

# Contents

<i>Introduction</i>	v
1. Basic concepts of the probability theory	1
1.1 The Definition of Probability . . . . .	1
1.1.1 Set of elementary events . . . . .	1
1.1.2 Probability model with a finite number of outcomes . . .	4
1.1.3 Relative-frequency definition of probability . . . . .	5
1.1.4 Classical definition of probability . . . . .	5
1.1.5 Geometrical definition of probability . . . . .	6
1.1.6 Exercises . . . . .	6
1.2 Basic Properties of Probability . . . . .	7
1.2.1 Addition of probabilities . . . . .	7
1.2.2 Nonindependent and independent events . . . . .	9
1.2.3 The Bayes formula and complete probability . . . . .	10
1.2.4 Exercises . . . . .	11
1.3 Distribution Functions . . . . .	11
1.3.1 Random variables . . . . .	11
1.3.2 Distribution function . . . . .	12
1.3.3 The density function . . . . .	14
1.3.4 The distribution and density of function of one random argument . . . . .	15
1.3.5 Random vectors . . . . .	16
1.3.6 Marginal and conditional distributions . . . . .	17
1.3.7 The distributive law of two random variables . . . . .	18
1.3.8 Exercises . . . . .	21
1.4 The Numerical Characteristics of Probability Distributions . . . .	23
1.4.1 Mathematical expectation . . . . .	23
1.4.2 Variance and correlation coefficients . . . . .	24
1.4.3 Quantiles . . . . .	26
1.4.4 Characteristics of a density function . . . . .	27

1.4.5	Exercises . . . . .	29
1.5	Characteristic and Generating Functions . . . . .	30
1.5.1	Moment generating function . . . . .	31
1.5.2	Probability generator . . . . .	32
1.5.3	Semi-invariants or cumulants . . . . .	33
1.5.4	Exercises . . . . .	33
1.6	The Limit Theorems . . . . .	34
1.6.1	Convergence in probability . . . . .	34
1.6.2	Chebyshev inequality . . . . .	34
1.6.3	The law of averages (Chebyshev's theorem) . . . . .	35
1.6.4	Generalised Chebyshev's theorem . . . . .	36
1.6.5	Markov's theorem . . . . .	36
1.6.6	Bernoulli theorem . . . . .	37
1.6.7	Poisson theorem . . . . .	37
1.6.8	The central limit theorem . . . . .	37
1.6.9	Exercises . . . . .	39
1.7	Discrete Distribution Functions . . . . .	39
1.7.1	Binomial distribution . . . . .	40
1.7.2	Poisson distribution . . . . .	41
1.7.3	Geometrical distribution . . . . .	42
1.7.4	Exercises . . . . .	43
1.8	Continuous Distributions . . . . .	45
1.8.1	Univariate normal distribution . . . . .	45
1.8.2	Multivariate normal distribution . . . . .	47
1.8.3	Uniform distribution . . . . .	51
1.8.4	$\chi^2$ -distribution . . . . .	51
1.8.5	Student's distribution ( <i>t</i> -distribution) . . . . .	52
1.8.6	Fisher distribution and <i>Z</i> -distribution . . . . .	53
1.8.7	Triangular distribution . . . . .	55
1.8.8	Beta distribution . . . . .	55
1.8.9	Exponential distribution . . . . .	55
1.8.10	Laplace distribution . . . . .	56
1.8.11	Cauchy distribution . . . . .	57
1.8.12	Logarithmic normal distribution . . . . .	57
1.8.13	Significance of the normal distribution . . . . .	58
1.8.14	Confidence intervals . . . . .	59
1.8.15	Exercises . . . . .	60
1.9	Information and Entropy . . . . .	62
1.9.1	Entropy of the set of discrete states of system . . . . .	62
1.9.2	Entropy of the complex system . . . . .	63
1.9.3	Shannon information (discrete case) . . . . .	64

1.9.4	Entropy and information for systems with a continuous set of states . . . . .	66
1.9.5	Fisher information . . . . .	69
1.9.6	Exercises . . . . .	72
1.10	Random Functions and its Properties . . . . .	72
1.10.1	Properties of random functions . . . . .	74
1.10.2	Properties of the correlation function . . . . .	76
1.10.3	Action of the linear operator on a random function . . . . .	77
1.10.4	Cross correlation function . . . . .	77
1.10.5	Wiener–Khinchin theorem and power spectrum . . . . .	78
1.10.6	Definition of estimations of the characteristics of random variables . . . . .	81
2.	Elements of mathematical statistics . . . . .	83
2.1	The Basic Concepts of the Decision Theory . . . . .	83
2.1.1	Distribution class of the statistical problem . . . . .	85
2.1.2	The structure of the decision space and the loss function . . . . .	85
2.1.3	Decision rule . . . . .	88
2.1.4	Sufficient statistic . . . . .	90
2.2	Estimate Properties . . . . .	91
2.2.1	Consistency . . . . .	92
2.2.2	Bias . . . . .	92
2.2.3	Rao–Cramer inequality. Efficiency . . . . .	94
2.2.4	Sufficiency . . . . .	97
2.2.5	Asymptotic normality . . . . .	97
2.2.6	Robustness . . . . .	98
3.	Models of measurement data . . . . .	99
3.1	Additive Model . . . . .	99
3.2	Models of the Quantitative Interpretation . . . . .	101
3.3	Regression Model . . . . .	102
3.4	The Models of Qualitative Interpretation . . . . .	104
3.5	The Models of Qualitative-Quantitative Interpretation . . . . .	105
3.6	Random Components of Model and its Properties . . . . .	106
3.7	Model with Random Parameters . . . . .	111
3.8	A Priori Information . . . . .	111
4.	The functional relationships of sounding signal fields and parameters of the medium . . . . .	115
4.1	Seismology and Seismic Prospecting . . . . .	115
4.2	Acoustics of the Ocean . . . . .	123

4.3	Wave Electromagnetic Fields in Geoelectrics and Ionospheric Sounding . . . . .	126
4.4	Atmospheric Sounding . . . . .	131
5.	Ray theory of wave field propagation . . . . .	135
5.1	Basis of the Ray Theory . . . . .	135
5.2	Ray Method for the Scalar Wave Equation . . . . .	138
5.3	Shortwave Asymptotic Form of the Solution of the One-Dimensional Helmholtz Equation (WKB Approximation) . . . . .	144
5.4	The Elements of Elastic Wave Ray Theory . . . . .	146
5.5	The Ray Description of Almost-Stratified Medium . . . . .	147
5.6	Surface Wave in Vertically Inhomogeneous Medium . . . . .	152
5.7	Ray Approximation of Electromagnetic Fields . . . . .	155
5.8	Statement of Problem of the Ray Kinematic Tomography . . . . .	159
6.	Methods for parameter estimation of geophysical objects . . . . .	163
6.1	The Method of Moments . . . . .	163
6.2	Maximum Likelihood Method . . . . .	164
6.3	The Newton–Le Cam Method . . . . .	165
6.4	The Least Squares Method . . . . .	167
6.5	LSM — Nonlinear Model . . . . .	169
6.6	LSM — Orthogonal Polynomials (Chebyshev Polynomials) . . . . .	170
6.7	LSM — Case of Linear Constraints . . . . .	172
6.8	Linear Estimation — Case of Nonstationary Model . . . . .	174
6.9	Bayes' Criterion and Method of Statistical Regularization . . . . .	175
6.10	Method of Maximum a Posteriori Probability . . . . .	176
6.11	The Recursion Algorithm of MAP . . . . .	178
6.12	Singular Analysis and Least Squares Method . . . . .	179
	6.12.1 Resolution matrix . . . . .	184
6.13	The Method of Least Modulus . . . . .	184
6.14	Robust Methods of Estimation . . . . .	186
	6.14.1 Reparametrization algorithm . . . . .	187
	6.14.2 Huber robust method . . . . .	188
	6.14.3 Andrews robust method . . . . .	189
6.15	Interval Estimation . . . . .	190
6.16	The Method of Backus and Gilbert of the Linear Inverse Problem Solution . . . . .	193
6.17	Genetic Algorithm . . . . .	196
	6.17.1 Coding . . . . .	196
	6.17.2 Selection . . . . .	197
	6.17.3 Crossover . . . . .	197
	6.17.4 Mutation . . . . .	197

6.17.5	Choice . . . . .	199
7.	Statistical criteria for choice of model	201
7.1	Test of Parametric Hypothesis . . . . .	201
7.2	Criterion of a Posteriori Probability Ratio . . . . .	202
7.3	The Signal Resolution Problem . . . . .	206
7.4	Information Criterion for the Choice of the Model . . . . .	208
7.5	The Method of the Separation of Interfering Signals . . . . .	211
8.	Algorithms of approximation of geophysical data	217
8.1	The Algorithm of Univariate Approximation by Cubic Splines . .	217
8.2	Periodic and Parametric Spline Functions . . . . .	223
8.3	Application of the Spline Functions for Histogram Smoothing . .	226
8.4	Algorithms for Approximation of Seismic Horizon Subject to Borehole Observations . . . . .	227
8.4.1	The Markovian type of correlation along the beds and no correlation between beds . . . . .	229
8.4.2	Markovian correlation between the beds and no correla- tion along bed . . . . .	230
8.4.3	Conformance inspection of seismic observation to borehole data concerning bed depth . . . . .	230
8.4.4	Incorporation of random nature of depth measurement using borehole data . . . . .	232
8.4.5	Application of a posteriori probability method to approximation of seismic horizon . . . . .	234
8.4.6	Case of uncorrelated components of random vector . . . .	235
8.4.7	Approximation of parameters of approximation horizon by the orthogonal polynomials . . . . .	237
8.4.8	Numerical examples of application of approximation algorithms . . . . .	238
8.5	Algorithm of Approximation of Formation Velocity with the Use of Areal Observations with Borehole Data . . . . .	241
9.	Elements of applied functional analysis for problem of estimation of the parameters of geophysical objects	247
9.1	Elements of Applied Functional Analysis . . . . .	247
9.2	Ill-Posed Problems . . . . .	264
9.3	Statistical Estimation in the Terms of the Functional Analysis . .	272
9.4	Elements of the Mathematical Design of Experiment . . . . .	289
10.	Construction and interpretation of tomographic functionals	293
10.1	Construction of the Model of Measurements . . . . .	293

10.2	Tomographic Functional . . . . .	297
10.3	Examples of Construction and Interpretation of Tomographic Functionals . . . . .	298
10.3.1	Scalar wave equation . . . . .	298
10.3.2	The Lamé equation in an isotropic infinite medium . . . . .	299
10.3.3	The transport equation of the stationary sounding signal . . . . .	306
10.3.4	The diffusion equation . . . . .	308
10.4	Ray Tomographic Functional in the Dynamic and Kinematic Interpretation of the Remote Sounding Data . . . . .	309
10.5	Construction of Incident and Reversed Wave Fields in Layered Reference Medium . . . . .	312
11.	Tomography methods of recovering the image of medium . . . . .	315
11.1	Elements of Linear Tomography . . . . .	315
11.1.1	Change of variables . . . . .	316
11.1.2	Differentiation of generalized function . . . . .	317
11.2	Connection of Radon Inversion with Diffraction Tomography . . . . .	322
11.3	Construction of Algorithms of Reconstruction Tomography . . . . .	327
11.4	Errors of Recovery, Resolving Length and Backus and Gilbert Method . . . . .	334
11.5	Back Projection in Diffraction Tomography . . . . .	340
11.6	Regularization Problems in 3-D Ray Tomography . . . . .	347
11.7	Construction of Green Function for Some Type of Sounding Signals . . . . .	351
11.7.1	Green function for the wave equation . . . . .	351
11.7.2	Green function for “Poisson equation” . . . . .	353
11.7.3	Green function for Lamé equation in uniform isotropic infinite medium . . . . .	354
11.7.4	Green function for diffusion equation . . . . .	358
11.7.5	Green function for operator equation of the second genus . . . . .	359
11.8	Examples of the Recovery of the Local Inhomogeneity Parameters by the Diffraction Tomography Method . . . . .	360
11.8.1	An estimation of the resolution . . . . .	360
11.8.2	An estimation of the recovery accuracy of inhomogeneities parameters . . . . .	361
12.	Methods of transforms and analysis of the geophysical data . . . . .	371
12.1	Fourier Transform . . . . .	371
12.1.1	Fourier series . . . . .	371
12.1.2	Fourier integral . . . . .	372
12.2	Laplace Transform . . . . .	374
12.3	Z-Transform . . . . .	375

12.4	Radon Transform for Seismogram Processing . . . . .	376
12.5	Gilbert Transform and Analytical Signal . . . . .	378
12.6	Cepstral Transform . . . . .	380
12.7	Bispectral Analysis . . . . .	381
12.8	Kalman Filtering . . . . .	382
12.9	Multifactor Analysis and Processing of Time Series . . . . .	384
12.10	Wiener Filter . . . . .	389
12.11	Predictive-Error Filter and Maximum Entropy Principle . . . . .	390
Appendix A Computer exercises		397
A.1	Statistical Methods . . . . .	397
A.1.1	Examples of numerical simulation of random values . . . . .	397
A.1.2	Construction of histogram . . . . .	398
A.1.3	Description of a random variable . . . . .	399
A.1.4	Computer simulation of random values . . . . .	404
A.1.5	Confidence intervals . . . . .	405
A.1.6	Time series . . . . .	405
A.2	Transforms . . . . .	406
A.2.1	Fourier transform . . . . .	406
A.2.2	Signals and their spectral characteristics . . . . .	406
A.2.3	Multifactor analysis . . . . .	407
A.2.4	Cepstral transform . . . . .	411
A.3	Direct and Inverse Problem Solution . . . . .	413
A.3.1	Computer simulation of gravitational attraction . . . . .	413
A.3.2	Computer simulation of magnetic field . . . . .	414
A.3.3	Computer simulation of the seismic field . . . . .	414
A.3.4	Deconvolution by the Wiener filter . . . . .	415
A.3.5	Quantitative interpretation . . . . .	417
A.3.6	Qualitative interpretation . . . . .	420
A.3.7	Diffraction tomography . . . . .	421
Appendix B Tables		425
<i>Bibliography</i>		427
<i>Index</i>		433