

# Contents

<i>Preface</i>	v
<b>GENERAL NOTIONS</b>	<b>1</b>
<i>LECTURE 1</i>	3
i) Definition of a group and examples . . . . .	3
<i>LECTURE 2</i>	7
i) Mapping and functions for sets . . . . .	7
ii) Isomorphisms and homomorphisms . . . . .	7
iii) $SU(2)$ and $SO(3)$ . . . . .	9
<i>LECTURE 3</i>	11
i) $SU(2)$ and $SO(3)$ (continued) . . . . .	11
ii) Subgroups . . . . .	12
iii) Cosets and invariant subgroups . . . . .	13
<i>LECTURE 4</i>	15
i) Cosets and invariant subgroups (continued) . . . . .	15
ii) Conjugate elements and classes . . . . .	17
iii) Simple groups . . . . .	17
<i>LECTURE 5</i>	19
i) Abelian and semi-simple groups . . . . .	19
ii) Representations of a group . . . . .	20
iii) The regular representations . . . . .	21
<i>LECTURE 6</i>	23
i) Reducibility of representations . . . . .	23

ii) Full or complete reducibility . . . . .	25
iii) Irreducible representations . . . . .	25
<b>LECTURE 7</b> . . . . .	<b>29</b>
i) Schur's lemma . . . . .	29
ii) Operations with representations and with groups . . . . .	31
iii) Direct product of representations . . . . .	33
iv) Direct products of groups . . . . .	35
 <b>FINITE GROUPS</b>	 <b>37</b>
 <i>LECTURE 8</i> . . . . .	 39
i) Definitions and simple results . . . . .	39
 <i>LECTURE 9</i> . . . . .	 43
i) Representation theory of finite groups . . . . .	43
ii) Character theory . . . . .	45
 <i>LECTURE 10</i> . . . . .	 49
i) Character theory (continued) . . . . .	49
ii) The regular representation of a finite group . . . . .	50
iii) The group or Frobenius algebra $\mathbb{C}G$ for a finite group $G$ . . . . .	51
iv) The symmetric or permutation group $S_n$ . . . . .	53
v) Cycles in $S_n$ . . . . .	54
 <i>LECTURE 11</i> . . . . .	 55
i) Cycles in $S_n$ (continued) . . . . .	55
ii) Reduction of regular representation of $S_n$ . . . . .	57
 <b>LIE GROUPS</b>	 <b>63</b>
 <i>LECTURE 12</i> . . . . .	 65
i) Definition and examples . . . . .	65
 <i>LECTURE 13</i> . . . . .	 69
i) More on $GL(n, \mathbb{C})$ and its subgroups . . . . .	69
 <i>LECTURE 14</i> . . . . .	 73
i) Continuous curves or paths in $G$ . . . . .	73
ii) Connected component to identity $G_0$ of a Lie group $G$ . . . . .	73
iii) Invariant integration . . . . .	75

<i>LECTURE 15</i>	79
i) Invariant integration (continued)	79
ii) The left regular representation $\Gamma^L$ on $H_L$	81
 <i>LECTURE 16</i>	 83
i) Lie algebras	83
ii) Equivalence of representations for Lie algebras	86
iii) Reducibility of representations for Lie algebras	87
iv) More on Lie groups	88
 <i>LECTURE 17</i>	 91
i) Lie algebras as differential operators	91
ii) More on Lie algebras as differential operators	94
 <i>LECTURE 18</i>	 95
i) On canonical realizations of Lie algebras	95
ii) On harmonic polynomials	96
 <i>LECTURE 19</i>	 99
i) Relation between Lie groups and Lie algebras	99
 <i>LECTURE 20</i>	 103
i) More on one-parameter subgroups 1	103
ii) More on one-parameter subgroups 2	104
 <i>LECTURE 21</i>	 107
i) Connectivity	107
ii) Summary of relations between Lie groups and Lie algebras	109
 <i>LECTURE 22</i>	 113
i) Representation theory	113
ii) The Lie algebra $L_{SO(3)}$ of $SO(3)$	114
iii) The Lie algebra $L_{SU(n)}$ of $SU(n)$	115
 <i>LECTURE 23</i>	 119
i) The center of a Lie algebra	119
ii) Casimir operators	119
iii) General considerations on the adjoint representation	120
iv) The Cartan-Killing form	122
v) Simple and semi-simple Lie algebras	123
 <i>LECTURE 24</i>	 125
i) Compact and simple Lie algebras	125

<i>LECTURE 25</i>	129
i) The rank of a Lie algebra . . . . .	129
ii) The irreducible representations of $SO(3)$ or $SU(2)$ Lie algebras . . . . .	130
<i>LECTURE 26</i>	135
i) Tensor methods for groups . . . . .	135
ii) The group $GL(n, \mathbb{C})$ . . . . .	137
<i>LECTURE 27</i>	139
i) Dimensions of irreducible subspaces of $GL(n, \mathbb{C})$ . . . . .	139
<i>LECTURE 28</i>	143
i) Classification of irreducible representations of $GL(n, \mathbb{C})$ . . . . .	143
ii) The group $GL(n, \mathbb{R})$ . . . . .	145
iii) The group $SL(n, \mathbb{C})$ . . . . .	145
iv) The group $SL(n, \mathbb{R})$ . . . . .	146
<i>LECTURE 29</i>	147
i) Tensor methods for compact Lie groups . . . . .	147
ii) The group $U(n)$ . . . . .	147
<i>LECTURE 30</i>	151
i) The group $SU(n)$ . . . . .	151
ii) The group $O(n)$ . . . . .	152
iii) The group $SO(n)$ . . . . .	153
<b>THE POINCARÉ GROUP</b>	<b>155</b>
<i>LECTURE 31</i>	157
i) The connectivity of $O(3, 1)$ . . . . .	157
<i>LECTURE 32</i>	163
i) The universal covering group of $\mathcal{L}_+^\uparrow$ . . . . .	163
ii) The universal covering group of $\mathcal{P}_+^\uparrow$ . . . . .	164
iii) The Lie algebra of $\mathcal{P}_+^\uparrow$ . . . . .	164
<i>LECTURE 33</i>	167
i) Galilei group . . . . .	167
ii) Group contraction . . . . .	168
iii) Other examples . . . . .	170

*LECTURE 34* 173  
 i) The UIRR's of  $\bar{\mathcal{P}}_+^\uparrow$  . . . . . 173  
 ii) The orbits  $O_{\hat{p}}$  and the little groups  $G_{\hat{p}}$  . . . . . 174  
 iii) Massive positive energy representations ( $I_a$ ) . . . . . 176  
 iv) Massless positive energy representations ( $II_a$ ) . . . . . 177

*LECTURE 35* 181  
 i) Reduction of the direct product of two UIRR's . . . . . 181  
 ii) Direct product of massive states . . . . . 181  
 iii) Direct product of massive and massless states . . . . . 186  
 iv) Direct product of two massless states . . . . . 187

*LECTURE 36* 189  
 i) Direct product of two identical particle states . . . . . 189  
 ii) Selection rules and the Landau-Yang theorem . . . . . 190

**HOPF ALGEBRAS IN PHYSICS 193**

*LECTURE 37* 195  
 i) On symmetries and symmetry groups . . . . . 195  
 ii) Groups as symmetries . . . . . 196

*LECTURE 38* 201  
 i) The coassociativity of the coproduct . . . . . 201

*LECTURE 39* 205  
 i) Example . . . . . 205

*LECTURE 40* 207  
 i) Further properties of  $CG$  . . . . . 207

*LECTURE 41* 209  
 i) Introducing Hopf algebras . . . . . 209

*LECTURE 42* 213  
 i) Identical particles and statistics . . . . . 213

*LECTURE 43* 217  
 i) The braid group and Yang-Baxter relations . . . . . 217

*LECTURE 44* 221  
 i) The Kuperberg diagrams . . . . . 221

<i>PROBLEMS</i>	229
<i>Bibliography</i>	243
<i>Index</i>	247