

# Preface

This text is devoted to game theory and its various applications. With the aim of being accessible to the widest audience the text is clearly separated into two parts with different levels of exposition.

1. The first part (Chapters 1-8) give an elementary but systematic exposition of the main ideas of modern game theory without any special prerequisites in mathematics (secondary school level should be quite sufficient). It requires from a reader only an inclination towards logical thinking, equations, some calculations, and an acceptance of the Greek letters  $\epsilon$  (epsilon),  $\delta$  (delta),  $\sigma$  (sigma),  $\eta$  (eta) and  $\Pi$  or  $\pi$  (pi). Nevertheless, not following the tradition of popular science books, our presentation is concise, with rigorous definitions, quantitative as well as qualitative results. On the other hand, due to the elementary nature of this presentation, it often stops just where really serious analysis begins, giving (as a compensation) lots of references for further developments.

2. The second part is devoted mostly to the mathematical methods of the theory. Having carefully separated from the first part of this exposition all higher mathematics, we give in Chapter 9 a concise presentation of mathematical aspects and techniques of game theory. This is supplemented by examples and exercises needed to get used to these techniques and to learn to apply them. The level of mathematics required in Chapter 9 is higher than in earlier chapters, but assumes an acquaintance with only basic notions of calculus, differential equations, linear algebra and probability. Chapter 10 presents several concrete (mostly original) game theoretic models and their analysis including game theoretic treatment of rainbow options in financial mathematics, advanced models of inspection, price war games, investment competitions, etc. The sections of this chapter are supplied with problems that can be used as a starting point for individual projects

students. The material from Chapters 1-9 fertilized by chosen models from Chapter 10 can serve as a basis for an introductory undergraduate course in game theory for both mathematics related degrees and university degrees with a minimal mathematics background (business, economics, biology, etc.). The last Chapter 11 is meant mostly for mathematics graduate and postgraduate students (and could be of interest to researchers), as it requires in places some mathematical culture, for instance some knowledge of functional analysis and stochastic processes. This chapter is devoted to selected topics of more advanced analysis (partially reflecting the authors' interests and research) including differential geometry approach (transversality and catastrophe) to stability, abstract dynamic system approach to the analysis of differential games, Bellman type equations for multi-criteria optimization, turnpikes for stochastic games, connections with recently becoming popular tropical (or idempotent) mathematics, as well as with statistical physics (interacting particles). Chapter 9 with chosen parts of the last two chapters can be used for various advanced courses on the mathematical methods of game theory.

The text is aimed primarily at undergraduate students and instructors in game theory as well as at postgraduates in mathematics, system biology and social sciences. We have tried to provide an entertaining and easy read for those wishing to get acquainted quickly with (or to refresh their knowledge of) the beauty of the basic ideas of the game theory, its wide range of applicability and some recent developments. We aimed at helping to make teaching and learning a more interesting and exciting process, supplementing a course by a variety of motivating examples, historical, cultural and general science excursions.

The authors believe that due to the elementary character of the first part, the book can be used also (i) by laymen (e.g. businessmen, politicians and everyone interested in scientific problems and methods), as it introduces rigorous quantitative methods that can be helpful for an assessment of a wide range of human interactions and provides some glimpses of relevant problems from biology, economics and psychology; and (ii) by teenagers, as it is aimed to interest them in the problems of science in general, to involve them in the process of logical thinking and to stimulate their interest in mathematics.

With game theory becoming popular, there is a variety of good textbooks on the theory. As examples of accessible and extensive general introductions one can recommend books [25] and [58]. For nonspecialists, introductions to particular areas of theory and applications (well reflected

in their titles) one can refer to [2], [12], [21], [24], [29], [30], [31], [32], [33], [39], [40], [42], [47], [51], [53], [56], [57], [135], [139], [143], [144], [145], [160], [180], [181] and [191]. More mathematically oriented introductions can be found in [14], [36], [43], [142], [179], [185] [156], [195] and [129]. The special character of the present text is the clear separation of elementary and more advanced material, the wide covering of the theme in combination with a concise and rigorous exposition, which at the same time is generously spiced with relevant glimpses from literature and history, and finally the discussion of some advanced topics that have not yet found their place in textbooks but have a potential to become a part of the scientific culture of the future.

Almost all chapters of the first part can be read independently (only some general notions introduced in Chapter 1 are used repeatedly). So, according to their wishes and tastes, readers can start from thinking about choosing the best president or prime minister in Chapter 6, or looking through Chapter 1 and then going to explore the biological context in Chapter 4, or touching the curious quantum world in Chapter 7, or going directly to the party games of Chapter 8. On the other hand, the material in each chapter is carefully organized in a logical order and it is advisable to read each chapter from the beginning (the exception being Chapters 10, 11, whose sections are devoted to various topics and perspectives).

The first part of this book is largely based on the Lecture Notes [88] prepared for the students of the Nottingham Trent University. Further work of the authors resulted in the Lecture Notes [99] aimed primarily at the students of St. Petersburg University. The present book is based on the best parts of [88] and [99], fully revised and updated.

To stimulate mathematical and scientific imagination and to add charm to the book, we illustrate it by carefully selecting artistic graphics of a world renowned mathematician and mathematics imaging artist A.T. Fomenko (to whom the authors express their deepest gratitude for allowing the use his works in this text). Though these works were originally designed to illustrate the geometric structure of the Universe, they fit nicely in the circle of ideas dealt with here. Readers who would like to see more of these graphics are referred to the album [55].

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