and the single particle drifts are useful tools for the description of the plasma dynamics. In all other cases one must refer to a *collective behavior* of the plasma which arises from the internal correlations between particles and fields even in the collisionless case. The plasma may then be considered not to consist of single particles but of particle fluids species. Each fluid can have its own density, bulk speed, pressure and temperature.

Such fluids when immersed into a magnetic field experience a *diamagnetic drift* which has been derived in Eq. (I.7.72). Obviously, this drift is a *collective effect* insofar as the collective particle pressure comes into play

$$\mathbf{v}_{dia,s} = \frac{\mathbf{B} \times \nabla_\perp p}{q_s n_s B^2} \tag{1.12}$$

Like the polarization drift, this bulk *pressure gradient drift* motion leads to currents, drift waves, may cause instability and nonlinear effects.

For completeness we mention that inhomogeneities and curvatures in the magnetic field generate additional drift motions in a plasma. In the present volume we will not make use of these, but refer the reader to the companion volume, *Basic Space Plasma Physics*, for reference.

1.3. Basic Kinetic Equations

Single particle effects, like the particle motion reviewed in the previous section, are often hidden in a plasma. In general, plasma dynamics cannot be described in such a simple way, but is determined by complicated correlations between particles and fields. The full set of basic equations of a plasma consists of the two *Maxwell equations*

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} \tag{1.13}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{1.14}$$

which must be completed by the two additional conditions, the absence of magnetic charges and Poisson's equation for the electric charge density, ρ

$$\nabla \cdot \mathbf{B} = 0 \tag{1.15}$$

$$\nabla \cdot \mathbf{E} = \rho/\epsilon_0 \tag{1.16}$$

The current and charge densities are defined as the sums over the current and charge densities of all species

$$\mathbf{j} = \sum_s q_s n_s \mathbf{v}_s \tag{1.17}$$

$$\rho = \sum_s q_s n_s \tag{1.18}$$

The bulk velocities and densities must be calculated from the basic equations determining the dynamics of the plasma. In a purely collisionless state the most fundamental equation describing the plasma dynamics is the *Vlasov equation*, taken separately for each species

$$\left[\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla + \frac{q_s}{m_s}(\mathbf{E} + \mathbf{v}\times\mathbf{B})\cdot\frac{\partial}{\partial \mathbf{v}}\right] f_s(\mathbf{x}, \mathbf{v}, t) = 0 \tag{1.19}$$

which is a scalar equation for the particle *distribution function*. For its justification and derivation see Chap. 6 of the companion volume, *Basic Space Plasma Physics*. The densities and bulk velocities entering the current and charges are determined as the *moments* of the distribution function, f_s, as solution of the Vlasov equation

$$n_s = \int d^3v\, f_s(\mathbf{x}, \mathbf{v}, t) \tag{1.20}$$

$$n_s\mathbf{v}_s = \int d^3v\, \mathbf{v} f_s(\mathbf{x}, \mathbf{v}, t) \tag{1.21}$$

The Vlasov equation together with the system of field equations and definitions of densities and currents turns out to be a highly nonlinear system of equations, in which the fields determine the behavior of the distribution function and the fields themselves are determined by the distribution function through the charges and currents.

This self-consistent system of equations forms the basis for collisionless plasma physics. In our companion volume we present a number of solutions of this system of equations for equilibrium and linear deviations from equilibrium. In the following we extend this approach to a number of unstable solutions and into the domain where nonlinearities become important.

The Vlasov equation (1.19) may be used to derive *fluid equations* for the different particle components. The methods of constructing fluid equations is given in Chap. 7 of the companion volume, *Basic Space Plasma Physics*. It is based on a moment integration technique of the Vlasov equation which is well known from general kinetic theory. One multiplies the Vlasov equation successively by rising powers of the velocity $\mathbf{v}$ and integrates the resulting equation over the entire velocity space. The system of hydrodynamic equations obtained consists of an infinite set for the infinitely many possible moments of the one-particle distribution function, f_s. The first two moment equations are the continuity equation for the particle density and the momentum conservation equation

$$\frac{\partial n_s}{\partial t} + \nabla\cdot(n_s\mathbf{v}_s) = 0 \tag{1.22}$$

$$\frac{\partial n_s\mathbf{v}_s}{\partial t} + \nabla\cdot(n_s\mathbf{v}_s\mathbf{v}_s) = n_s\frac{q_s}{m_s}(\mathbf{E} + \mathbf{v}_s\times\mathbf{B}) - \frac{1}{m_s}\nabla p_s \tag{1.23}$$

where, for simplicity, the pressure has been assumed to be isotropic. These equations have to be completed by another equation for the pressure or by an energy law.

In the following chapters of the present book we will frequently take advantage of the more simple fluid approach in spite of the simplifying assumptions underlying any fluid theory. In fact, many of the nonlinear problems cannot be treated in the framework of the precise kinetic approximation. In these cases one is forced to return to the fluid description in order to treat the nonlinear effects at least in a approximate way.

1.4. Plasma Waves

The system of Vlasov-Maxwell equations or its hydrodynamic simplifications allow for the propagation of disturbances on the background of the plasma. Generally, these disturbances are nonlinear time-varying states the plasma can assume. But as long as their amplitudes are small when compared with the undisturbed field and particle variables, they can be treated in a linear approximation as small disturbances. This condition can be written as $|\delta A(\mathbf{x}, t)| \ll |A_0(\mathbf{x}, t)|$, where δA is the amplitude of the variation of some quantity $A(\mathbf{x}, t)$, and A_0 is its equilibrium undisturbed value which may also vary in time and space. In the linear approximation such disturbances of the plasma state represent propagating waves of frequency, $\omega(\mathbf{k})$, and wavenumber, $\mathbf{k}$. As usual, the phase and group velocities of these waves are defined as

$$\mathbf{v}_{ph} = \frac{\omega(\mathbf{k})}{k^2}\mathbf{k} \tag{1.24}$$

$$\mathbf{v}_{gr} = \frac{\partial\omega(\mathbf{k})}{\partial\mathbf{k}} = \nabla_{\mathbf{k}}\omega(\mathbf{k}) \tag{1.25}$$

The *phase velocity* is directed parallel to $\mathbf{k}$ and gives direction and speed of the propagation of the wave front or phase

$$\phi(\mathbf{x}, t) = \mathbf{k}\cdot\mathbf{x} - \omega(\mathbf{k})t \tag{1.26}$$

while the *group velocity* can point into a direction different from the phase velocity. It gives the direction of the flow of energy and information contained in the wave. Both can be calculated from knowledge of the frequency. The latter is the solution of the wave dispersion relation in both the linear approximation and the full nonlinear theory.

In the linear approximation the dispersion relation is particularly simple to derive. Because of the linear approximation, the full set of Maxwell-Vlasov or Maxwell-hydrodynamic equations contains only linear disturbances. Thus the system can be reduced to a set of linear algebraic equations with vanishing determinant

$$D(\omega, \mathbf{k}) = 0 \tag{1.27}$$

the *dispersion relation*. The analytical form of the dispersion relation is obtained from the linearized wave equation (I.9.45)

$$\nabla^2\delta\mathbf{E} - \nabla(\nabla\cdot\delta\mathbf{E}) - \epsilon_0\mu_0\frac{\partial^2\delta\mathbf{E}}{\partial t^2} = \mu_0\frac{\partial\delta\mathbf{j}}{\partial t} \tag{1.28}$$