Introduction

Symmetries are of fundamental importance in the description of physical phenomena. In the realm of particle physics symmetries are believed to permit ultimately a classification of all observed particles. A fundamental symmetry of particle physics, which has been firmly established both theoretically and experimentally is that of the Poincaré group, *i.e.* of rotations and translations in four-dimensional Minkowski space. Besides this fundamental symmetry there are other so-called internal symmetries (such as the symmetry of the $SU(3)$ flavor group) which have also been firmly established over the last few decades, although their manifestation in Nature is not exact. As is well known, the consistent search for more fundamental symmetries led to the development of non-Abelian gauge theories and the spectacular experimental confirmation of several predictions of the latter in recent years.

In the course of time several attempts have been made to unify the spacetime symmetry of the Poincaré group with the symmetry of some internal group ([22], [81], [82], [83]). Such attempts have, however, been shown to be futile if the theory, which necessarily has to be a quantum field theory, is expected to satisfy certain basic requirements. In fact, the so-called "no-go"-theorem of S. Coleman and J. Mandula [23] shows that if one makes the plausible assumptions of locality, causality, positivity of energy and finiteness of the number of particles (and one more technical assumption) the invariance group of the theory can at best be the direct product of the Poincaré group and a compact internal group, and this therefore does not offer a genuine unification of one group with the other.

The generators of the Poincaré group satisfy well known commutation relations and Noether's theorem relates these to conserved currents. In their turn these conserved currents are functions of relativistic fields. The commutation relations of the field operators which quantize these fields are therefore directly related to those of the generators. It was realized by J. Wess and B. Zumino [124], [125] that if one allows also anticommutation relations of generators of supersymmetry transformations which transform bosons into fermions and *vice versa*, then the unification of the spacetime symmetries of the Poincaré group with this internal symmetry can be achieved. The formal proof of this discovery, *i.e.* the proof that anticommuting generators which respect the other assumptions of the theorem of S. Coleman and J. Mandula [23] do exist, was established by R. Haag, J.T. Łopuszański and M.F. Sohnius [54].

Supersymmetry thus arises as a symmetry which combines bosons and fermions in the same representation or multiplet of the enlarged group which encompasses both the transformations of the Poincaré group and the appropriate supersymmetry transformations. Thus every bosonic particle must have a fermionic partner and *vice versa*. In view of the fact that such a spectrum of particles is not compatible with observation, supersymmetry must be badly broken at the level of presently available energies. Clearly only experimental observation can decide whether supersymmetry is indeed inherent in Nature. It can be argued that one of the most immediate ways to observe evidence of supersymmetry is to see if there is a missing energy and momentum in the final e^+e^- spectrum of the reaction

$$e^+ + e^- \longrightarrow \gamma \longrightarrow \widetilde{e}^+ + \widetilde{e}^- \longrightarrow e^+ + e^- + \widetilde{\gamma} + \widetilde{\gamma}$$

where $\tilde{e}^+, \tilde{e}^-$ and $\tilde{\gamma}$ are the supersymmetric partners of e^+, e^- and γ respectively. If there is such a missing energy and momentum it could be that carried away by the neutral photino $\widetilde{\gamma}$. Charged supersymmetry particles at energies presently available would have been detected long ago. Since supersymmetry must be broken, the photinos $\widetilde{\gamma}$ would not be massless.

However, supersymmetry does not only open the possibility of a much more complex spectrum of particles than heretofore envisaged; supersymmetry also has some intriguing theoretical consequences which could make it a desirable theory. It is well kown that a realistic quantum field theory in the traditional sense is plagued by the problem of ultraviolet divergences and the consequent necessity of renormalization. Supersymmetry, however, provides a mechanism for the cancellation of such divergences in view of the same number of bosonic and fermionic degrees of freedom in each particle multiplet. Clearly, such a built-in cancellation of divergent terms is a highly desirable feature of a quantum field theory.

In Chapter 1 we begin with a recapitulation of basic aspects of the Lorentz group, including a discussion of Casimir operators and the classification of representations in terms of their eigenvalues. We then consider the group $SL(2,\mathbb{C})$ and its basic representations, *i.e.* the self-representation and the complex conjugate self-representation. The elements of the appropriate representation spaces are the undotted and dotted Weyl spinors. In view of the importance of Weyl spinors throughout the entire text, we consider these here in more detail than is generally done in the literature. We then introduce the concept of Grassmann numbers and perform some basic manipulations involving Weyl spinors, thereby deriving a number of useful formulas. In the subsequent section the connection between the special linear group $SL(2, \mathbb{C})$ and the proper orthochronous Lorentz group is established. It is then natural to discuss four-component Dirac spinors and the Weyl representation. The connection with two-component Weyl spinors is obtained by introducing four-component Majorana spinors. Then again various formulas are derived which are useful in later calculations.

Chapter 2 begins with a discussion of the "no-go" theorems of Coleman and Mandula [23] and Haag, Łopuszański and Sohnius [54]. The latter leads to a consideration of graded Lie algebras which we approach in successive steps by defining first the characteristics of a Lie algebra, then those of a graded algebra and finally those of a graded Lie algebra, *i.e.* the properties of *grading*, *supersymmetrization* and *generalized Jacobi identities*. As an example we construct the graded Lie algebra of the algebra $su(2, \mathbb{C})$. The final section of Chapter 2 deals with graded matrices and their properties.

The following Chapter 3 deals with the grading, *i.e.* supersymmetrization of the Poincaré algebra. We demonstrate explicitly that for the grading chosen all possible Jacobi identities are satisfied. This turns out to be a crucial point of consistency of a grading. Having established the algebra of the Super–Poincaré group with the fermionic generators in the Dirac four-component form, we then decompose it into the appropriate relations of the two-component Weyl formalism.

In Chapter 4 we use the method of Casimir operators to classify the irreducible representations of the Super–Poincaré algebra, and it is shown that supersymmetry implies an equal number of bosonic and fermionic degrees of freedom.

Chapter 5 deals with the most immediate field theoretical realization of the Super–Poincaré algebra, the *Wess–Zumino model*, which is a field theory involving a scalar field, a pseudoscalar field and one spinor field, all with the same mass. We demonstrate by explicit calculation that the spinor charges of the theory, considered as linear operators in Fock space, satisfy the commutation and anticommutation relations of the Super–Poincaré algebra.

In Chapter 6 we introduce the concepts of superspace and superfields, and define differentiation with respect to Grassmann numbers. Then three different but related operators are constructed which describe three different but equivalent actions of the supersymmetry group on functions in superspace. These operators define three different types of superfields. By considering infinitesimal supersymmetry transformations we obtain the corresponding three differential operator representations of the fermionic generators of the

Super–Poincaré group. Then covariant derivatives are introduced as a prerequisite for the construction of manifestly supersymmetric action integrals. These covariant derivatives also permit the definition of projection operators. The search for irreducible representations of the Super–Poincaré algebra then becomes a search for solutions of constraint equations expressed in terms of these projection operators. The final section of Chapter 6 is devoted to the derivation of the explicit supersymmetry transformations of the component fields of the supermultiplet. In this context it is seen that the highest order component field always transforms into a total Minkowski derivative and thus is a candidate for a supersymmetric Lagrangian density.

In Chapter 7 we begin with an investigation of the constraint equations which define left-handed and right-handed chiral superfields (also known as scalar superfields). Then vector superfields are defined by an appropriate constraint equation, and the supersymmetric generalization of the Abelian gauge transformation is discussed. Finally, left-handed and right-handed spinor superfields are discussed which represent the components of the supersymmetric field strength for an arbitrary vector superfield.

Chapter 8 deals with the construction of supersymmetric action integrals. We begin with the definition of integration over Grassmann numbers. Then Lagrangians are constructed from scalar superfields and from vector superfields (*i.e.* the supersymmetric field strength). The case of the former is shown to contain the Wess–Zumino model as a special case, whereas the case of the latter yields the supersymmetric generalization of the pure Maxwell theory (*i.e.* with no interaction with matter fields) which contains in addition to the massless vector field also the massless spinor field of the photino.

Chapter 9 deals with the spontaneous breaking of supersymmetry. For the convenience of discussions the concept of superpotential is introduced. In view of the necessity of evaluating action integrals over superspace an equivalent and convenient Grassmann projection technique is developed. Some general aspects of spontaneous symmetry breaking are then discussed and, in particular, the Goldstone theorem is established for the general case of the breaking of supersymmetry and some other symmetry. Finally, the O'Raifeartaigh model, which is a specific theory involving three scalar superfields, is considered and the spectrum resulting from the spontaneous breaking of supersymmetry is investigated. In this case supersymmetry breaking results from the nonvanishing vacuum expectation value of some auxiliary field of a superfield.

Finally, in Chapter 10, we consider supersymmetric gauge theories. Introducing first global and local $U(1)$ gauge transformations of scalar superfields and the corresponding supersymmetric version of minimal coupling, we consider super quantum electrodynamics. We then investigate the Fayet–Iliopoulos mechanism of spontaneous breaking of supersymmetry in which the latter results from the nonvanishing vacuum expectation value of the highest order component of a vector superfield. The last section contains a brief introduction to non-Abelian gauge transformations for superfields with the appropriate tensorial transformation properties.