

# Preface

This small volume introduces several important methods for calculating approximations to the periodic solutions of “truly nonlinear” (TNL) oscillator differential equations. This class of equations take the form

$$\ddot{x} + g(x) = \epsilon F(x, \dot{x}),$$

where  $g(x)$  has no linear approximation at  $x = 0$ . During the past several decades a broad range of calculational procedures for solving such differential equations have been created by an internationally based group of researchers. These techniques appear under headings such as

- averaging
- combined and linearization
- harmonic balance
- homotopy perturbation
- iteration
- parameter expansion
- variational iteration methods.

Further, these methodologies have not only been applied to TNL oscillators, but also to strongly nonlinear oscillations where a parameter may take on large values. Most of these techniques have undergone Darwinian type evolution and, as a consequence, a large number of papers are published each year on specializations of a particular method. While we have been thorough in our personal examination of the research literature, only those papers having an immediate connection to the topic under discussion are cited because of the magnitude of existing publications and because an interested user of this volume can easily locate the relevant materials from various websites.

We have written this book for the individual who wishes to learn, understand, and apply available techniques for analyzing and solving problems involving TNL oscillations. It is assumed that the reader of this volume has a background preparation that includes knowledge of perturbation methods for the standard oscillatory systems modeled by the equation

$$\ddot{x} + x = \epsilon F(x, \dot{x}).$$

In particular, this includes an understanding of concepts such as secular terms, limit-cycles, uniformly-valid approximations, and the elements of Fourier series.

The basic style and presentation of the material in this book is heuristic rather than rigorous. The references at the end of each chapter, along with an examination of relevant websites, will allow the reader to fully comprehend what is currently known about a particular technique. However, the reader should also realize that the creation and development of most of the methods discussed in this book do not derive from rigorous mathematical derivations. This task is a future project for those who have the interests and necessary background to carry out these procedures. However, these efforts are clearly not relevant for our present needs.

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The book consists of seven chapters and several appendices. Chapter 1 offers an overview of the book. In particular, it presents a definition of TNL equations, introduces the concept of odd-parity systems, and calculates the exact solutions to four TNL oscillatory systems.

Chapter 2 provides a brief discussion of several procedures for a priori determining whether a given TNL differential equation has periodic and/or oscillatory solutions. The next four chapters present introductions to most of the significant procedures for calculating analytical approximations to the solutions of TNL differential equations. These chapters discuss, respectively, harmonic balance, parameter expansion, iteration, and averaging methods. Each chapter gives not only the basic methodology for each technique, but also provides a range of worked examples illustrating their use.

The last chapter considers six TNL oscillator equations and compares results obtained by all the methods that are applicable to each. It ends with general comments on TNL oscillators and provides a short listing of unresolved research problems.

We also include a number of appendices covering topics relevant to understanding the general issues covered in this book. The topics discussed range from certain mathematical relations to basic results on linear second-order differential equations having constant coefficients. Brief presentations

are given on Fourier series, the Lindstedt-Poincaré perturbation method, and the standard first-order method of averaging. A final appendix, “Discrete Models of Two TNL Oscillators,” illustrates the complexities that may arise when one attempts to construct discretizations to calculate numerical solutions.

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*Ronald E. Mickens*  
Atlanta, GA  
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