Introduction

Having taught several years in a row the course on distributions at the Ecole Normale Supérieure (Takaddoum) of Rabat (Morocco), we find it useful to write an introductory book in an accessible and self-contained context; presenting at the same time some useful strong tools of the theory. We have, in adequate situations, produced calculations on regular distributions in order to justify the introduction of certain notions. We have made an effort to make the proofs the more transparent possible. The theory of distributions has many applications and in various domains. Hence they do not, all of them, have a place in a first course. We have limited ourselves here to the utilization of Fourier and Laplace transformations in the resolution of some differential equations.

We indicate briefly the content of this work. Other comments are given at the beginning of the chapters.

The fundamental spaces, which play an essential role in the theory of distributions, are presented in Chapter 1. General properties of distributions are given in Chapter 2. We also study, in the latter, the first classical examples (regular distributions, measures, principal value of Cauchy, finite parts of Hadamard, ...). For more clarity, we have reserved a chapter to tensor products (Chapter 3). The convolution product is the subject of Chapter 4. The notion of an allowed family is also discussed. Diverse classical equations (partial differential equations with constant coefficients, finite difference equations with constant coefficients and Volterra equations) are seen as convolution equations in suitable algebras (said to be of convolution). For convenience and due to the importance of the Fourier transformation, we reserved Chapter 5 to it, and this in $L^1(\mathbb{R}^n)$. We also give, in the latter, the transfer theorem and the preparatory formula of Riesz. In Chapter 6, we present and study the space $\mathcal{S}(\mathbb{R}^n)$ of Schwartz, the elements of which are the rapidly decreasing functions of class C^∞ as well as their derivatives of any order. We also study the space of tempered distributions which is the topological dual $\mathcal{S}'(\mathbb{R}^n)$ of $\mathcal{S}(\mathbb{R}^n)$. At the end of this chapter the notion of ultradistribution is discussed. Structures of some distributions are described in Chapter 7. In the eighth chapter we determine Fourier ranges of some subspaces of $\mathcal{S}'(\mathbb{R}^n)$. Essential properties of Laplace transformation are discussed in Chapter 9. We also present simple applications of Laplace transform in the resolution of partial differential equations. Different notions of kernels (regular, regularizing and very regular) are the subject of Chapter 10. They are used in the resolution of partial differential equations. Finally, in Chapter 11, we introduce Sobolev spaces $H^s(\mathbb{R}^n)$, $s \in \mathbb{R}$, and we present the injection theorem of Sobolev-Rellich.

Discussions are provided at the end of each chapter. It is a kind of brief comments on the results in relation with others, as well as their importance. Indications are given concerning the presentation in comparison with books in the Bibliography. Ideas on the introduction of the notions, the choice of the methods and the driving of the proofs are pointed out whenever possible. We also suggest further readings from the references according to the content.

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