Preface

This text, unlike others, did not grow out of seminar or classroom lectures; instead it grew out of the author's conviction that present day physicists and mathematicians should know the basics of string and superstring theories, just as they know calculus, linear algebra, geometry and analysis. To reach this goal, however, the theory that is pursued at research level in selected schools has to be made available to a wider audience. This is possible only if the teaching (and a text) focuses not only on the string and superstring theories but also provides: (i) the elements of all the prerequisites; (ii) an overview of other great theories that have preceded it (since it uses their phenomenology); and (iii) a motivational thread to reach the end goal.

The present text is organized to fulfill these objectives. Since the target here is a much larger population of physicists and mathematicians, we avoid the mathematical rigor. No theorems (with few exceptions) are proved in the main text, in fact they are stated as 'Results' and 'Facts,' and their proofs (in important cases) are given as solutions to the exercises at the end of a section. This offers the reader (the teacher) the option of choosing the preferred level of in-depth/non-depth coverage (in a class). Besides this, the material is often presented using the two points of view (mathematics and physics) which makes the subject easily comprehensible. The first six chapters provide the mathematical background needed for the theory and thus fulfill the criterion (i).

Chapter 0 gives the definitions in topology, differentiable manifolds, analysis and algebraic topology, and Chapter 1 explains the basics of the theory of complex functions, Riemann surfaces, and two-dimensional conformal field theory.

In Chapter 2 a quick review of group theory which includes algebraic, topological and Lie groups is given. A brief description of bundle theory from two points of view (mathematics and physics) is also part of this chapter. Chapter 3 is devoted to elementary operator theory with emphasis on spectral decomposition of Hermitian and unitary operators, on generalized Schrödinger and Dirac operators, and on the operators formed by the generators of the groups $SU(2)$ and $SU(3)$.

Chapter 4 deals with the basics of finite-dimensional algebras. Solvable, semi-simple and simple Lie algebras along with their representations are studied, objects such as weights and root systems, Weyl groups, Cartan matrices and Dynkin diagrams are defined. Chapter 5 explains the intricacies of infinite-dimensional algebras, in particular those of Kac-Moody algebras, and Heisenberg algebras. Using the Dynkin indices the generalized Casimir operator is defined, and vertex operators (needed especially in string theory) are introduced.

Chapter 6, devoted to several aspects of symmetry (e.g., global and gauge) in nature and to symmetry breaking phenomena, is the first chapter that exposes the reader to particle physics. Examples based on different types of Lagrangians are used to explain various gauge theories, namely Maxwell's, Yang-Mills' and GSW's. Chapter 7 is a brief review of all those objects that have emerged, ever since the notion of supersymmetry gained credence amongst physicists and mathematicians. The chapter begins with the definitions of $\mathbf{Z}_2$-graded algebras (superalgebras), Lie superalgebras, Clifford algebras and spinors (Dirac, Majorana and Weyl). The concepts of supersymmetry transformations, superspace, supermanifold, superscalar field and supervector field, etc., are introduced in order to write a Lorentz-invariant super Lagrangian. The question of renormalizability is addressed, though rather briefly (by using the Wess-Zumino gauge). The form calculus on supermanifolds and Berezin integrals are also included in this chapter.

Chapter 8 gives an overview of the theories of gravitation, relativity and black holes. Beginning with Newton's laws and Einstein's free float frame, the principles of general relativity are explained. Well-known exact solutions of Einstein's equation (e.g., Schwarzschild), the singularity theorems of Penrose and Hawking's black holes are then studied.

Chapter 9 is devoted to quantum theories. Due to the vastness of the subject, it includes four appendices. It introduces the reader to principles of quantum mechanics, the so-called Schrödinger and Heisenberg pictures, the Dirac equation in a non-relativistic as well as relativistic field. The chapter then develops into Feynman's path integral formalism. The Feynman propagator, Green's function, the action principles in quantum mechanics and some examples based on path integrals are given. In Appendix D a brief review of quantum groups is also given.

Chapter 10 is an introduction to Yang-Mills (YM) and Yang-Mills-Higgs (YMH) theories, in particular to those aspects that have resulted from applications of index theory, algebraic geometry, and algebraic topology to these two theories. A qualitative study of solutions of YM and YMH equations (known as instantons vortices and monopoles) is done in this chapter. A section on anomalies is also included.

Chapter 11, devoted to strings and superstrings, introduces the reader to Regge trajectories, the Nambu-Goto action of a string, bosonic strings and their quantization, DDF operators, the No-ghost theorem, the Fadeev-Popov ghosts and ghosts in bosonic theory. Some global aspects of string world sheet, the world sheet supersymmetry and super Virasoro operators in string theory are described. The super-Virasoro algebra, the anomaly, and superstrings as a theory of unification are briefly overviewed.

From the above descriptions it should be apparent that Chapters 6, 8, 9 and 10 are in accordance with (ii). Chapter 7, on the other hand, provides the tools for describing superstring theory. Finally, the solved exercises and examples (approximately 250 in number), more than 60 illustrations, explanatory footnotes and appendices, and a large number of references provide the motivational thread towards our main goal: learning a synthesized theory of 'Mathematical Physics of the 21st Century.'

The book fails to be 'consistent' with 'symbols.' The diversity of covered subjects made the 'consistency in symbols' rather impractical. Sometimes alphabets and notations have been used to represent different objects, whereas at other times the very same object (e.g., hermitian conjugate ≡ h.c.) is denoted differently. A list of notations chapterwise and adequate footnotes for describing the symbols (wherever required) should help to alleviate this problem.

Since the book comprises of chapters devoted to different subjects, one may have the impression that the chapters are separate entities. This, however, is not the case as one would find a large number of cross-references spread through these chapters. The repeated references which are indicated by decimal notation (for instance, Ref. 10.[5] in Chapter 3 stands for Ref [5] of Chapter 10) are another proof of a thread that binds the chapters.

Every attempt has been made to make the book self-contained. A few concepts that remained undefined in former chapters as well as concepts e.g. p-branes, D-branes, and dualities that led to 'Second Superstring Revolution' and black holes in string theory, are explained at the end of the book in Appendix 11.B. Similarly some recent titles of interest and the references not covered earlier are added in the form of Reference Addendum at the end of 11.B.

The original inspiration for writing the book came from Raoul Bott's remark to the author. He thought the relation that existed between string theory and Schwartzian derivative used by the author in her work on projective structures was worth examining. A greater motivational inspiration from Isadore Singer that led to the planning of the book followed soon after; the author is indebted to him for sponsoring a visiting position at the MIT mathematics department.

The author acknowledges the encouragement received from Sigurdur Helgason, Victor G. Kac, R. A. Gangolli, J. N. Kapur, M. Sharma, Jim Eggleston, and David Ferriero, and the help from Dennis Porche in administrative matters, from Sylvie Besett, Suli Rocha, and Nini Wang in partial typing, from Hong Shu in proof-reading and preparing the references, and from Hayden library staff in searching the material during the course of this long project. Heartfelt thanks are due to Ulrich Gerlach and Scott Axelrod for their precious time to give comments on portions of the book, as well as to those whose excellent texts and original works helped in writing the book.

A conversation with Gerlach, Witten and Canizares, that led to the section on black holes is thankfully acknowledged.

The author also wishes to acknowledge Giuseppe Castellacci's help in editing, and Jean Morris' outstanding typing of the final version of the book, and Angela Chang's help in typing the subject index. In spite of their good work, and author's commitment to accuracy of information there will still be some errors, for which the latter is fully responsible; she would appreciate, if they are brought to her attention.

Finally, a word of gratitude to the staff of Tata McGraw-Hill, New Delhi for their patience and continued support during the writing of the book. It is hoped that the book in spite of its shortcomings will be a useful addition to the scientific literature.

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