Introduction

The study of the structure of the Earth and near-earth space requires simultaneous processing and interpretation of a large volume of experimental data. The presence of a random error, which one is inevitable at measurings, and the presence of a random noise in the wave fields, which distorts useful signals, leads to a necessity to use the probability-statistical methods for the analysis and interpretation of the geophysical information (Goltsman, 1971; Ryzhikov and Troyan, 1994; Troyan, 1982; Troyan and Sokolov, 1989; Fedorov, 1972; Sheriff and Geldart, 1995; Troyan and Kiselev, 2000). The digital recording of the geophysical fields and the computer-aided data processing with the subsequent interpretation now is widely used. The formalization of the interpretation process requires the links of a measured field with the parameters and a state of an investigated object, that is called frequently as the model of an experimental material. The similar model includes the scheme of the experiment together with the formalization of links of an observable geophysical field with parameters and the state of the medium, and also the presence of a random deviation between the experimental field and "idealized" one obtained as a result of a solution of the direct problem. The main goal of the processing and interpretation of the geophysical data consists in the restoration of desired parameters and the state of the medium. For the solution of this intricate problem it is necessary to use a maximum of a priori information about the medium, which can be obtained on the basis of the previous experiments. The systems of the processing and interpretation of the geophysical data in practice, as a rule, are on-line, that allows along with strict mathematical methods and routines as an unformalized device to include an intuition and an experience of the geophysicist.

The suggested textbook is based on the courses of lectures "Statistical methods of processing of the geophysical data" and "Inverse problems in geophysics", which are given by the authors for master's and post-graduate students of the Chair of Physics of the Earth at Physical Faculty of Saint Petersburg State University during last ten years.

In the first chapter the basic concepts of the probability theory are given. The space of elementary events and the relative-frequency, classic and geometrical definitions of probability are introduced. The formula of the total probability and

the Bayes formula are given (Populis and Pillai, 2002; Cramer, 1946; Kolmogorov, 1956; Pugachev, 1984). The cumulative distribution function is introduced and its properties are considered (Brandt, 1998; Pugachev, 1984). Numerical characteristics of a distribution of probabilities are analyzed in details: an expectation, a variance, a coefficient of correlation, a median, the initial and central moments, an asymmetry coefficient and an excess. The characteristic functions and their properties are considered. The limit theorems of the probability theory are given (Populis and Pillai, 2002; Cramer, 1946). Various types of probability distributions are considered: binomial, Poisson, geometrical, normal, uniform, Student, Fisher, exponential, Laplace, Cauchy, logarithmic normal, χ^2–distribution etc. (Pugachev, 1984; Rao, 1972). The concept of the entropy and the information is introduced. The informations of Shannon and Fisher are considered; the possibilities of their use for an exposition of the interpretation quality of the geophysical data are analyzed (Rao, 1972). The properties of random functions are given. The autocorrelation and cross-correlation functions are introduced. The connection of the autocorrelation function with the power spectrum is considered (Pugachev, 1984).

The second chapter is devoted to an account of basic elements of the mathematical statistics. The basic concepts of the theory of decisions are introduced: structure of a decision space, a loss function, a resolution rule and sufficient statistics. The attention to the properties of estimates (consistency, bias, effectiveness, sufficiency, normality, robustness) are given. The examples of an estimation of the accuracy and reliability of the interpretation of geophysical fields are surveyed (Johnson and Lion, 1977; Goltsman, 1971; Cramer, 1946; Nikitin, 1986; Pugachev, 1984; Troyan and Sokolov, 1989; Fedorov, 1972).

In the third chapter the concept of the model of the measurement data is introduced. This model is a functional relationship between the observations and with the state and parameters of an investigated medium. The random noise is a very important part of the model. The distinctive feature of the statistical theory of interpretation is the assumption about a stochastic nature of an observed geophysical field. By depending on the statement of problem and purpose of interpretation, the models of an experimental material are subdivided into the quantitative interpretation, when the problem consists in a determination of the estimates of the desired parameters of a medium, the qualitative interpretation, when the problem consists in a choice of a state of the object (test of hypothesis) and the qualitative-quantitative interpretation, when the parameters and the state of the object are estimated simultaneously. The important points of a description of the model are the representation of properties of the random component and taking into account correctly of a priori information about properties of an investigated medium (Goltsman, 1971; Troyan, 1982; Troyan and Sokolov, 1989).

The fourth chapter is devoted to the description of the perfect relationship of the sounding signals (geophysical fields) with the parameters of a medium (examples of the solution of the direct geophysical problem). The elastic wave fields, which are

used for the reconstruction of the Earth structure in the problems of the seismology and seismic exploration are surveyed in details. The basic equations of acoustics used at study the oceanic column and oceanic sedimentary cover are introduced. The mathematical model of propagation of the electromagnetic signals in an earth-crust and ionosphere is described. The transport equation for the problem of remote sensing of an atmosphere is introduced (Aki and Richards, 2002; Jensen *et al.*, 2000; Alekseev *et al.*, 1983; Kravtsov and Apresyan, 1996; Brekhovskikh and Godin, 1998; Brekhovskikh and Lysanov, 1991; Kravtsov, 2005; Petrashen *et al.*, 1985; Petrashen and Nakhamkin, 1973; Petrashen, 1978, 1980; Petrashen and Kashtan, 1984; Bleistein *et al.*, 2000).

In the fifth chapter the elements of the ray method, which is used widely for the solution of the problem of wave propagation is introduced. The geometrical optics method (Babic and Buldyrev, 1991; Babic *et al.*, 1999; Kravtsov, 2005; Bleistein *et al.*, 2000) is considered. This method is a short-wave asymptotics of the wave field in smoothly inhomogeneous, stationary and weakly conservative media (the reference heterogeneities are much greater than wavelength). The spatio-temporal approximation of the solution of the scalar wave equation is given (Babic *et al.*, 1999). The short-wave asymptotics for the Helmholtz homogeneous equation is introduced (WKBJ approximation). The ray method for propagation of elastic waves is considered in an assumption of the faster change of the characteristics of the wave process in a direction of a normal to the wave front in comparison with a change of characteristics of the medium (Kravtsov and Apresyan, 1996). For the description of the propagation of acoustic waves at ocean it is offered to use the ray approximation of the propagation in the almost stratified medium, i.e. in a medium with smoothly (in comparison with depth) varying velocity of propagation of a signal in a horizontal plane. The ray method for the description of the surface waves in a vertical inhomogeneous medium is introduced. The considered ray approximation of the propagation of electromagnetic fields is a basis for the description of the propagation processes in inhomogeneous media, and also has the significant methodological importance. On the basis of this description it is possible to get the transport equation and to establish connection of the statistical parameters of a medium with the parameters of the phenomenological theory of a transport of electromagnetic waves.

In the sixth chapter the methods of a parameter estimation of geophysical objects are described (Goltsman, 1971; Nikitin, 1986; Petrashen *et al.*, 1985; Ryzhikov and Troyan, 1994; Stratonovich, 1975; Troyan and Sokolov, 1989). The algorithms and examples of applying of the basic methods of the parameter estimation which have obtained widespread occurrence at the solution of the geophysical problems are introduced: the method of moments, the maximum likelihood method, the Newton-Le Cam method, various modifications of the least squares method, the Bayes criterion and method of the statistical regularization, criterion of the a posteriori probability ratio, the singular analysis, the least modules method, the ro-

bust methods (reparametrization method, Huber's method, Andrews's method), the method of Backus-Gilbert, the interval estimation method and the genetic algorithm. All methods, introduced in this chapter, can be used at problems of quantitative interpretation.

In the seventh chapter the statistical criteria for a choice of the model are given, which are meant for the solution of the problems of qualitative interpretation (Goltsman, 1971; Troyan and Sokolov, 1989). The problem of the test of parametric hypotheses and the a posteriori probabilities ratio are surveyed in details. The special attention is given to the signal resolution problem for the signals of a various geophysical nature. The information criterion of a choice of the model, which is basing on the maximum likelihood method and Shannon's information, is introduced. The iterative procedure of an estimation of parameters of interfering signals with simultaneous definition of a number of signals is represented (Akaike, 1974).

The eight chapter is devoted to the tasks of approximation of geophysical fields (Troyan and Sokolov, 1989). The methods of the spline approximation are introduced. An algorithm of one-dimensional approximation by cubic spline, periodic and parametric spline functions, two-dimensional spline, application of spline functions for a smoothing of histograms are considered. The algorithms of approximation of the seismic horizon and the velocity law by piecewise-polynomials together with the well observations are represented.

In the ninth chapter the mathematical notions in the terms of the functional analysis for a problem of the parameters estimation of geophysical objects are introduced (Ryzhikov and Troyan, 1994). The basic concepts and relations of the applied functional analysis (which are used in 10-th and 11 chapters) are briefly described. The definition of the ill-posed problems and the methods of their solution are considered. Some statistical criteria in the terms of the functional analysis are introduced. On the basis of the information approach the setting of the problem of the mathematical design of the geophysical experiment are tendered: a choice of the frequency and temporal intervals of measurings, choice of a spread the sources and receivers, and also their number.

The tenth chapter is devoted to a problem of a creation and interpretation of the tomography functionals, which make sense of the functions of the influence of various spatial domains of a medium on a separate measuring (Ryzhikov and Troyan, 1994). The norm of the tomography functional is determined by the intensity of the interaction of the incident and reversed fields. The examples of a build-up and the interpretation the the tomography functionals for the scalar wave equation, Lame equation, for the transport equation of a stationary sounding signal and for a diffusion equation are introduced. The build-up of the incident and reversed fields in a layered reference medium in the conformity with problems of the propagation of elastic, acoustic and electromagnetic waves are surveyed.

In the eleventh chapter the tomography methods of an image reconstruction of a medium (Ryzhikov and Troyan, 1994) are introduced. An algorithm of the re-

constructive tomography is proposed on the basis of the statistical regularization method. The comparison of this method with the Backus–Hilbert method is held. The notion of the information sensitivity is considered. The measure of the information sensitivity of the observation field concerning a linear functional of parameters of a field can be the effective tool of a choice of the physically justified model. The original algorithm of the regularization for the problems of the three-dimensional ray tomography is introduced.

The twelfth chapter is devoted to the transforms and analysis of geophysical signals. The traditional transforms, such as the Fourier transform, Laplace transform, the Radon and Hilbert transforms with the reference to the analysis of seismograms, and rather new methods the cepstral and bispectra analysis are introduced. The traditional algorithms of the Wiener filtration, inverse filtration, dynamic filtration (Kalman filter) are surveyed. The original algorithm of the factor analysis is introduced, which can be successfully applied to searching latent periodicities and for a wide range of the interpretating problems.

In Appendix the tasks and computer exercises with a description of the required programs implemented under the MATLAB package are introduced. The realization of these computer exercises will allow the reader more deeply to understand a content and possibilities of the methods for the analysis and processing of the geophysical information which are introduced in the book.

The textbook is intended for the students, master students, post-graduate students of geophysical specialities as well engeneers and geoscientists. However it can be suit to the students and post-graduate students of other specialities, which are concerned to the analysis and handling of signals of any physical nature (radiophysics, optics, astrophysics, physical medicine, etc.).

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