# Introduction

One of the principal objectives of theoretical research in any department of knowledge is to find the point of view from which the subject appears in its greatest simplicity.

—J. Willard Gibbs
(acceptance letter of Rumford Medal, 1881)

Failures of many engineering structures fall into one of two simple categories: (1) material failure and (2) structural instability. The first type of failure, treated in introductory courses on the strength of materials and structural mechanics, can usually be adequately predicted by analyzing the structure on the basis of equilibrium conditions or equations of motion that are written for the initial, undeformed configuration of the structure. By contrast, the prediction of failures due to structural instability requires equations of equilibrium or motion to be formulated on the basis of the deformed configuration of the structure. Since the deformed configuration is not known in advance but depends on the deflections to be solved, the problem is in principle nonlinear, although frequently it can be linearized in order to facilitate analysis.

Structural failures caused by failure of the material are governed, in the simplest approach, by the value of the material strength or yield limit, which is independent of structural geometry and size. By contrast, the load at which a structure becomes unstable can be, in the simplest approach, regarded as independent of the material strength or yield limit; it depends on structural geometry and size, especially slenderness, and is governed primarily by the stiffness of the material, characterized, for example, by the elastic modulus. Failures of elastic structures due to structural instability have their primary cause in geometric effects: the geometry of deformation introduces nonlinearities that amplify the stresses calculated on the basis of the initial undeformed configuration of the structure.

The stability of elastic structures is a classical problem which forms the primary content of most existing textbooks. We will devote about half the present treatise to this topic (Part I, Chapters 1–7).

We begin our study of structural stability with the analysis of buckling of elastic columns and frames, a bread-and-butter problem for structural engineers. Although this is a classical research field, we cover in some detail various recent advances dealing with the analysis of very large regular frames with many members, which are finding increasing applications in tall buildings as well as certain designs for space structures.

The study of structural stability is often confusing because the definition of structural stability itself is unstable. Various definitions may serve a useful purpose for different problems. However, one definition of stability—the dynamic definition—is fundamental and applicable to all structural stability problems. Dynamic stability analysis is essential for structures subjected to nonconservative loads, such as wind or pulsating forces. Structures loaded in this manner may falsely appear to be stable according to static analysis while in reality they fail through vibrations of ever increasing amplitude or some other accelerated motion. Because of the importance of this problem in modern structural engineering we will include a thorough treatment of the dynamic approach to stability in Chapter 3. We will see that the static approach yields correct critical loads only for conservative structural systems, but even for these it cannot answer the question of stability completely.

The question of stability may be most effectively answered on the basis of the energy criterion of stability, which follows from the dynamic definition if the system is conservative. We will treat the energy methods for discrete and discretized systems in Chapter 4 and those for continuous structures in Chapter 5, in which we will also focus on the approximate energy methods that simplify the stability analysis of continuous structures.

In Chapters 6 and 7 we will apply the equilibrium and energy methods to stability analysis of more complicated thin structures such as thin-wall beams, the analysis of which can still be made one-dimensionally, and of two-dimensional structures such as plates and shells. Because many excellent detailed books deal with these problems, and also because the solution of these problems is tedious, requiring lengthy derivations and mathematical exercises that add little to the basic understanding of the behavior of the structure, we limit the treatment of these complex problems to the basic, prototype situations. At the same time we emphasize special features and approaches, including an explanation of the direct and indirect variational methods, the effect of imperfections, the postcritical behavior, and load capacity. In our computer era, the value of the complicated analytical solutions of shells and other thin-wall structures is diminishing, since the solutions can be obtained by finite elements, the treatment of which is outside the scope of the present treatise.

While the first half of the book (Part I, Chaps. 1–7) represents a fairly classical choice of topics and coverage for a textbook on structural stability, the second half of the book (Part II, Chaps. 8–13), devoted to inelastic and damage theories of structural stability, attempts to synthesize the latest trends in research. Inelastic behavior comprises not only plasticity (or elastoplasticity), treated in Chapters 8 and 10, but also creep (viscoelastic as well as viscoplastic), treated in in Chapter 9, while damage comprises not only strain-softening damage, treated in Chapter 13, but also fracture, which represents the special or limiting case of localized damage, treated in Chapter 12. Whereas the chapters dealing with plasticity and creep present for the most part relatively well-established theories, Chapters 10–13, dealing with thermodynamic concepts and finite strain effects in three dimensions, as well as fracture, damage, and friction, present mostly fresh results of recent researches that might in the future be subject to reinterpretations and updates.

Inelastic behavior tends to destabilize structures and generally blurs the aforementioned distinction between material failures and stability failures. Its effect can be twofold: (1) it can merely reduce the critical load, while instability is still caused by nonlinear geometric effects and cannot occur in their absence—this is typical of plasticity and creep (with no softening or damage); or (2) it can cause instability by itself, even in the absence of nonlinear geometric effects in the structure—this is typical of fracture, strain-softening damage, and friction and currently represents a hot research subject. An example of this behavior is fracture mechanics. In this theory (outlined in Chapter 12), structural failure is treated as a consequence of unstable crack propagation, the instability being caused by the global structural action (in which the cause of instability is the release of energy from the structure into the crack front) rather than the nonlinear geometric effects.

Stability analysis of structures that are not elastic is complicated by the fact that the principle of minimum potential energy, the basic tool for elastic structures, is inapplicable. Stability can, of course, be analyzed dynamically, but that too is complicated, especially for inelastic behavior. However, as we will see in Chapter 10, energy analysis of stability is possible on the basis of the second law of thermodynamics. To aid the reader, we will include in Chapter 10 a thorough discussion of the necessary thermodynamic principles and will then apply them in a number of examples.

Irreversibility, which is the salient characteristic of nonelastic behavior, produces a new phenomenon: the bifurcation of equilibrium path need not be associated with stability loss but can typically occur in a stable manner and at a load that is substantially smaller than the stability limit. This phenomenon, which is not found in elastic structures, will come to light in Chapter 8 (dealing with elastoplastic columns) and will reappear in Chapters 12 and 13 in various problems of damage and fracture. A surprising feature of such bifurcations is that the states on more than one postbifurcation branch of the equilibrium path can be stable, which is impossible for elastic structures or reversible systems in general. To determine the postbifurcation path that will actually be followed by the structure, we will need to introduce in Chapter 10 a new concept of stable path, which, as it turns out, must be distinct from the concept of stable state. We will present a general thermodynamic criterion that makes it possible to identify the stable path.

The stability implications of the time-dependent material behavior, broadly termed creep, also include some characteristic phenomena, which will be explained in Chapter 9. In dealing with imperfect viscoelastic structures under permanent loads, we will have to take into account the asymptotic deflections as the time tends to infinity, and we will see that the long-time (asymptotic) critical load is less than the instantaneous (elastic) critical load. In imperfect viscoelastic structures, the deflections can approach infinity at a finite critical time, and can again do so under a load that is less than the instantaneous critical load. For creep buckling of concrete structures, we will further have to take into account the profound effect of age on creep exhibited by this complex material.

The most important consequence of the instabilities caused by fracture or damage rather than by geometric effects is that they produce size effect, that is, the structure size affects the nominal stress at failure. By contrast, no size effect exists according to the traditional concepts of strength, yield limit, and yield surface in the stress or strain space. Neither does it according to elastic stability theory. The most severe and also the simplest size effect is caused by failures due to propagation of a sharp fracture where the fracture process happens at a point. A less severe size effect, which represents a transition from failures governed by strength or yield criteria to failures governed by instability of sharp fractures, is produced by instability modes consisting either of propagation of a fracture with a large fracture process zone (Chap. 12) or of damage localization (Chap. 13). As a special highlight of the present treatise, these modern problems are treated in detail in the last two chapters.

The practical design of metallic or concrete columns and other structures is an important topic in any stability course. In this text, the code specifications and design approaches are dispersed through a number of chapters instead of being presented compactly in one place. This presentation is motivated by an effort to avoid a cookbook style and present each aspect of design only after the pertinent theory has been thoroughly explained, but not later than that. It is for this reason, and also because fundamental understanding of inelastic behavior is important, that the exposition of column design is not completed until Chapters 8 and 9, which also include detailed critical discussions of the current practice.

The guiding principle in the presentation that follows is to advance by induction, from special and simple to general and complex. This is one reason why we choose not to start the book with general differential equations in three dimensions and thermodynamic principles, which would then be reduced to special cases. (The general three-dimensional differential equations governing stability with respect to nonlinear geometric effects do not appear in the book until Chap. 11.) There is also another reason—the three-dimensional analysis of stability is not necessary for slender or thin structures made of structural materials such as steel or concrete, which are relatively stiff. It is only necessary for dealing with incremental deformations of massive inelastic structures or structures made of highly anisotropic or composite materials which can be strained to such a high level that some of the tangential moduli of the material are reduced to values that are of the same order of magnitude as the stresses.

As another interesting phenomenon, which we will see in Chapter 11, various possible choices of the finite-strain tensor lead to different expressions for the critical loads of massive bodies. It turns out that the stability formulations corresponding to different choices of the finite-strain tensor are all equivalent, but for each such formulation the tangential moduli tensor of the material has a different physical meaning and must be determined from experimental data in a different manner. If this is not done, then three-dimensional finite-strain stability analysis makes no sense.

As we live in the new era of computers, stability of almost any given structure could, at least in principle, be analyzed by geometrically nonlinear finite element codes with incremental loading. This could be done in the presence of complex nonlinear behavior of the material as well. Powerful though this approach is, the value of simple analytical solutions that can be worked out by hand must not be underestimated. This book attempts to concentrate on such solutions. It is these solutions that enhance our understanding and also must be used as test cases for the finite element programs.