Chapter I

The Nature of Light

Since the dawn of history man has been fascinated by the universe around him, most of which he senses by means of his eyes. He has therefore always considered light as one of the most important phenomena.

1.1 ORIGINS

The ancient world was already familiar with many optical principles and devices, among them the observation that light travels in straight lines, and the law of specular reflection for a ray of light, e.g. that the angles of incidence and reflection lie in a plane determined by the incident ray and a line perpendicular to the surface at the point where the incident ray intersects the reflecting surface, and that the angles of incidence and reflection are equal. See Fig. 1.1, where i is the angle of incidence and r is the angle of reflection:

$$i = r \tag{1.1}$$

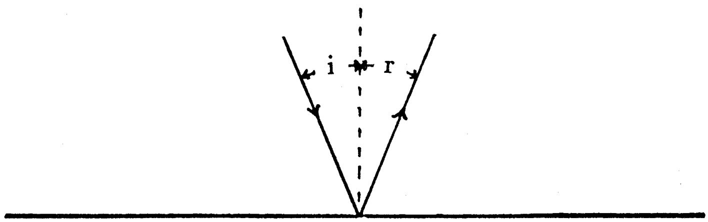

Fig. 1.1. The Law of Reflection.

(Although refraction was also known in classical antiquity, the correct law of refraction was not discovered until 2000 years later during the 17th Century). Plane and spherical mirrors were also known to the ancients and the latter as well as simple lenses were used to focus the sun's rays for starting fires.

The Greek philosopher Empedocles of Agrigentum (c. 490–430 B.C.) was one of the first to put forth a theory of light, teaching essentially correct ideas of vision.

Euclid, besides developing the famous axioms and theorems of geometry, was the author of a treatise *Optics* in which the fundamentals of geometric optics are set forth, probably for the first time.

Archimedes (c. 287–217 B.C.), known for his famous Principle of Flotation, is said to have used concave mirrors to set the Roman ships afire during the Roman siege of Syracuse in 212 B.C.

The actual nature of light was, however, unknown to the ancients and, for example, it was generally believed that its speed was infinite. After the close of the ancient period, roughly coinciding with the last western Roman Emperor, Romulus Augustulus in 475, until the beginning of the Renaissance, which is often considered to have begun with the fall of the Eastern Roman Empire when Constantinople fell to the Ottoman Turks under Mehmet the Conqueror in 1453, there is a gap of nearly a thousand years during which very little occurred in the development of optical science.

Spectacles, however, were first depicted by the religious scribes in Europe during the 13th century, and were probably known in China long before.

1.2 REFRACTION

The revival of learning which occurred in Europe during the Renaissance did not neglect the study of light. Willebrord Snell (c. 1620) and René Descartes (1637) independently discovered the correct law of refraction for a ray of light incident at the boundary of two transparent media, see Fig. 1.2:

$$n_1 \sin i = n_2 \sin t \tag{1.2}$$

where n_1 and n_2 are quantities that depend on the properties of the two media, and are called their *indices of refraction* and i and t are the angles measured from the perpendicular of a ray of light in each medium. As with reflection, the two rays lie in a plane determined by the incident ray and the line perpendicular to the surface at the point where the incident ray contacts the surface. From this deceptively simple law, together with the law of reflection, almost all the principles of modern lens and optical mirror design can be derived.

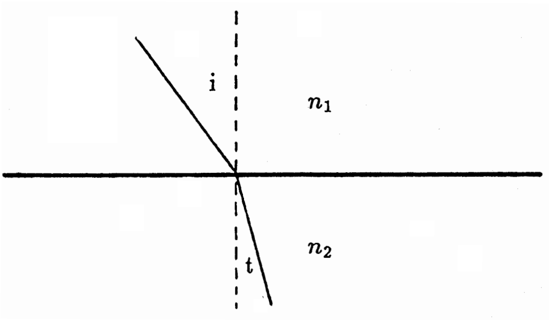

Fig. 1.2. The Law of Refraction.

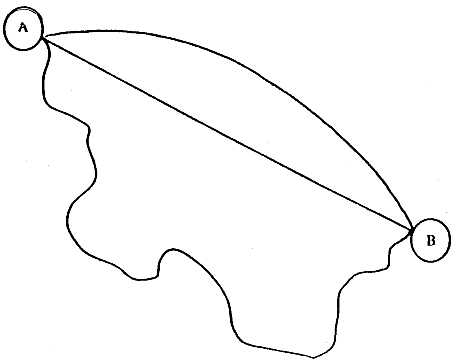

Fig. 1.3. Fermat's Principle.

1.3 FERMAT'S PRINCIPLE

Both the laws of refraction and reflection, together with that of rectilinear propagation, can, however, be derived from an even more fundamental theorem, foreshadowed by Heron of Alexandria in Antiquity (c. 62 A.D.), and put forth by Pierre de Fermat about 1660 — *a ray of light follows the path of "least time"*. In other words if a ray of light is to pass from point A to point B (see Fig. 1.3), it will follow the path which will take the least time (in the case shown, the straight line). The laws of rectilinear propagation and reflection follow immediately, and the law of refraction can also be shown, with a little calculation, to follow, since the speed of light in a medium is inversely proportional to the medium's index of refraction.

Fermat's Theorem implies something that had long been suspected but never proven that the speed of light is not infinite, but that light propagates with a definite speed that could, in principle, be measured.

1.4 THE SPEED OF LIGHT

Galileo (1564–1632) had apparently tried to measure the speed of light by uncovering a lantern on a mountain and having an assistant on a neighboring mountain uncover his lantern as soon as he saw the light from the first lantern. The experiment failed because the speed of light was far too great to measure in that way.

Ole Römer, while developing a method to determine longitudes by using the rotations of the moons of the planet Jupiter, an idea originally suggested by Galileo, discovered that the speed of light was finite, and about 2.998×10^8 m/s. At present, the speed of light is among the most accurately known values, having been determined to better than one part in 10^{12}.

1.5 THE WAVE NATURE OF LIGHT

1.5.1 What Are Waves?

Waves are familiar to all of us. Perhaps the best known are the *gravity* *waves* that form on the surface of liquids such as the water in the sea. These are of many forms. When they are rather low in the open water surface they have almost perfect sine shapes. When they grow large and approach the shore they change form and the tops curl over, breaking as surf. It turns out that all shapes of waves can be considered as combinations of simple sine waves, so for the time being, we will describe all waves as though they have the shape of the sine function, as shown in Fig. 1.4.

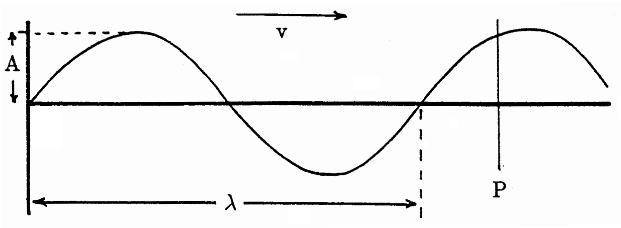

Fig. 1.4. A wave.

1.5.2 Wave Characteristics

Waves are characterized by certain parameters including their *amplitude, velocity, wavelength, frequency,* and *period*. In Fig. 1.4, *A* is the amplitude, and λ is the wavelength. Notice that the wavelength is the distance along the wave from a point to where the wave begins to repeat. When the wave moves with velocity, *v*, a number of waves pass a given point, P for example, in a given time. This number is the frequency, *f*. The period, *T*, is the reciprocal of the frequency, 1/*f*, and is expressed in units of time, usually seconds. There is a relation between the velocity, the wavelength, and the frequency which can be expressed as:

The wavelength equals the velocity divided by the frequency.

Algebraically it is written as:

$$\lambda = v/f \tag{1.3}$$

where λ is the wavelength, *v* the velocity and *f* the frequency.

Let us look first at the units to see if they make sense. (This is a good idea whenever we deal with an equation, not only to help understand what is going on, but also to check whether the relation is correct). Using SI units, the wavelength will be in meters, the velocity in meters divided by seconds, and the frequency in number of cycles, which has no units, divided by seconds. Putting these units together we will have a *units* equation. The units must turn out to be the same on both sides of the equation (apples equal apples):

meters = (meters/seconds) divided by 1/seconds

inverting the denominator and multiplying:

meters = (meters/seconds) × seconds

The seconds cancel out, and we have meters = meters, which checks.

The wave equation can be written in several equivalent forms. Since they are often refered to, they are listed here:

$$\lambda = v/f \qquad f = v/\lambda \qquad v = f\,\lambda$$
$$\lambda = vT \qquad T = \lambda/v \qquad v = \lambda/T \tag{1.4}$$

In the second set of three, the period, T, which is the reciprocal of the frequency has been inserted.

Examples:

1. The velocity of a water surface gravity wave is 2 meters per second. A small anchored boat is bobbing up and down once every 4 seconds. What is the wavelength of the waves?

Answer: v, the velocity is given as 2 meters/second. The period, T, is the time between cycles of the boat's bobbing or 4 seconds. We use the equation relating velocity, period and wavelength which is solved for wavelength:

$$\lambda = v\mathrm{T} \tag{1.5}$$

Substituting values, we find that λ, the wavelength, equals the velocity, 2 meters per second, times the period, 4 seconds, from which we obtain 8 meters for λ.

The question as to whether light consists of waves or bullet-light particles occupied the attention of scientists for almost the next two centuries. Christian Huygens (1620–1605), a Dutch astronomer, led those who believed that light consisted of waves, while Isaac Newton (1642–1726) championed the particle or *corpuscular* model. The corpuscular model turned out to be by far the more widely accepted during the next century and a half. This was because light seemed to travel in strict straight lines (except for an apparently unimportant small spreading which occurred when the light passed through a very small hole — more about this in a moment), while the wave phenomena familiar at the time, e.g. sound, water waves and the like, all were characterized by noticeable spreading. Bullets, of course, travel in straight lines, so the corpuscular model appealed to those guided by "common sense." Newton's overwhelming stature in science must also have contributed to the popularity of the corpuscular theory.

There was, however, one phenomenon which stubbornly refused to fit into any sort of corpuscular model. This was *double refraction*. It was known that, if oriented in a certain way, some crystals, especially of calcite (also called iceland spar) produced *two* refracted beams when a single beam of light was directed at them, one (the *ordinary ray*) which followed the Snell-Descartes refraction rule, and another which did not (the *extraordinary ray*), see Fig. 1.5), where the ordinary ray is marked *O* and the extraordinary *E*. (The extraordinary ray angle is somewhat exaggerated in

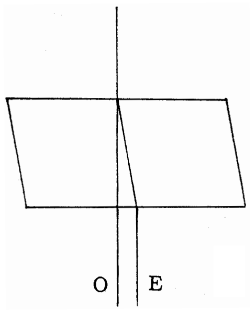

Fig. 1.5. Double refraction.

the figure). This was explained by Huygens using a wave model for light, but it could not be explained using a corpuscular theory).

Finally in 1802 the British physician *cum* physicist Thomas Young (1773–1829) performed a definitive experiment which showed not only that light must have wave properties, but allowed him to measure the wavelength itself. The experiment consisted of passing light from a small distant source through two neighboring slits and observing the resulting pattern on a screen (see Fig. 1.6). Upon the screen were regularly-spaced regions, or *fringes* of greater and lesser light intensity, which could only be explained by the *interference* of waves. Where the waves from the two slits arrive in step

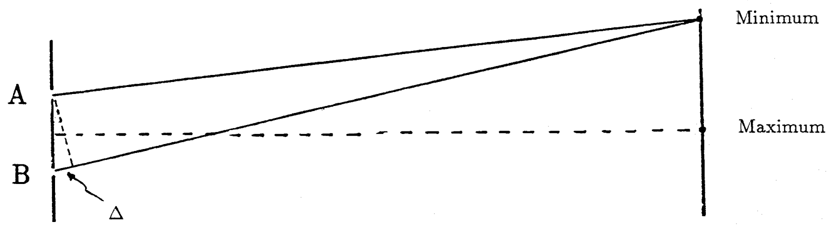

Fig. 1.6. Young's experiment.

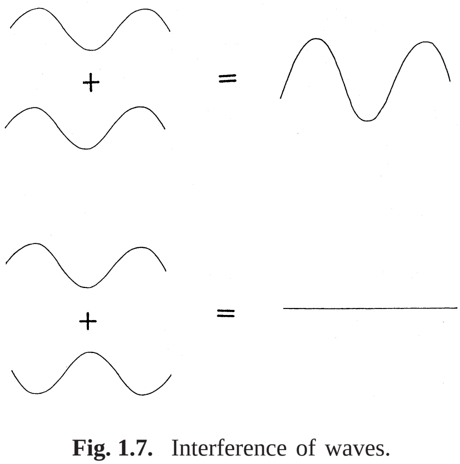

Fig. 1.7. Interference of waves.

(in phase) they add together, and where they are out of step (out of phase) they subtract and tend to cancel each other, see Fig. 1.7. In Young's day it was difficult to see the fringes since the light was very weak, but today the experiment is easy to perform with a laser. Referring again to Fig. 1.6, notice that when the path difference, D, from slit A to slit B is zero or an integral (by integral, we mean the integers 1, 2, 3, …) number. n, of wavelengths, $n \times \lambda$, there are intensity maxima, since in these cases, the waves arrive at the screen exactly in step. When the path difference, on the other hand is an integral multiple of a half wavelength, $n \times \lambda/2$, the waves arrive out of step and they tend to cancel each other.

1.6 DIFFRACTION AND RESOLUTION LIMITS

The basis for a wave model was further reinforced by the theoretical work of the French physicist Augustin Fresnel (1788–1827). He was able to use waves to explain the fact that when light passes through a small aperture it tends to spread out in angle, such that the smaller the aperture, the greater the spread. This is known as *diffraction* and was first described by the Italian physicist Grimaldi (1618–1663). At the time of Grimaldi's discovery the wavelength of light was unknown, so the significance of diffraction in supporting the wave theory was ignored (diffraction is an example of one of the fundamental laws of physics, the famous *Heisenberg Uncertainty Principle*).

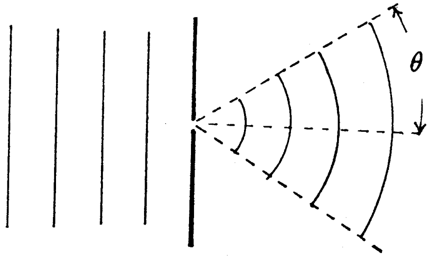

Fig. 1.8. Diffraction at a slit.

To simplify the situation, let us consider a two-dimensional case (see Fig. 1.8). Here a ray of light passes through a slit which has a breadth W. The actual pattern of the light after it passes through the slit is often complex, but one can consider the angular width of the resulting pattern to be between the places where the intensity has dropped to a certain fraction, usually taken to be about 80% of the maximum at the center. This is considered the condition where two equal neighboring images are just clearly distinguishable, and is called the *Rayleigh criterion* after Lord Rayleigh (1842–1919). It is commonly used to evaluate the resolving power of optical instruments such as the microscope, telescope and spectrometer. In the case above, when we have a slit with width W, the spread in the angle, θ, of the light after passing through the slit, as measured between the 80% intensity points is given by the simple relation:

$$\theta = \lambda / \mathrm{W} \tag{1.6}$$

where λ is the wavelength of the light and θ is measured in radians.

For a circular aperture, the situation is very much the same, but a correction factor has to be used, so the expression for θ_s becomes:

$$\theta_s = 1.22\ \lambda / \mathrm{R} \tag{1.7}$$

where R is the diameter of the aperture.

Since for most optical instruments (microscopes, telescopes, cameras and the human eye), the apertures (often simply the outer edges of the lenses) are circular, it is the second form of the diffraction relationship which is the most commonly used.

Let us consider a few examples:

(a) The human eye

The wavelengths of light for which the human eye is sensitive extend from about 400 nanometers to about 750 nm. For convenience, let us take 550 nm (which produces the sensation of green light) as a typical value. The pupil of the eye varies in diameter depending on the brightness of light from about 1 millimeter in very bright light to 7 mm in dim light. We can take 4 mm (or 4 × 10^6 nm) for an average value. Substituting 550 and 4 × 10^6 for λ and W in Equation 1.7 above we can calculate the angle of spread due to diffraction for light passing into the eye. Since diffraction tends to confuse an image this will give the angular limit of resolution, assuming a perfect eye:

$$\theta = 1.22 \times 550/4 \times 10^6 \qquad (1.8)$$

or $\theta = 1.68 \times 10^{-4}$ radians. (A radian is a convenient measure of angle. It is equal to $360/2\pi$ or about 57.3 degrees, so that 1.68×10^{-4} radian corresponds to about 1/120th of a degree.)

To see what this means, let us consider a pattern of black and white stripes (see Fig. 1.9). Since the diffraction angle in radians is about 1.7 parts in 10,000, this means that if the stripes are 1.7 cm wide, and the pattern is at a distance, *d*, 10,000 cm away from the eye (100 meters), the stripes could theoretically just be discerned. (Actually the human eye is not quite optically perfect; the best vision is about half this good. A person with 20–20 vision would just be able to distinguish the pattern if it were about 50 meters away.)

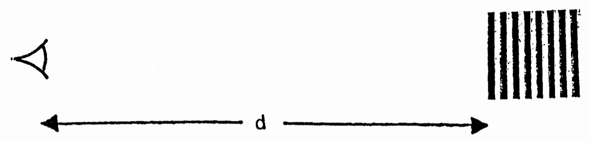

Fig. 1.9. Black and white stripes for diffraction.

(b) A small telescope

Assuming that the telescope would be used visually, we can take the wavelength to be same as for the human eye, 550 nm. A typical telescope might have an objective with a diameter of 8 cm, or about 3 inches. This is 20 times the value of 4 mm we have taken for the eye in the first example. Thus the limiting angle, θ, would be 20 times smaller than for the human eye, or 8.5×10^{-6} radian. This would mean that the pattern of 1.7 cm stripes would be visible 20 times further than in the example of the unaided eye, or about 2 kilometers away.

(c) A large telescope

The Mount Palomar reflector has a mirror with a diameter of 200 inches, or 508 cm. This is 1270 times larger than the value we have taken for the eye which makes the resulting angle so small that, assuming perfect imaging, the pattern would be discernible at a distance of 127 kilometers! (Unfortunately, no earth-based telescope can do as well as this because the atmosphere is not homogenous enough. This is one of the reasons for establishing the Hubble laboratory in space). Diffraction also limits the resolution of the microscope and the spectroscope.

1.7 THE QUANTUM THEORY AND PHOTONS

The idea that energy exists in small discrete pieces, *quanta*, was first proposed in 1900 by Max Planck to explain the radiation from a black body. A black body is one which absorbs all the radiation incident upon it. No perfect black body exists, but a good approximation is made by a hole drilled into a spherical or cylindrical shell. It turns out that the wavelength distribution of the radiation emitted by such a black body depends only on its temperature and not on its material, hence its usefulness. Fig. 1.10 shows

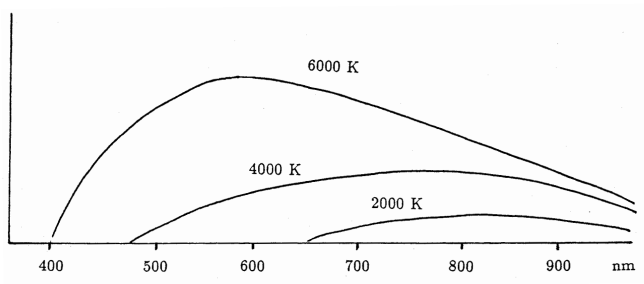

Fig. 1.10. The black body spectrum.

the spectrum (radiated energy plotted against wavelength) of the emission from a black body heated to various temperatures. The explanation of the shape of these curves remained a mystery at the close of the 19th century. After several theoretical attempts by a number of other researchers Planck finally succeeded in calculating the radiation distribution, but only by making the seemingly absurd assumption that the light must be emitted by myriads of radiators in the walls of the black body only in tiny discrete lumps or *quanta*. He was forced to assume that the amount of energy in each such quantum depends only on the wavelength, and obeys the following relation:

$$E = \frac{hc}{\lambda} \tag{1.9}$$

where h is a constant equal to 6.625×10^{-34} joule-sec., c is the speed of light in a vacuum, and λ is the wavelength of the radiation. (A joule is a measure of energy and is equal to one watt expanded during one second.)

In the hands of Planck, the quantum of energy was nothing more than an *ad hoc* assumption which was necessary to derive the black body radiation curve, and had no general physical significance.

It was Albert Einstein who in 1905 brought the quantum into the mainstream of scientific thought, and he did it by suggesting that light was not to be considered just as waves after all, but that it also had to be provided with particle properties. In other words Newton's corpuscular theory of light, which had been apparently put to rest by Young, Fresnel, and all the wave theorists had, in a new sense, to be resurrected. This is not to say that the wave theory is all wrong — light has wave properties to be sure — but light turns out to be more complex than that; it also has particle properties.

The phenomenon that suggested the particle picture of light to Einstein was the interaction of light with a solid surface, the photoelectric effect which was first noticed by Heinrich Hertz (1857–1894) when he was in the process of discovering radio waves c. 1887. The photoelectric effect is that when light strikes a solid in a vacuum, electrons are ejected from the surface (Fig. 1.11). (There are other forms of the photoelectric effect

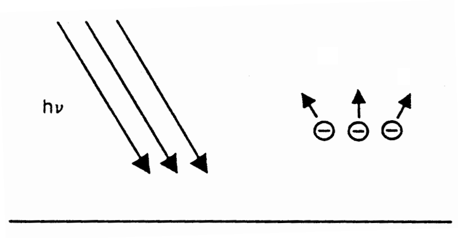

Fig. 1.11. The photoelectric effect.

involving gases and even liquids instead of solids, but we will not be concerned with them here). Various metals and semiconductors are more or less efficient photoelectrically, the alkali metal cesium, being especially so. The quantity and energy of the electrons emitted from the surface can be measured since they form an electric current. A surprising result, and the one that Einstein finally explained, is that the energy of the electrons ejected from the surface depends *not at all on the intensity of the light falling on the metal surface, but only on its wavelength*. This cannot be consistent with a pure wave model for light since the amplitude of the waves would certainly increase the energy of the particles (electrons) with which they interact (compare this with the effect of large and small ocean waves "exciting" pebbles on the beach).

To escape from this dilemma, Einstein enlisted the help of Planck's quanta, whose energy, as we have seen, depends only on their wavelength. Einstein theorized that *when interacting with matter* light, which otherwise acts like waves, acts as though it consists of particles, which he called *photons*, each of which has the energy of one of Planck's oscillators, hc/λ. When the photons interact with the electrons in the surface, their energy is transferred to the electrons, which neatly explains the wavelength dependence of the photoelectrons' energy. (Einstein received the Nobel prize in 1921, partially for this discovery.)

To describe light, we must therefore not only use a wave model but a particle model as well — waves during the transit of light from one place to another and particles when light is either emitted or absorbed. This dual nature of light turned out to be not as unique as it was thought at first. The constituents of ordinary matter, that is protons, neutrons and electrons, ordinarily thought of as particles, are now known also to have both particle and wave properties. In the case of these matter waves, the wavelength is given by:

$$\lambda = \frac{h}{p} \qquad (1.10)$$

where p is the momentum (mass multiplied by velocity) of the particle. This expression was first suggested by Louis de Broglie (1892–1987) in 1924. Their wavelength varies inversely with the momentum, p, of the photon.

According to Einstein's theory of relativity, the momentum of a photon is related to its energy, E, by the expression:

$$E = pc \qquad (1.11)$$

where c is the speed of light. Combining Equations 1.10 and 1.11, we again have Equation 1.9:

$$E = hc/\lambda. \qquad (1.12)$$

1.8 THE PRODUCTION OF LIGHT

As we have pointed out, the photon nature of light is encountered whenever light is emitted or absorbed. To emit light, a source of energy must be present such that the photons can be provided with their necessary energy. In almost every case we shall be interested in, this energy is provided by electrons.

1.8.1 The Energy of Bound Electrons

When electrons are attached, or *bound* to atoms and molecules, they can only exist stably with definite amounts (levels) of energy. The simplest picture of the atom which exhibits these levels of electron energy is the *Bohr model*, described in 1912 by Niels N.D. Bohr (1885–1962). He assumed that the atom was, as already (1911) had been suggested by Ernest

Rutherford (1871–1937), formed resembling the solar system with light negative electrons revolving around a heavy positive nucleus. Bohr also made the logical assumption that, in place of the gravitational attraction which binds the planets to the sun, the atom is held together by the electrical attraction between the negative electrons and the positive nucleus.

Where Bohr's approach was revolutionary was in his next two assumptions:

The first of these was that the *angular momentum*, p_ϕ of the electrons in their orbits (the electron's angular momentum is equal to the time rate of change of the angle around the orbit times the mass of the electron times the radius of the orbit) were confined to integral multiples, l, of $h/2\hbar\pi$.

$$p_\phi = l\hbar \tag{1.13}$$

Calculation from this assumption has the result that an electron circling a positive nucleus could exist only in orbits of definite energy according to the following law:

$$E = RZ^2/n^2 \tag{1.14}$$

where Z is the number of positive charges in the nucleus (one for hydrogen) and R is a constant called the Rydberg which Bohr derived from fundamental quantities and is given by:

$$R = 2\pi^2 me^4/ch^3 \tag{1.15}$$

where m is the mass of the electron, e is its electric charge, c is the speed of light and h is Planck's constant. Thus according to the Bohr model, the electrons orbit the nucleus in a manner similar to the artificial satellites around the earth but unlike such satellites, they are confined to orbits with definite amounts of energy, (see Fig. 1.12). It is a bit as though NASA's shuttle could be in an orbit at 100, 150, 200 and 300 kilometers above the earth and nowhere in between.

The second of Bohr's revolutionary assumptions was that the rotating electrons do not radiate as they would be expected to do according to classical physics but only give rise to radiation when they "jump" from one

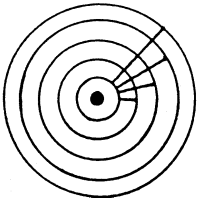

Fig. 1.12. The Bohr model for hydrogen.

orbit to another. If such a jump results in a decrease in electron energy, a photon is created whose energy equals the difference in the electron's energy. (There also are electron jumps which do not result in the emission of a photon; in this case the resulting energy is dissipated in other ways.) When a photon is produced, if the change in electron energy is ΔE, then the photon has a wavelength, λ, such that:

$$\lambda = hc/\Delta E. \tag{1.16}$$

By measuring the wavelengths (the spectrum) of the light emitted by the atom, the spacing of its energy levels can therefore be determined. Thus one may produce an *energy level diagram*, such as is shown for hydrogen in Fig. 1.13. Here some of the jumps or *transitions* between energy levels are shown, together with the photons emitted.

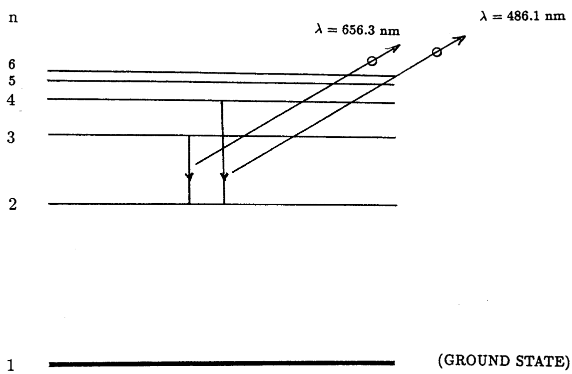

Fig. 1.13. Energy level diagram for hydrogen.

After de Broglie's discovery that electrons have wave properties, a new form of the theory of the atom was developed by E. Schrödinger (1887–1961), P.A.M. Dirac (1902–1984) and others which allowed more complete calculations than were possible using Bohr's model. This is now called *quantum mechanics* and it has provided an unparalleled expansion of our knowledge of atoms, molecules and virtually all aspects of the natural world.

For atoms with more than one electron, the situation is more complex but under ordinary conditions the electron transitions producing photons are performed only by the outer or *valence* electrons, usually numbering between one and four, so that the appearance of the energy level diagrams is similar to that of hydrogen, but often more complex. Sodium with 11 electrons, has only one electron beyond a closed shell (the valence electron) the lowest levels of which are shown in Fig. 1.14.

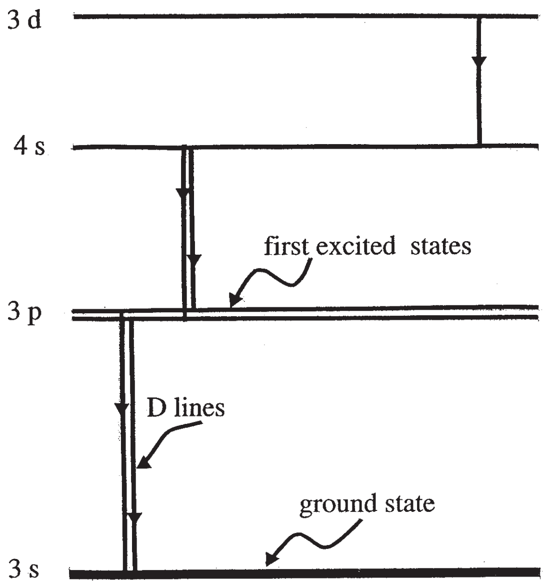

Fig. 1.14. Sodium energy levels. Adapted from H.E. White.

1.8.2 Electron Spin

The appearance in the spectrum of sodium and other elements of pairs of closely-spaced energy levels suggested that the electron acts like a tiny magnet and that its orientation in the atoms might be limited or quantized in two directions, explaining the small differences in energy. S. Goudsmit and G.E. Uhlenbeck proposed that the magnetic behavior of electrons could be explained if they were spinning. A spinning charged body acts as though it is circled with an electric current and an electric current since the time of Oerstead (1777–1851) has been known to produce a magnetic field.

According to quantum mechanics the angular momentum of the electron spin is $\hbar$ multiplied by the number 1/2, and it can exist in one of two orientations with respect to an external field.

1.9 THE PLANCK EQUATION AND PHOTOCHEMICAL REACTIONS

The energy characteristic of a monochromatic beam of radiation is related to its wavelength, λ or its frequency, v, as expressed in Equation 1.9.

When dealing with photochemical reactions the unit most commonly used is the mole, consisting of N molecules, where N is Avogadro's number, or 6.023×10^{23}. For radiation, a similar unit is used, called the *photon mole* (previously called the *einstein*) containing N photons. It follows that one photon mole has the radiant energy equal to Nh*n*.

$$1 \text{ photon mole} = 1.19629 \times 10^8 \text{ J}/\lambda \tag{1.17}$$

where λ is expressed in nanometers, nm.

As described by Equations 1.9 and 1.17, the energy of the photon mole decreases enormously as we pass from the shortest wavelengths of the electromagnetic spectrum to the longest, as shown in Table 1 of which the near ultraviolet and visible regions are the most important in photobiology. When these wavelengths are absorbed by biomolecules, the outermost electrons, e.g. those involved in atom binding are those affected. As a result chemical changes can occur during light absorption.

Table 1

Region of Spectrum	Typical wavelength in nm	Approx. photon moles in Joules
AM Radio	3×10^{11}	4×10^{-4}
Radar	5×10^{7}	2.4
Infrared	5×10^{3}	2.4×10^{4}
Visible	550	2×10^{5}
Near UV	320	3.75×10^{5}
X-rays	10	1.2×10^{7}
γ rays	10^{-3}	1.2×10^{11}

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