

PREFACE

Quantum theory is one of the most important intellectual developments in the early twentieth century. The mathematical theory of quantum physics emerged largely from works, pioneered by John von Neumann, on the spectral theory of linear operators on a Hilbert space in the mid 1920s, and many important developments of the last eighty years are owed to a strong interplay between theoretical physics and mathematics. Moreover, in the last twenty to thirty years, there have been phenomenal advances in which mathematicians found new tools and motivations through physical concepts and physicists exploited ideas and techniques that were originally developed for the use in pure mathematics.

It was therefore felt timely that we should raise interest in mathematical physics among researchers and students around the world, and in particular at the National University of Singapore (NUS) as well as elsewhere in Singapore, by organizing a mathematical workshop on various mathematical aspects of quantum theory. It was also felt that such a collaboration between scientists of different backgrounds, different expertise, and different scientific culture could bear fruit on the research of all participants by intellectual cross-fertilization. So, one of the major objectives of the workshop was to bring together mathematicians, whose work has a bearing on quantum physics, with researchers from mathematical and theoretical physics. This book arose from such a workshop.

The eventual theme for the workshop was “Mathematical Horizons for Quantum Physics” and the event was co-organized by the Institute for Mathematical Sciences (IMS) and the Centre for Quantum Technologies (CQT) at NUS and held on the premises of IMS over an eight-week period

in 2008. In order to allow leading experts to mingle and discuss freely with young researchers and students, the workshop was organized with fewer than usual lectures and a lot of informal, interactive tutorials and discussions.

The eight-week period from July 28 to 21 September 2008 was divided into four sessions, each lasting three weeks, with an overlap of one or two weeks between successive sessions. The overlap periods gave opportunities for mutual interactions between participants from different sessions. At the end of the workshop, there were more than eighty active participants who had contributed to the discussions in one way or another.

In addition to the workshop activities, there were also two very interesting public talks: Burkhard Kümmeler spoke on “Knot or Not Knot” and Reinhard Werner delivered a talk that addressed the question “Are Quantum Computers the Next Generation of Supercomputers?” During the workshop, one of the organizers, Huzihiro Araki, also gave two presentations: the first one on the lives and careers of the Japanese Nobel Laureates Hideki Yukawa and Sin-Itiro Tomonaga at the Faculty of Science, NUS, and a second one concerning the history and mathematics of the Tomita–Takesaki theory for operator algebras at the Department of Mathematics, NUS.

During each session of the workshop, a number of pedagogical lectures, accessible to young researchers and graduate students, were provided. This book compiles the written accounts of some of these lectures. We therefore hope that this compilation will prove to be useful for graduate students and researchers who would like to start their research in an area covered in the workshop and also for researchers who require solid introductory materials and surveys of the status of the field.

The themes of the four sessions of the workshop and the respective lectures are as follows:

Session 1. **Quantum Control and Dynamics**

The central topics discussed were

- Quantum control of the alignment and orientation of polar molecules;
- Quantum chaos;
- Laser-driven models in quantum computing systems.

Arne Keller’s treatment of *Control of the Molecular Alignment or Orientation by Laser Pulses* and the contribution of *Quantum Computing and Devices: A Short Introduction* by

Zhigang Zhang, Viswanath Ramakrishna, and Goong Chen as well as Hans-Rudolf Jauslin's and Dominique Sugny's account of *Dynamics of Mixed Classical-Quantum Systems, Geometric Quantization and Coherent States* are the three book chapters that originated in this session.

Session 2. **Operator Algebras in Quantum Information**

The discussions focused mostly on

- Entropy in quantum channels and the problem of additivity of quantum capacity;
- Stability of quantum algorithms in the presence of external noise;
- Entanglement of multipartite and infinite systems.

These topics are covered in another three book chapters: *Quantum Memories as Open Systems* by Robert Alicki, *Two Mathematical Problems in Quantum Information Theory* by Alexander S. Holevo, and *Dissipatively Induced Bipartite Entanglement* by Fabio Benatti.

Session 3. **Non-Equilibrium Statistical Mechanics**

In their discussions the participants tried to answer the questions

- Is there a large deviation theory for quantum fluctuations?
- How can one construct non-equilibrium steady states?

Jan Dereziński's essay on *Scattering in Non-Relativistic Quantum Field Theory* is the book chapter for this session.

Session 4. **Strongly Interacting Many-Particle Systems**

The discussions addressed

- The theory of large atoms, molecules, and solids;
- The mathematical description of the radiation field and its interaction with matter.

Volker Bach's chapter on *Mathematical Theory of Atoms and Molecules* deals with these matters.

This volume would not have been possible without the immense efforts and contributions from the lecturers who agreed to prepare, present, and then write up their lectures at the workshop. As the organizers for the workshop, we would like to take this opportunity to thank all authors for their hard work. We are confident that this volume conveys the exciting atmosphere of all those stimulating discussions.

We would also like to thank Prof. Louis Chen, Director of IMS, A/Prof. Leung Ka Hin, the then Deputy Director of IMS, and Prof. K. K. Phua for their strong encouragement and precious advice during all stages of planning and conducting this workshop. We are grateful to the Lee Foundation and the Dean of the Faculty of Science, NUS, Prof. Andrew Wee, for their generous financial support. Without the budget contribution from the Lee Foundation, certain events — such as public talks and interaction sessions held during the workshop — would not have been possible.

We are equally thankful for the clerical support provided by the IMS secretariat, with special thanks to Agnes Wu and Claire Tan. During the workshop Stephen Auyong, IT manager at IMS, rendered invaluable support to the participants. Many others have also contributed to the success of the workshop. In particular, we would like to single out Evon Tan and Bess Fang from CQT who took splendid care of so many errands.

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