Preface

For several years, one of the authors (A. P. B.) taught a one-semester graduate course in group theory to second-year graduate students in physics at Syracuse University. The students in this course were expected to have a prior knowledge of the familiar tools of mathematical physics such as linear algebra and complex analysis and a good background in classical and quantum mechanics. These students were preparing to engage in such diverse fields of research as condensed matter and particle physics, and the course was designed to give them a broad culture in group theory.

In 1984, Balachandran and Trahern published a book entitled "Lectures on Group Theory for Physicists". It was based on these courses and addressed to second-year graduate students in physics with a reasonable understanding of linear vector spaces, classical mechanics and quantum mechanics who seeks a general education in group theory and to teachers of such a course as well. Classroom experience showed that the material of this book can be covered without difficulty in one semester. This was an elementary book for physicists so that no attempt was made at rigor. Furthermore, since it was not addressed to students in one or another particular field, there was also a conscious attempt to treat the subject from a broad perspective and to avoid lengthy discourses on specialized topics like finite groups or Lie algebras.

Recently, Balachandran and his colleagues Sang Jo and Giuseppe Marmo decided to revise this book. One reason for our decision has been the emergent significance of Hopf algebras or quantum symmetries in fundamental physics. Symmetry groups provide simple, but particular examples of Hopf algebras. The latter, however, are more general and yet retain the features required of a symmetry in quantum theory. Quantum symmetries can thus be based on Hopf algebras which do not come from groups. Hopf algebras such as $SU(2)_q$ with features transcending those of groups first made their appearance in physics during the study of integrable models and conformal field theories. Recently, they have also assumed a central role in quantum field theories on noncommutative spacetimes. But there is still no widespread appreciation of the far-reaching importance of Hopf algebras for fundamental physics. It seemed to us that a revision of the 1984 book which includes a simple introduction to Hopf algebras is now appropriate.

On the other hand, we did not want to make the unduly long. Our intention has been to write a book which can be used for a one-semester course like the old one, or for a two-semester course which also treats relatively advanced topics.

The book is divided into five parts and still maintains the lecture note style. The first three essentially comprise the material of the old version. In part IV, besides relatively minor additions and explanations of the old version, we have now included a discussion of the Galilei group and its relation to the Poincaré group via group contractions. We have also included a discussion of the reduction of the direct product representations of the Poincaré group and the derivation of the Landau-Yang theorem on the non-existence of the $Z^0 \to 2\gamma$ from there. This theorem is a striking group theoretical result which does not rely on quantum field theory at all.

Part V contains an introductory treatment of Hopf algebras. The literature on Hopf algebras and their applications is extensive. Our intention is not to give an in-depth review of this subject. Rather we want to explain why it is progressively recognized that quantum symmetries need not be based on groups and instead can be based on Hopf algebras and their variants, and then introduce the reader to their elementary properties. We wish also to illustrate the applications of these algebras in physics using examples from quantum field theories on noncommutative spacetimes.

There are other beautiful topics of interest to physicists in Lie group theory which we do not discuss here. There is for example a whole body of literature which discuss coadjoint orbits of the Lie groups. They are symplectic manifolds, and in favorable cases, all unitary representations of the parent group can be obtained by quantizing them. We have discussed this material in detail elsewhere.[1,2] It can be used to supplement this part.

[1] A. P. Balachandran, G. Marmo, B.-S. Skagerstam and A. Stern, *Gauge Symmetries and Fibre Bundles* (Springer-Verlag, Berlin, 1983).

[2] A. P. Balachandran, G. Marmo, B.-S. Skagerstam and A. Stern, *Classical Topology and Quantum States* (World Scientific, Singapore, 1991).

There is also a sophisticated literature on Kac-Moody and Virasoro algebras which has a fundamental role in conformal field theories and string physics. We refer the reader to specialized treatises for their treatment.[3,4]

The first three parts of the book, like its old version, should be accessible to a second year student working towards a doctoral degree on a topic in theoretical physics. Parts IV and V are somewhat more demanding and require greater mathematical sophistication. Part V on Hopf algebras in particular contains introductory material to active research areas in physics and mathematics.

We thank C. G. Trahern for generously granting us permission to use the material of the old book in this new version, and Francesco del Franco and Bibliopolis for waiving copyright. This book would have been impossible to write without their cooperation.

[3]See for example P. D. Francesco, P. Mathieu and D. Sénéchal, *Conformal Field Theory* (Springer-Verlag, Berlin, 1997).

[4]See for example M. Green, J. H. Schwarz and E. Witten, *Superstring Theory*, volumes 1 and 2 (Cambridge University Press, 1987).