Preface

Tits buildings or simple *buildings* are combinatorial constructions successfully exploited to study various types of groups (classical, simple algebraic, Kac–Moody). One of historical backgrounds of this concept is Cartan's well-known classification of simple Lie groups. We refer to [Abramenko (1996); Brown (1989); Garrett (1997); Ronan (1989); Scharlau (1995); Tits (1974)] for various aspects of building theory.

Buildings can be obtained from groups admitting Tits systems. Such groups form a sufficiently wide class which contains classical groups, reductive algebraic groups and others. The formal definition of a building is pure combinatorial and does not depend on a group. In [Tits (1974)] a building is defined as a simplicial complex with a family of subcomplexes called *apartments* and satisfying certain axioms. All apartments are isomorphic to the simplicial complex obtained from a Coxeter system which defines the building type.

The vertex set of a building can be labeled by the nodes of the diagram of the associated Coxeter system. The set of all vertices corresponding to the same node is called a *Grassmannian* (more general objects were investigated in [Pasini (1994)]). This term is motivated by the fact that every building of type A_n is isomorphic to the flag complex of an $(n+1)$-dimensional vector space and the Grassmannians of the building can be identified with the Grassmannians of this vector space. Every building Grassmannian has a natural structure of a partial linear space (point-line geometry); this partial linear space is called the *Grassmann space* associated with the Grassmannian.

The aim of this book is to present both classical and more recent results on Grassmannians of buildings of classical types ($\mathsf{A}_n, \mathsf{B}_n = \mathsf{C}_n, \mathsf{D}_n$). These results will be formulated in terms of point-line geometry. A large portion of them is a part of the area known as *characterizations of geometrical transformations under mild hypotheses.* Roughly speaking, we want to show that some mappings of Grassmannians can be extended to mappings of the associated buildings. Other results are related with structural properties of apartments. Also we show that our methods work for some geometric constructions non-related with buildings — Grassmannians of infinity-dimensional vector spaces, the sets of conjugate linear involutions and Grassmannians of exchange spaces.

The book is self-contained and prospective audience includes researchers working in algebra, combinatorics and geometry, as well as, graduate and advanced undergraduate students. The requirement to the reader is knowledge of basics of algebra and graph theory.

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Mark Pankov