapter 6 is devoted to symmetries. The discussion starts with spatial rotations and the internal symmetry of isospin, and it builds on the analysis of SU(2) in chapter 3. Here isospin is developed in terms of global SU(2) transformations of the nucleon field $\psi = (\psi_p, \psi_n)$. The internal symmetry is generalized to SU(3) within the framework of the Sakata model with a baryon field $\psi = (\psi_p, \psi_n, \psi_\Lambda)$.[2]

It is also shown in chapter 6 how the imposition of invariance under local phase transformations of the charged Dirac field, where the transformation parameter depends on the space-time point x, necessitates the introduction of a photon (gauge) field $A_\mu(x)$ and leads to quantum electrodynamics (QED), the most accurate theory known. Yang-Mills theory, which extends this idea to invariance under local internal symmetry transformations of the Dirac field, and necessitates the introduction of corresponding gauge bosons, is developed in detail. These gauge bosons must be massless, and to

[2] Wigner's supermultiplet theory based on internal SU(4) transformations of the nucleon field $\psi = (\psi_{p\uparrow}, \psi_{p\downarrow}, \psi_{n\uparrow}, \psi_{n\downarrow})$ is also touched on.

understand the very successful physical application of Yang-Mills theories, it is necessary to understand how mass is generated in relativistic quantum field theories.[3]

We do this within the framework of the σ-model, a very simple model which has had a profound effect on the development of modern physics. A massless Dirac field has an additional chiral invariance under a global transformation that also mixes the components of the Dirac field. The corresponding conserved axial-vector current, which augments the conserved vector current arising from global isospin invariance, corresponds closely to what is observed experimentally in the weak interactions. The σ-model extends the massless Dirac lagrangian through a chiral-invariant interaction with a pion and scalar field $(\boldsymbol{\pi}, \sigma)$. A choice of shape of the chiral-invariant meson potential $\mathcal{V}(\boldsymbol{\pi}^2+\sigma^2)$ then leads to a vacuum expectation value for the scalar field that gives rise to a mass for the Dirac particle while maintaining chiral invariance of the lagrangian. This *spontaneous symmetry breaking* illustrates how observed states do not necessarily reflect the symmetry of the underlying lagrangian. Generating mass through the expectation value of a scalar field, in one way or another, now underlies most modern theories of particle interactions.[4]

The most fundamental symmetry in nature is Lorentz invariance. One must obtain the same physics in any Lorentz frame. The Lorentz transformation properties of the scalar and Dirac fields are detailed in an appendix. Some very useful tools are provided in another appendix devoted to the irreducible representations of SU(n).

Chapter 7 is concerned with the derivation of the Feynman rules, and to focus on the method, they are developed for the simplest theory of a Dirac particle interacting with a neutral, massive, scalar field. Wick's theorem is proven. This allows one to convert a time-ordered product of fields in the interaction picture, where the time dependence is that of free fields, into a normal-ordered product where the destruction operators sit to the right of the creation operators for all times. It is the time-ordered product that occurs naturally in the scattering operator, and it is the normal-ordered product from which it is straightforward to compute any required matrix elements. Wick's theorem introduces the vacuum expectation value

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[3]Both quantum chromodynamics (QCD) and the Standard Model of electroweak interactions are Yang-Mills theories built on internal symmetry groups, the former on an internal color $SU(3)_C$ symmetry and the latter on an internal weak $SU(2)_W \otimes U(1)_W$.

[4]In the Standard Model, it provides the basis for the "Higgs mechanism" (see, for example, [Walecka (2004)]).

of the time-ordered product of pairs of interaction-picture fields— these are the *Feynman propagators.* An appendix provides a thorough discussion of these Green's functions, as well as other singular functions, for the scalar, Dirac, and electromagnetic fields.[5] The lowest-order scattering amplitudes, self-energies, and vacuum amplitude are all calculated for the Dirac-scalar theory, and then interpreted in terms of Feynman diagrams and Feynman rules. The cancellation of the disconnected diagrams is demonstrated in this chapter, as is the requisite procedure for mass renormalization.

In chapter 8 these techniques are applied to a theory with immediate experimental implications. That theory is quantum electrodynamics (QED), where the fine-structure constant $\alpha = e^2/4\pi\hbar c\varepsilon_0 = 1/137.04$ provides a meaningful dimensionless expansion parameter. The point of departure here is the derived QED hamiltonian in the Coulomb gauge, where $\boldsymbol{\nabla} \cdot \mathbf{A}(x) = 0$ and there is a one-to-one correspondence between the degrees of freedom in the vector potential and transverse photons. The interaction of the electron current and vector potential is combined with the instantaneous Coulomb interaction to produce a photon propagator, and then conservation of the interaction-picture current is invoked to reduce this to an effective photon propagator with a Fourier transform in Minkowski space of $\tilde{D}_{\mu\nu}(q) = \delta_{\mu\nu}/q^2$. One thereby recovers covariance and gauge invariance in the electromagnetic interaction.

The steps leading from an S-matrix element to a cross section are covered in detail in two examples, $\mu^- + e^- \to \mu^- + e^-$ and $e^+ + e^- \to \mu^+ + \mu^-$. Expressions are obtained in the center-of-momentum (C-M) frame that are exact to $O(\alpha^2)$. The scattering operator is extended to include an interaction with a specified external field, and the lowest-order amplitudes for bremsstrahlung and pair production are obtained. The Feynman diagrams and Feynman rules in these examples serve to provide us with the Feynman diagrams and Feynman rules for QED.

Chapter 9 presents an introduction to the calculation of various virtual processes in relativistic quantum field theory, and again, to keep close contact with experiment, we focus on QED. Calculations of the $O(\alpha)$ corrections to the scattering amplitude for an electron in an external field provide an introduction to the relevant lowest-order "loop" contributions, where there is an integral over one virtual four-momentum. The insertions here are characterized through the electron self-energy, vertex modification, and vacuum polarization (photon self-energy) diagrams.

[5]The neutral, massive vector meson field is covered in the problems.

Dimensional regularization, detailed in an appendix, serves as a technique that gives mathematical meaning to originally ill-defined integrals. Here one works in the complex n-plane, where n is the dimension, and any potential singularity is then isolated at the point $n \to 4$. The contribution of each of the above diagrams is cast into a general form that isolates such singular pieces and leaves additional well-defined convergent expressions.

Some care must be taken with the contribution of the self-energy insertions on the external legs ("wavefunction renormalization"), and we do so. It is then shown how Ward's identity, which relates the electron self-energy and vertex insertion, leads to a *cancellation* in the scattering amplitude of the singular parts of these insertions. Vacuum polarization then leads to a shielding of the charge in QED and to charge renormalization. The two remaining singular terms in the theory are removed by mass and charge renormalization, and if the scattering amplitude is consistently expressed in terms of the renormalized mass and charge (m, e) one is left with finite, calculable, $O(\alpha)$ corrections to the scattering amplitude. The Schwinger term in the anomalous magnetic moment of the electron is calculated here. Higher-order corrections are summarized in terms of Dyson's and Ward's equations, and it is demonstrated through Ward's identities how the multiplicative renormalizability of QED holds to all orders.

With the techniques developed in chapter 9, one has the tools with which to examine loop contributions in any relativistic quantum field theory.

Chapter 10 is on path integrals. There are many reasons for becoming familiar with the techniques here, which underly much of what now goes on in theoretical physics, for example: this approach provides an alternative to canonical quantization, which, with derivative couplings, can become prohibitively difficult; here one deals entirely with classical quantities, in particular the classical lagrangian and classical action; and the classical limit $\hbar \to 0$ leads immediately to Hamilton's principle of stationary action.

We start from the analysis of a non-relativistic particle moving in a potential in one dimension and show how the quantum mechanical transition amplitude can be exactly expressed as an integral over all possible paths between the initial and final space-time points.[6] We then make the transition to a system with many degrees of freedom, and then to field theory.

The addition of an arbitrary source term, together with the crucial theorem of Abers and Lee, allows one to construct the generating functional as a ratio of two path integrals, one a transition amplitude containing the

[6]It is shown in a problem how the partition function of statistical mechanics in the microcanonical ensemble can also be expressed as a path integral.

source and the second a vacuum-vacuum amplitude without it. The connected Green's functions can then be determined from the generating functional by functional differentiation with respect to the source, as detailed here. The generating functional is calculated for the free scalar field using gaussian integration, and it is shown how the Feynman propagator and Wick's theorem are reproduced in this case. The treatment of the Dirac field necessitates the introduction of Grassmann variables, which are anti-commuting c-numbers. The generating functional is computed for the free Dirac field, and the Feynman propagator and Wick's theorem again recovered. It is shown how to include interactions and express the full generating functional in terms of those already computed.

An appendix describes how one uses the Faddeev-Popov method in a gauge theory, at least for QED, to factor the measure in the path integral into one part that is an integral over all gauge functions and a second part that is gauge invariant. With a gauge-invariant action, the path integral over the gauge functions then factors and cancels in the generating functional ratio. It is shown how the accompanying Faddeev-Popov determinant can be expressed in terms of ghost fields, which also factor and disappear from the generating functional in the case of QED. The generating functional for the free electromagnetic field is calculated here.

Although abbreviated, the discussion in chapter 10 should allow one to use path integrals with some facility, and to read with some understanding material that starts from path integrals.

The final chapter 11 deals with canonical transformations for quantum systems. Chapter 11 of Vol. I provides an introduction to the properties of superfluid Bose systems and superconducting Fermi systems. In both cases, in order to obtain a theoretical description of the properties of the quantum fluids, it is necessary to include interactions. A technique that has proven invaluable for the treatment of such systems is that of canonical transformations. Here one makes use of the fact that the properties of the creation and destruction operators follow entirely from the canonical (anti)commutation relations in the abstract Hilbert space. By introducing new "quasiparticle" operators that are linear combinations of the original operators, and that preserve these (anti)commutation relations, one is able to obtain exact descriptions of some interacting systems, both in model problems and in a starting hamiltonian.

The problem of a weakly interacting Bose gas with a repulsive interaction between the particles is solved with the Bogoliubov transformation. A phonon spectrum is obtained for the many-body system, which, as shown in Vol. I, allows one to understand superfluidity. Motivated by the Cooper pairs obtained in Vol. I, a Fermi system with an attractive interaction between those particles at the Fermi surface is analyzed with the Bogoliubov-Valatin transformation. The very successful BCS theory of superconductivity is obtained in the case that the residual quasiparticle interactions can be neglected.

A problem takes the reader through the Bloch-Nordsieck transformation, which examines the quantized electromagnetic field interacting with a specified, time-independent current source. A key insight into the infrared problem in QED is thereby obtained. A second problem guides the reader through the analysis of a quantized, massive, neutral scalar field interacting with a classical, specified, time-independent source. The result is an exact derivation of the Yukawa interaction of nuclear physics.

This book is designed to further the goals of Vol. I and to build on the foundation laid there. Volume II covers in more depth those topics that form the essential framework of modern theoretical physics.[7] Readers should now be in a position to go on to more advanced texts, such as [Bjorken and Drell (1964); Bjorken and Drell (1965); Schiff (1968); Itzykson and Zuber(1980); Cheng and Li (1984); Donoghue, Golowich, and Holstein (1993); Merzbacher (1998); Fetter and Walecka (2003a); Walecka (2004); Banks (2008)], with a deeper sense of appreciation and understanding.

Modern theoretical physics provides a basic understanding of the physical world and serves as a platform for future developments. When finished with this book, readers should have an elementary working knowledge in the principal areas of theoretical physics of the twentieth-century.

[7] The author considered also including in Vol. II a chapter on solutions to the Einstein field equations in general relativity; however, given the existence of [Walecka (2007)], it was deemed sufficient to simply refer readers to that book.