Preface

This book is designed as an investigation of certain aspects of quantum mechanics and general relativity. The approach is based largely on equations and mathematical structures from which one is able to extract some "meaning" in physics (whence the title). The mathematics can be regarded as a basically simple language (it can be self taught for example) but questions of physical meaning are often unclear, or inadequately developed in the mathematical context chosen. Partly for this reason mathematics is itself a developing language of course and it is often enlarged to handle emergent physical phenomena, as well as feeding on itself for development. In any event various features or aspects of physics are displayed, based on recent literature and the author's interpretation of these matters. Some original work is included, based on [**170, 171, 172, 173, 179, 185**], relating the Bohmian quantum potential to gravity and Ricci flow, but generally we will extract from known results and conjectures with explicit references to the sources (although we will not trace historical origins). Much of the material is taken from the electronic bulletin boards at arXiv.org and publication information can often be found there if it is omitted in the bibliography here; we hope to have portrayed a small amount of the range, versatility, and significance of net activity. The material is more or less up to date in a number of areas and hopefully will contribute to further discoveries. As this seems to be an age of emergence, this theme will be emphasized in various places, especially in the quantum-classical spirit, but also e.g. in the connections between gravity and thermodynamics. The book is written in the language of mathematics and the interaction with physics, along with the resulting emergent points of view, are best expressed via equations. Given that the Schrödinger equation can arise from e.g. subquantum oscillations, or diffusion, or hydrodynamics it still is not quantum mechanics since there remains a firm linking of quantum mechanics to eigenvalues and other algebraic notions (e.g. C^* algebras and K-theory). There still seem to be many mysteries which we will not try to discuss involving the vacuum, the cosmological constant, dark energy and matter, etc. We will devote considerable attention to Bohmian mechanics and to semi-classical theory as well since these arenas interact with both classical and quantum regimes. In particular we will emphasize certain aspects of the quantum potential (QP) whose basic nature is best characterized via the quantum equivalence principle QEP) of Faraggi-Matone (cf. [**97, 170, 297, 589, 590**] and see also Section 1.3 and Chapter 8). We also discuss phase space techniques involving symplectic ideas which involve quantum mechanics via the metaplectic group for example. On the other hand the development of non-commutative geometry

(NCG), via C^* algebras, K-theory, etc. involves a minimal coupling of quantum mechanics and gravity, and firmly establishes quantum mechanics with its algebraic structure as a seriously fundamental theory. We sketch this in Chapter 7 and indicate a little bit of the magnificent edifice involving the standard model and non-commutative geometry. Deep algebraic connections of quantum mechanics to number theory and algebraic geometry are known and flourishing and we will mention "arithmetic" at various times with the faintest suggestion that meaning (in a fundamental sense) really lies in number theory.

Thermodynamic ideas have assumed a definitive role in many areas of physics (including gravity) and suitable adjustments have to be made in partition functions for example when dealing with quantum particles (due to Bose-Einstein or Fermi-Dirac statistics, Pauli exclusion principles, etc.). Information theory can also be given a prominent role in many areas of physics and there is an emerging quantum information theory of possible use in computing as well (see e.g. [**146, 147, 197, 199, 547**]). In any event the SE applies to many situations where it can be modeled on momentum perturbations $\delta p = \nabla|\psi|/|\psi$ (where ψ satisfies the SE) and corresponding position perturbations δq will involve the Heisenberg inequality. This is lovely and can be obtained via Bohmian mechanics and there is no need to postulate any discrete spectrum of energies, or Hilbert space, or C^* algebras, etc. So the Schrödinger and/or Heisenberg pictures have some meaning independent of the algebraic structure arising from atomic frequencies, etc. There is also the idea of quantum chaos which can be discussed in various manners and we consider semi-classical methods leading to trace formulas à la Gutzwiller (cf. [**396**]). This again suggests number theory, zeta functions, etc.

The work of Dürr, Goldstein, and Zanghi, in collaboration with e.g. Berndl, Dauman, Peruzzi, Struyve, and Tumulka, (see e.g. [**274, 364**]) has been very important in developing the deBroglie-Bohm theory, and we mention also fundamental contributions of Bacciagaluppi ([**56**]), Brown ([**132, 133**]), deGosson ([**369**]), Hiley ([**436**]), Holland ([**440**]), Valentini ([**848, 849, 850**]), etc. (cf. [**173**] for more references). Questions of ontology and epistimology will be largely ignored (except when citing results) but it is hoped that a philosophical flavor arises concerning "meaning" as arising from the mathematics of physics and perhaps the physics of mathematics. Generally it is clear that Bohmian mechanics is not the same as quantum mechanics and the quantum potential is best viewed via the QEP (see also comments below on trajectories) but the Bohmian approach has many extraordinary features applying to both the Schrödinger and Heisenberg pictures which lead to results of major interest (the Bell inequalities are a non-issue for us). We refer also to [**95, 137, 145, 187, 213, 406, 502, 597, 778, 903**] for information on time-energy uncertainty. In dealing with trajectories however there are striking differences between dBB trajectories and the classical ones and this is revealed beautifully in papers of Matzkin, Lombardi, and Nurok (cf. [**593, 594, 595**]). The dBB trajectories tend to follow the streamlines of the probability flow and are generically nonclassical but the Floydian type trajectories stemming from the QEP are generically quantum in nature (cf. [**177, 187, 297, 310, 311, 312, 590, 890**]).

Semiclassical systems are quantum systems that display the manifestation of classical trajectories; the wave function and the observable properties of such systems depend on the trajectories of the classical counterpart of the quantum system. Thus we will study semiclassical systems and phase space methods in some detail including the Gutzwiller trace formula (cf. also [**360, 361, 362**]). There is an interesting occurance of the Schwarzian derivative arising in [**595**] and the Schwarzian derivative is a fundamental object in the beautiful theory of Bertoldi, Farragi and Matone et al about the quantum equivalence principal (cf. [**97, 98, 177, 175, 176, 297, 589, 590**]). In this theory quantization (and eigenvalues) arise via the quantum stationary Hamilton Jacobi equation (QSHJE) which characterizes quantum solutions of the Schrödinger equation (SE). The Gutzwiller trace formula connects quantum energy eigenvalues with periodic orbits of the associated classical system and the techniques involve the metaplectic representation; however we do not try to cover the related arithmetic features of quantization which are entangled with the Riemann zeta function, the Selberg trace formula, etc. (see e.g. [**25, 128, 231, 233, 423, 505, 576, 749, 808, 809, 847**]) and recall the NCG approach leading also to zeta functions, etc. Given that Bohmian trajectories do not seem to correspond generally to classical ones while, via Gutzwiller, classical orbits are connected with the arithmetic of quantum mechanics (via eigenvalues), there may be no way to extract arithmetic information from the Bohmian theory (see [**36, 37, 360, 361, 362, 364, 557, 593, 594, 595, 597, 713, 781, 890, 903**] for more on trajectories). There is also a strong connection of the Bohmian quantum potential to Fisher information and thence to Riemannian geometry as developed in Chapter 2 and the question can arise of seeking suitable arithmetic in Riemannian geometry (cf. [**97, 98, 591, 592**]).

Chapter 2 contains some original work of the author on the quantum potential, Weyl geometry, and Ricci flow extracted from [**170, 172, 173, 185**] for example. There are also results in Chapter 1 concerning the quantum potential, Fisher information, and thermalization based on the author's work ([**173**]) as well as some comments on stability and trajectories. We remark that in the process of citing information from various sources we will often use the appelation "one" to refer to the author or authors of the source in question (instead of boringly repeating names or citations). The source is always clearly indicated and this practice involves no proprietary claim on my part to the information. Accurate rendering of mathematical or physical arguments can often involve repetition of words as well as equations. Chapter 3 (Sections 1-2) and the first two sections of Chapter 6 deal with some work of Elze, t'Hooft, Isidro, et al on emergence (cf. [**2, 290, 444, 445, 446, 456, 457, 458, 459, 461, 462**]). This becomes involved with Ricci flow and Perelman entropy (cf. [**170, 172, 506, 507, 621, 693, 836**]. Section 3 of Chapter 3 is deals with work of H. Yang et al involving gravity as a collective phenomenon emerging from gauge fields of electromagnetism living in a fuzzy space-time (cf. [**893, 894, 895, 896**]). Chapter 4 is about Kaluza-Klein and 5 dimensional physics and there is material on electrodynamics, induced matter, the zero point field, Klein-Gordon and Dirac equations, etc. following Mashoon, Ponce de Leon, Wesson, et al (cf. [**585, 586, 717, 718, 719, 868, 869, 870,**

871, 872] etc.). Chapter 5 (Sections 1-5) is about connections of thermodynamics and gravity and gives a derivation of the Einstein equations from entropy ideas following Padmanabhan et al; this is later expanded enormously in Sections 3-5 of Chapter 6 (see e.g. [**15, 16, 149, 178, 471, 474, 476, 523, 524, 525, 620, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 776, 860, 861**]). The theory is developed beyond GR into Lanczos-Lovelock gravity for example and provides a gravitational connection to thermodynamical laws. In Sections 6-8 of Chapter 5 we sketch some work of L. Glinka et al on thermodynamics and quantum gravity (cf. [**353, 354, 355, 356, 357, 358**]). This involves bosonic strings and quantum field theory (QFT) and is speculative (but very interesting). Chapter 7 starts with some ideas about emergent time and the Connes-Rovelli approach (see [**234**]) via C^* algebras compels us to pursue these themes via NCG to arrive at the Chamseddine-Connes-Kreimer-Marcolli universe where the standard model and gravity are united in NCG. Only a brief sketch of this is given for obvious reasons and we refer to [**205, 206, 207, 208, 230, 231, 232, 233, 236, 576**] for details. In Section 4 we sketch some work of Singh on noncommutative geometry and Klein-Gordon equations; this is followed by a sketch of recent results of Jejjala, Kavic, Minic, and Tze on quantum mechanics, gravity, time, and geometry. Chapter 8 is first a summary of material on gravity and the quantum potential (with some repetition from previous chapters). Then, following Matone [**590**] the QEP is shown to generate quantization via the QSHJE.

In this book I have tried to bring together many different "strings" of mathematics and physics involving many points of view and many authors (incidentally string theory and loop quantum gravity are not covered and it is assumed in a few places that very elementary string theory is known - see [**716**] for strings and [**509, 750, 831**] for quantum gravity). In particular, in various sections it has been indicated how many originally distinct concepts are related to one another at a fundamental level (often via mathematical identifications) or how a theory "emerges" or "arises" from another theory via some manipulations or some added factors (such as information loss). It is hoped that this exposure will stimulate new insights, combinations, and developments. The formulas and equations may seem daunting at times but the mathematical topics are easily available in books and university courses.

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