1

Early Observations

The Sun dominates the sky by day but the stars and the Moon dominate the sky by night. It is true that the Moon can sometimes be seen by day but only very faintly.[1] The Moon and stars in their various phases can be studied comfortably whereas the Sun, with its bright and fiery appearance, is very difficult to study directly. Consequently, early observations were concerned more with the Moon and stars than with the Sun. We give here a very short review of the early work in so far as it refers to planetary science.

1.1. Stars and Planets

On a clear moonless night a person with normal eyesight can expect to see between five and six thousand stars. They remain in a standard set of fixed patterns across the sky, and have been portrayed since antiquity in terms of groups called the *constellations*. The stars of any group are generally not really linked together but appear linked only as seen from the Earth. They are, in fact, at very different distances from us but seem related rather like trees in a forest. Very careful observations show that the stars do, in fact, move to a small extent. This is called the *proper motion*. These very tiny movements were first detected by Halley in 1718. He showed that the positions of the stars Aldebaran and Arcturus had changed since the positions were catalogued in antiquity. The movements of the stars are generally very small (often some $0.1''$ of arc or less) and have been detected widely only since the accuracy of astrometry has been improved by the introduction of modern astronomical instruments. In excess of 100,000 such cases are now known. The proper motions of the nearer stars have recently been measured with high accuracy in the Hipparchus space mission of the

[1]A bright star whose position in the sky is known can also be seen by day through a small telescope against the bright background of the sky.

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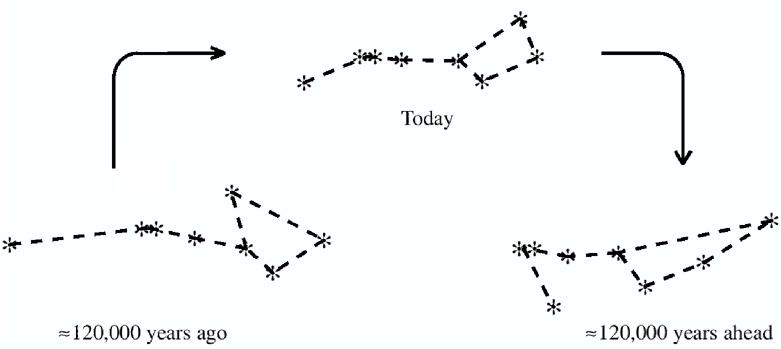

Fig. 1.1. The changing face of Ursa Major over a period of about 240,000 years. The top arrangement is as seen today, bottom left as it would have appeared 120,000 years ago and bottom right as it will appear 120,000 years in the future. Its everyday use as a guide to the north pole is only temporary.

European Space Agency. It has shown that the stars are rushing through space often at speeds that are very high by our everyday standards.

The small proper motions of the stars obviously will cause the constellations to change their shapes over time, although the time scale may be very large. As one example, Fig. 1.1 portrays the changes of the constellation Ursæ Majoris (the Great Bear or Big Dipper) over the period of some 240,000 years. The shape well known today was not there in the past and will not be there in the future. It can be noticed, however, that one pair of stars in the constellation are actually related and keep their link through the changes. This is a *double star.* The use of the constellation as a guide to the northern pole is only temporary on a cosmic time scale, and the marking of the pole by a star is also a transient feature. There is no star marking the southern pole at the present time. The same temporary form applies to the other constellations as well. The familiar pattern of the heavens was not the same for our very earliest ancestors and will not be the same for our distant successors. The changes are, however, so slow that they remain fixed to all intents and purposes from one generation to the next.

There are five points of light in the sky, separate from the stars, that the ancient observers realised to be special because they do, in fact, move noticeably across the constellations and are clearly not part of them. These were called then the *wandering stars* or the *planets* as we know them today (the word planet comes from the Greek word for wanderer). Whereas the stars flicker in the sky, it is said that the planets tend not to and that they can be recognised in practice immediately this way. They move across the sky following very much the same path as the Sun, called the *ecliptic.*

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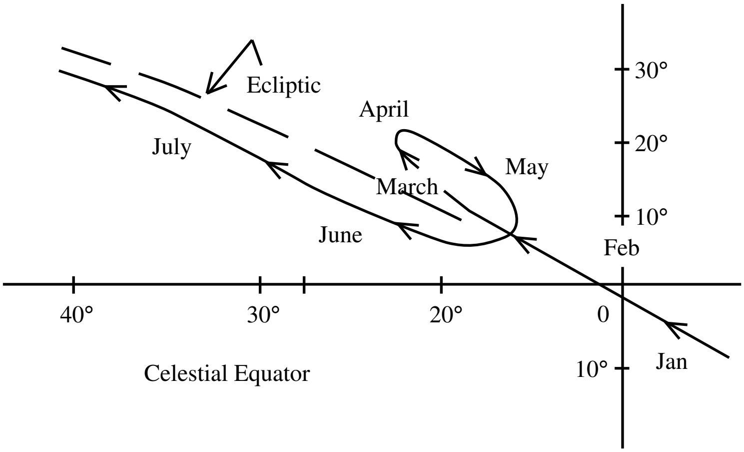

Fig. 1.2. The celestial path of Venus over a period from January to July. The motion is from right to left and moves from south to north of the celestial equator.

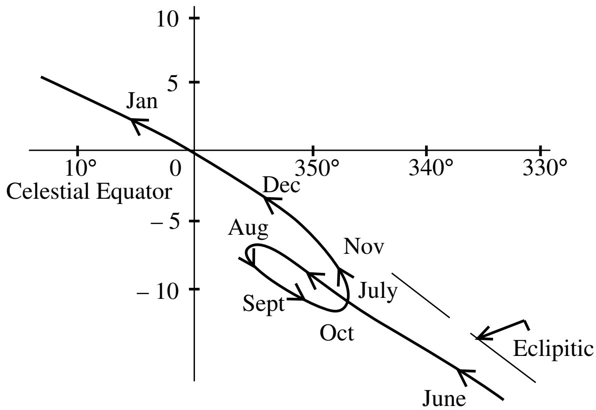

Fig. 1.3. The trajectory of Mars during the period June to January. The motion is from right to left. The ecliptic is also shown.

Whereas the Sun moves across the constellations in a simple path, that for the planets is more complicated. The path for Venus is shown in Fig. 1.2 and that for Mars in Fig. 1.3. There is an interesting feature. Although each planet moves from east to west in the sky, there is in each planetary path a part, lasting several months, where the motion is retrograde, that is moving the other way, from west to east. The retrograde motions of the two planets differ. Whereas that for Venus covers four months, that for Mars covers three. The different speeds across the sky allow the completion of a passage across the sky in different times for the two planets. The speed of

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Venus across the sky allows it to complete one passage in 250 days but Mars takes 465 days. Jupiter and Saturn take much longer times, respectively 5 and 10 years. The movement of the planets through the constellations is not the same for them all and this suggests they are not linked together.

Another set of observations is significant here. This describes the relations between the planets and the Sun. Whereas the motions of Jupiter and Saturn appear to move independently of the Sun though remaining on the ecliptic, the motions of Mercury, Venus and Mars seem linked to the annual movement of the Sun. The planet Mercury always stays close to the Sun as it moves across the sky, rising barely half an hour before sunrise or setting less than a half an hour after sunset. This makes the planet very difficult to observe by eye.[2]

1.2. Interpretations of the Observations

The earliest star chart yet known, but which unfortunately hasn't survived, is that of Hipparchus (c 127 BC). The earliest catalogue that has survived is the Almagest (c 137 AD) of Ptolemy with 1028 entries, including the five wandering stars. He attempted an explanation of the heavens as he saw them and offered a model of the observed motions of the Sun, Moon and planets which survived more than 1000 years. He made four assertions. The first is that the Earth is at rest (this seemed very obvious and was hardly an assertion to him). The second was that the Earth is at the centre of the firmament, as indeed it appears to be. The third that the Moon orbits the Earth and beyond that Mercury, Venus, Mars, the Sun, Jupiter and Saturn (in that order). The orbits are all circles lying in a common plane about the Earth. Finally, the stars in their constellations lie on the surface of a sphere, centred about the Earth, beyond the Sun and planets. This is the celestial sphere which rotated due to a supernatural action. The first of Ptolemy's assumptions seems so eminently reasonable — surely, any motion of the Earth would be recognised immediately — people would be knocked over and every moveable thing would move of its own accord. The third assumption involved the culture of the times which regarded the circle as the perfect figure, following the wisdom of the classical Greeks. The fourth assumption was obvious because there was no concept at that time of depthto the realm of the stars — they do appear to lie on the inside of a spherical ceiling. This is yet another example of the fact that you cannot always trust your eyes.

[2]A number of eminent observers of the past admitted to never actually having seen it.

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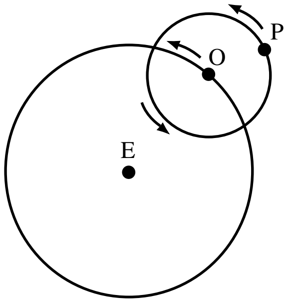

Fig. 1.4. A planetary circular path, with centre E, augmented by the epicycle with centre O. The observed motion of the planet is point P on the epicycle.

The orbits of the Sun and Moon appear simple to an observer on the Earth but we have seen that the motions of the planets involve the retrograde motions over part of their paths.[3] These special features had to be accounted for. This was done by introducing a so-called epicycle structure for the orbit, with the planet following a circular path whose centre is itself constrained to follow a circular path centred on the Earth.

Such a scheme is shown in Fig. 1.4. The selection of the orbits and the epicylces were chosen purely to fit the observations as closely as possible. This is an empirical approach with no theoretical background. Two cycles alone may not be able to reproduce the observations with very high accuracy. Better accuracy could have been obtained by adding more epicycles but this generally was not done.

It is interesting to notice that had Ptolemy moved consistently along this approach he would have achieved a very close empirical fit to the motions he sought and very likely have discovered the analysis which is now called the Fourier expansion.[4] It represents an actual curve by a series of sine or cosine terms with harmonic frequencies and with a magnitude for each term designed to fit the initial curve as closely as possible. This could have guided the analyses to a true path for the planet and would have advanced this branch of mathematics by 1,700 years but it was not to be. Mathematics, like language, is dependent on the perceptions of the times.

The Ptolemy model might appear artificial but the arrangement of the Sun, Moon and planets was made according to the times they take to move through the constellations. It remained the accepted wisdom until the 15th century. It was then that the Polish cleric Nicholas Copernicus made the first

[3]The paths of Jupiter and Saturn show similar effects.

[4]It is interesting that Pythagorus (6th century BC) introduced the idea of celestial harmonics.

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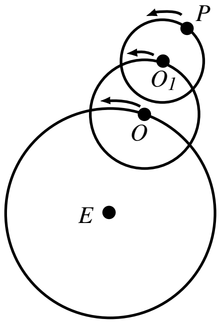

Fig. 1.5. Multiple epicycles to account more accurately for the motion of planet P.

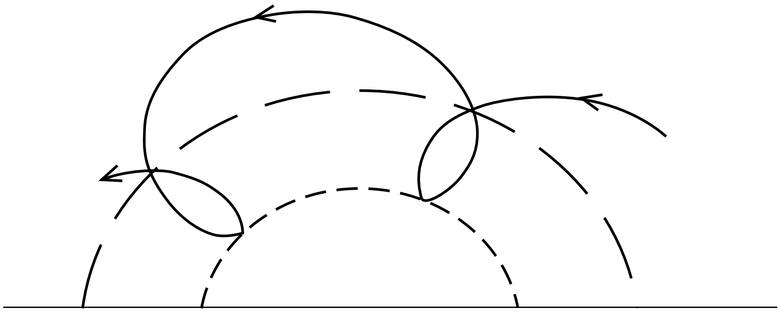

Fig. 1.6. The epicycle system for Venus.

moves to rearrange the model. The essential feature was to recognise that the retrograde motions of the planets are apparent and not real and are due to the motion of the Earth itself about the Sun. It follows at once that a moving Earth cannot be at the centre of the System. The differing appearance of the paths of Venus and Mars suggested that Venus should be placed *inside* the Earth orbit and Mars *outside*. The epicycle structure become considerably simplified if the Sun and Earth were interchanged, making the Sun the centre of the System orbited by the planets. The Earth then became the third body outwards orbiting the Sun. Copernicus got the order of the plants correctly and was even able to predict the correct relative distance scale for the System using geometrical arguments. It was not possible to determine the actual distances between the bodies by simply using geometrical arguments, an annoying feature which is still true today. When gravity was later recognised as the controlling force for the system of the planets it was realised that it is necessary to make only one accurate measurement of the distance between two

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objects in the System to be able to find the rest. The opportunity to do this came four centuries later, in 1932, with the determination of the distance of the asteroid Eros. Today the distances are found directly by radar measurements — the first using this technique was, in fact, the Earth–Venus distance.

The Copernicus model still supposed the planets to move in circular orbits about the Sun in the same plane with the Moon orbiting the Earth. There were still some discrepancies between calculated data and those from observations. As a result, it was still necessary to include some form of epicycles to predict the motions of the planets with some accuracy. The distinction between the Ptolemy approach and that of Copernicus was in many ways philosophical and open to argument. The correctness of the Copernicus arguments could only be established finally by observations using a telescope. This had to wait until the early 17th century.[5]

1.3. Sun, Moon and Earth

The ancients knew that, wherever they were, very occasionally, the Moon moves "behind" the Earth, into the shadow. The brightness of the Moon fades and this is seen everywhere as an *eclipse of the Moon*. It is not an uncommon sight for the Moon to be dimmed from its normal brightness. Alternatively, the Moon sometimes moves in front of the Sun cutting out its light and converting day into night for a few minutes. This is an *eclipse of the Sun* and the circumstances vary from one eclipse to the next. The duration of the dark period is quite variable but is always quite short. The full eclipse is seen only locally: a partial eclipse, where only part of the Sun is covered, is seen over a wider area. The eclipse forms a wide path across a region of the Earth although the path of full obscurity is narrow. The solar eclipse arises from a fortuitous relation between the sizes of the Sun and Moon and their distance apart. This coincidence is unique to the Sun and Moon and is not repeated anywhere else in the Solar System.

The disc of the Moon fits very closely over the disc of the Sun but not quite. The result is the temporary blocking out of nearly all of the Sun's light. A small quantity does reach us from round the edges but the amount varies from one eclipse to the next. Two examples are given in Figs. 1.7a and 1.7b. The diameter of the Moon appears rather larger in Fig. 1.7a when it covers much of the solar disc. The result is the appearance of bright

[5]The crucial observation was to find that Venus can show a crescent form but Mars never does. This means the Venus orbit is inside the Earth orbit while that of Mars is outside. Such an observation could not be made without at least a low-powered telescope.

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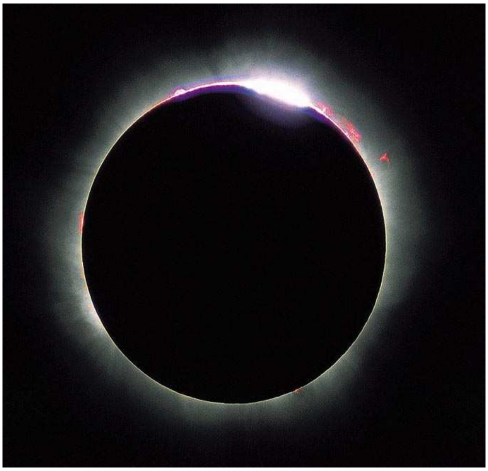

Fig. 1.7a. The solar eclipse of 1999 showing the diameter of the Moon is closely the same as that of the Sun, as seen from Earth (the image was taken by Lviatour of the French expedition).

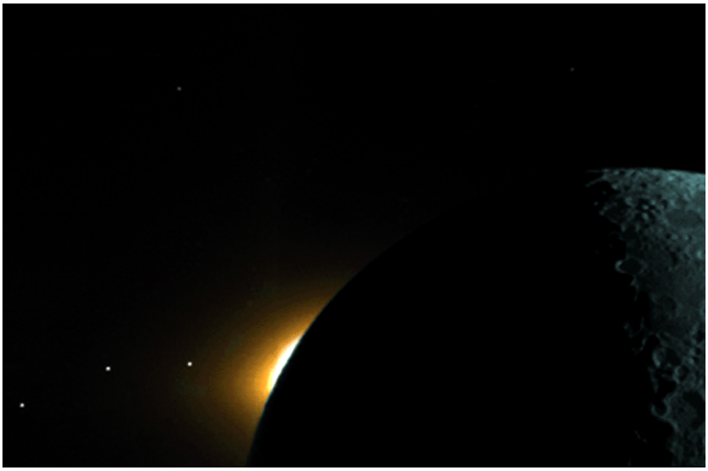

Fig. 1.7b. A photograph taken during the Clementine Mission of the Moon (lit by reflected light from Earth) with the Sun behind showing just the corona. Also visible are the planets Mercury (nearest to the Sun), then Mars and finally Saturn. It is clear that all these bodies lie closely in a single plane, the ecliptic. (BMDO & NASA)

streamers and "beads" of light (Bayley's beads) around the circular edge. In early times eclipses were the only times when the outer atmosphere of the Sun was visible for study. This difficulty persisted until the advent of automatic space vehicles when it became possible to observe the Sun from space

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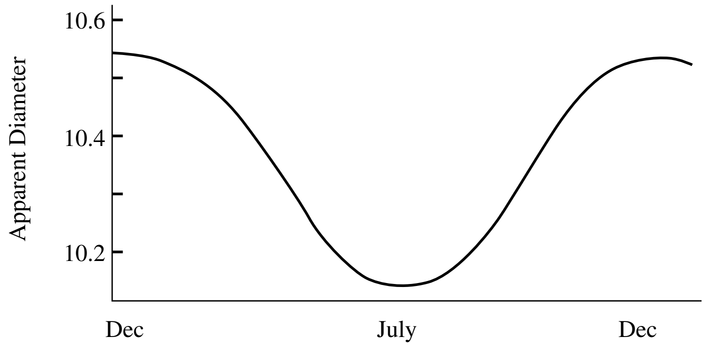

Fig. 1.8. Measurements of the apparent size of the Sun, made in London. It is seen that the maximum diameter occurs in the winter in the northern hemisphere.

without obscuration by the Earth's atmosphere. The Sun has been under constant investigation from space using the automatic Solar and Helioscopic Observatory (SOHO) of the European Space Agency. Measurements are made 24 hours of the day, 365 days of the year. Our knowledge of the Sun and its environment has grown enormously from these measurements.

The different appearances of eclipses suggest that the relative spacings of the Earth, Moon and Sun might change with time. In fact, measurements of the diameter of the Sun from Earth show that there is a periodical change with season during the year. It is found that the size increases and decreases about the mean periodically during the course of the year, every year. This can only mean that the distance of the Sun from the Earth decreases and then increases during the year from December to December. This is seen in Fig. 1.8 showing measurements made in London over the course of one year, from January to January. If the size of the Sun is fixed these measurements can only mean that the distance between the Earth and the Sun changes during the year. From the measurements it seems that the total change is about 3.8%, or 1.9% about a mean value.

Measurements of the apparent size of the Sun during the course of a year as observed from the Earth show clear differences between summer and winter. The diameter is less in the northern hemisphere during the summer months, and smaller in the winter. This implies that the Earth is nearer to the Sun in the winter and further away in the summer. It might have been expected to be the other way round — it is in the southern hemisphere.[6]

[6]Comparable measurements can be made for the Moon showing the size of that body varies throughout the lunar month. The diameter of the Moon varies by about 4.3% about the mean diameter during a lunar month corresponding to an elliptic orbit with eccentricity about 0.041. The phases of the Moon complicate the measurements of the diameter.

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1.4. The Shapes of the Orbits

We have seen that the circular path of the planets was accepted up to the 17^{th} century because the classical Greek thinkers had said this is a perfect shape and the Universe must be perfect. It is clear from Fig. 1.8, however, that the path of the Earth relative to the Sun cannot be a perfect circle; otherwise the plot there would be a straight line and not a periodic curve. In the 17^{th} century, Johannes Kepler realised that the shape could actually be found from observations. He devised a beautifully simple geometrical way of achieving this. The path turned out not to be a circle.

(a) The Earth and Mars

Kepler showed that a study of the location of the planet Mars over a Martian year allows the shape of the Earth's orbit to be found. A study of the location of the Earth then allows the shape of the orbit of Mars to be found. We follow Kepler and consider the Earth first.

The starting point is when the Sun, Earth and Mars lie on a single straight line, a situation said to be one of opposition (see Fig. 1.9, point S, E_0 and M_0). The time for one Earth orbit is 365 days, meaning the Earth covers $360/365 = 0.98° \approx 1°$ per day, on the average. The Earth moves substantially faster than does Mars. The time for one Mars orbit is 687 days, giving a mean speed of $360/687 \approx 0.53°$ per day. In the time it takes for one Mars orbit Earth will have completed $687/365 = 1.88$ orbits or $676.8°$. This is

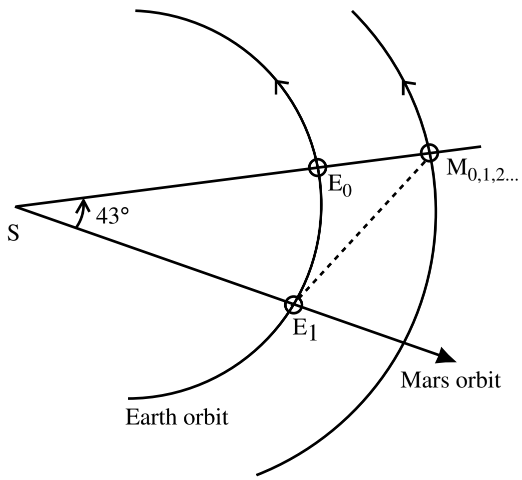

Fig. 1.9. The geometry of the orbits of Earth and Mars.

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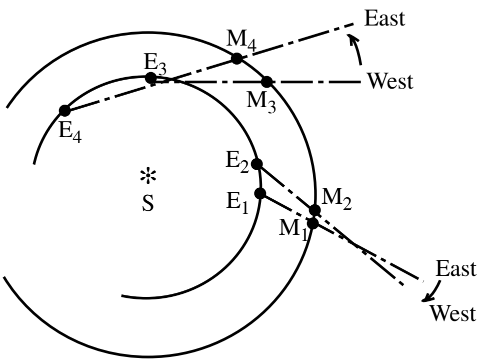

Fig. 1.10. The motions of the Earth and Mars, accounting for the retrograde motions.

closely 43° less than two Earth orbits. At the end of one Mars orbit the Earth is at the position E_1 while Mars is again at M_0. $\angle SM_0E_1$ can be inferred because the position of the Sun is known and that of Mars can be measured. $\angle E_1SM_0$ is 43° so $\angle SM_0E_1$ follows immediately. The place where the line from M_0 with $\angle SM_0E_1$ crosses the line SE_1 gives the location of the Earth. The same method is applied again and again so building up the shape of the orbit of the Earth. While the shape follows from this procedure the distance scale remains arbitrary because the distance SE_1 is itself arbitrary. The shape of the Mars orbit follows in a comparable way, but using Earth as the datum. The starting point is again the opposition for Earth and Mars but it is now Earth that is held at a fixed location. The shape of the Mars orbit follows and again the distance scale is arbitrary.

(b) The Results

Kepler found that the orbit of the Earth is closely circular but with the Sun not at the centre of the circle. His calculations using the observations of Tycho Brahe gave the value $\frac{d}{R} \approx 1.8 \times 10^{-2}$. The situation is shown in Fig. 1.11. The Earth is represented by the point P on the circle with radius R and centre C. The Sun is located at point S with $SC = d \ll R$. Select the point Q along the line SC extended such that $CQ = SC = d$. The point Q is called the equant. A motion of P about Q with constant angular speed ν provides a rotation of P about the point S of almost exactly the observed form for the Earth about the Sun. This construction was first introduced by Ptolemy. The rate of sweeping out area by the line SP is very closely constant throughout the orbit. For motion about Q this is $\frac{1}{2}\nu R^2$. For the perihelion point A it is $\frac{1}{2}\nu(R-d)(R+d) = \frac{1}{2}\nu(R^2 - d^2)$. For the aphelion point B it is

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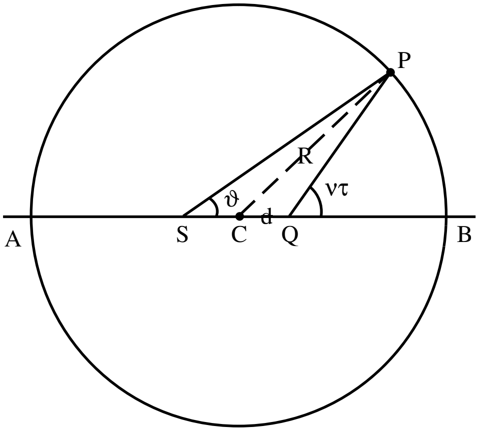

Fig. 1.11. The equant for the Earth.

$\frac{1}{2}\nu(R+d)(R-d) = \frac{1}{2}\nu(R^2-d^2)$ which is the same. At each of the two points on the orbit perpendicular to C it is $\frac{1}{2}\nu(R^2+d^2)$. Because $\frac{d}{R} \approx 3 \times 10^{-4}$ it follows that these four expression are the same to very good approximation. The motion of the point P about the gravitating centre S, and which mirrors closely that of the Earth about the Sun, sweeps out orbital area uniformly to all intents and purposes. The non-central location of the Sun is strange. Kepler applied the same equant approach to Tycho Brahe's data for Mars but found small though important discrepancies between calculations and observations. Clearly, the Mars orbit departs too much from the circular form and he abandoned the equant approach for that case. A more general approach is needed.

(c) Enter the Ellipse

The task of finding the curve that will satisfy the orbital measurements is formidable if it is tackled in an ad hoc way. This, in fact, is the way Kepler approached the problem. This is surprising with hindsight because Kepler was well aware of Greek geometry and the analyses gave several clues to guide him had he realised. The ellipse is, after all, a very special form of a circle. Cutting a cylinder perpendicular to the axis will provide a circular cut: cutting it at an angle will provide an ellipse. Alternatively, cutting through a right circular cone parallel to the base will provide a circular cut whereas a cut at an angle provides an ellipse. The elliptic shape is shown in Fig. 1.12.

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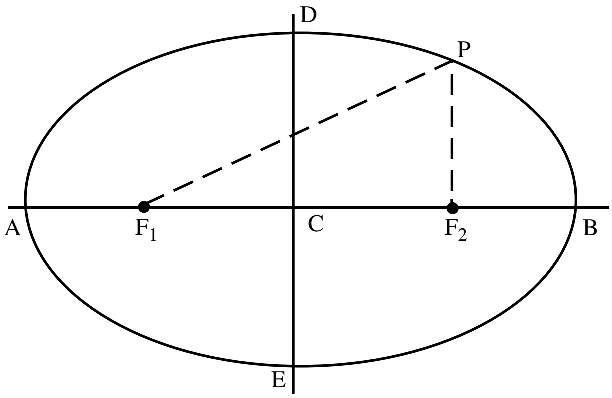

Fig. 1.12. An ellipse with major axis AB and minor axis DE. F_1 and F_2 are the focii. If AB = DE the shape is a circle.

The long axis AB marks the major axes with semi-major components ($\equiv a$) AC and BC. The perpendicular minor axis DE has semi-minor components ($\equiv b$) EC and DC. The points F_1 and F_2 are the focii from which the ellipse can be generated: the distance F_1PF_2 (where P is any point on the curve) is the same for any point on the ellipse. The ratio $(a - b)/a \equiv e$ is called the eccentricity. If $e = 0$ the ellipse degenerates to a circle: if $e = 1$ the ellipse becomes a straight line. It follows that $F_1C = F_2C = a(1 - e)$ while $AF_1 = BF_2 = ae$. It follows from the figure that $F_1PF_2 = 2a$.

1.5. Kepler's Laws of Planetary Motion

Kepler showed that the Copernican view of the Solar System is correct and that the motion of Earth and Mars about the Sun can be represented to high approximation by an elliptical path without the need to introduce epicycles. Over a period of rather more than two decades he studied with untiring patience the motions of the then known planets and devised three laws describing the motion of a planet about the Sun.[7]

Law 1. The path of a planet about the Sun is an ellipse lying in one plane with the Sun at one of the foci.

The orbit may be closely circular but the Sun is not located at the centre of the circle unless the orbit is actually circular when the two foci merge.

[7]The 1st and 2nd laws were published in 1609 and the 3rd was published in 1619.

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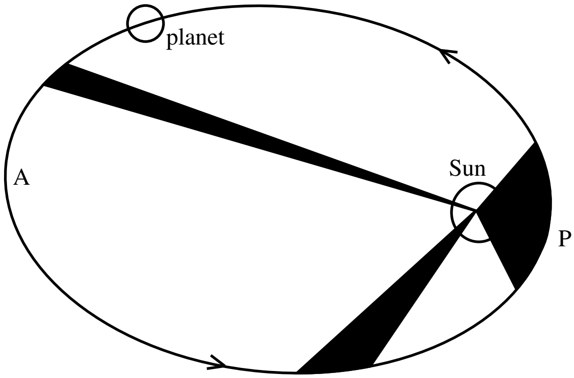

Fig. 1.13. Kepler's Second Law. The line joining the planet to the Sun sweeps out equal areas in equal times. The perihelion point is P while the aphelion point is A.

Law 2. The motion of the planet in its orbit is not uniform. The motion is such that the line joining the planet to the Sun sweeps out equal areas in equal times.

This describes the fact that the planet moves fastest at perihelion, when it is closest to the Sun, and slowest at aphelion when it furthest away. This is shown in Fig. 1.13. The difference in motion between the region of perihelion and the region of aphelion is very clear. The distinction becomes less, of course, as the eccentricity of the ellipse decreases, the shape become more circular.

Law 3. The square of the period to execute one orbit is proportional to the cube of the semi-major axis of the orbit.

The first and second laws are qualitative but the third law is quantitative, making a specific result applicable to all cases. There is no need to resort to epicycles to obtain the symbolic form: if T is the period of the orbit while a is the semi-major axis, the law says that $T^2 = Aa^3$, where A is a constant, the same for all planets of the Solar System.

1.6. Galileo's Law of Inertia: Newton's Laws of Motion

Independently of the empirical work on planetary orbits in Europe, Galileo, in Italy, made a fundamental discovery. He conducted experiments involving the rolling of a sphere under the force of gravity down planes of different slopes. He found the sphere accelerates under gravity down the slope but that the acceleration is less the smaller the angle. The local gravity is, of

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course, vertically downward but it is the component down the plane that is relevant to the motion. The component becomes zero when the angle of the slope becomes zero, that is when the plane is horizontal. In that case Galileo found that the acceleration of the sphere becomes zero as the force of gravity becomes zero. He recognized that the same would result from the action of any other force. This led him to generalize his results and enunciate the *Law of Inertia* according to which a mass will remain at rest or move with constant speed in the same straight line if it is not acted on by a force. The correctness of this law was known to ice skaters in the past but space missions have shown it to be universally true. This law represents a major advance in our understanding of nature and is still central to many arguments.[8]

A generation later, Newton developed Galileo's work. He accepted the law of inertia as the basis for the behaviour of a particle not acted on by a force. This is now called his 1st law of motion or perhaps more properly the Galileo-Newton law. With that as the standard he then postulated the effect of the action of a force. It is to accelerate a particle. What is the scale of the effect? Newton said that the force is balanced by a corresponding change of momentum with time, in the same direction as the force. If m is the mass of the particle and $\mathbf{v}$ its velocity (with magnitude and direction), its momentum is $m\mathbf{v}$. The change of momentum, $d(m\mathbf{v})$ during some vanishingly small time interval dt per unit time interval is $d(m\mathbf{v})/dt$. Newton defined the force, $\mathbf{F}$, acting on the particle as equal to this change of momentum per unit time, that is

$$\mathbf{F} = \frac{d(m\mathbf{v})}{dt} = m\mathbf{a}\,,$$

where $\mathbf{a} = dv/dt$. This is known as Newton's second law of motion. The second form of the law follows because the mass m is a constant.

The world is not made up of point particles but of bodies with size. In order to move his argument to include such bodies he introduced a 3rd law. This can be stated as requiring a reaction to every action. The consequence is that every particle in a finite volume balances the action of every other particle to provide a stable, finite, body. It is possible to find a centre where all the mass can be supposed to reside, called the centre of mass and to apply the other laws of motion to this finite body as if it were a point. Other concepts appear because the finite body can spin as one example but these effects can be accounted for.

[8]Kepler could not take his laws further because he did not know about inertia.

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The laws of Galileo and of Newton are descriptions of how the world is found to behave and are just as empirical as Kepler's laws of planetary motion. The test of their usefulness is whether they lead to predictions of events that agree with what is observed. Galileo's law of inertia has survived but Newton's world picture, with an absolute three-dimensional space and a separate absolute (universal) time has not for arguments of high accuracy. Nevertheless, for planetary problems the Newtonian world is fully adequate and provides appropriately accurate descriptions of the world.

1.7. Newton's Law of Gravitation

One force at the Earth's surface known before Newton is gravitation and it is important to include this in the armory of mechanics. A major advance was made by Isaac Newton in his proposed law of universal gravitation. This is expressed by a mathematical formula that was offered for universal application. The law has proved accurate to a high degree and is central to planetary studies involving the motion of planetary bodies. It was already clear before Newton's day that the force must fall off with distance as the inverse square of the distance from a source. The gravitational intensity passing through an enclosing spherical source will remain the same as the surface expands because the source is unaltered. The area of the sphere of radius r is πr^2 so the intensity per unit area must decrease as $1/r^2$. The problem then is to specify a proportionality factor to make this dependence precise. The novel thing about the law is that the force is mutual between two masses, m_1 and m_2 and that the mass for gravitation is the mass found from inertia experiments. The equality of gravitational and inertial masses is an assumption for Newtonian theory that is supported entirely by the many experiments that have been conducted over the last three hundred years. This means that the proportionality factor will depend on $m_1 m_2/r^2$. This has the dimensions $(\text{mass})^2/(\text{length})^2$ and not that of force. To correct the dimensions Newton introduced the universal constant of gravitation G with dimensions $(\text{length})^3/(\text{mass})(\text{time})^2$. The full law of gravitation then becomes

$$F = -\frac{Gm_1m_2}{r^2}\,.$$

The force acts along the line joining the centres of mass of the particles. The negative sign is present to denote an attractive force. The three laws of

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motion together with the law of force provide the basis for the quantitative description of the gravitational interaction between bodies. The force accelerates each mass. For the mass m_1 and mass m_2 the accelerations are

$$a_1 = \frac{Gm_2}{r^2}, \quad a_2 = \frac{Gm_1}{r^2}.$$

The larger mass is accelerated less than the smaller mass.

1.8. A Passing Encounter without Capture

Kepler's laws as usually stated apply to a closed orbit where one body orbits another under gravity. The gravitational energy is greater than the kinetic energy of the motion so the kinetic energy of the orbiting body is not sufficient to allow it to break away. There is the alternative case when one body passes another of greater mass but with a kinetic energy that is always greater than the gravitational energy of their mutual interaction. What path will the passing body follow? It will certainly be attracted to the more massive body provided its distance of separation is not always too great.

The whole encounter will be described by the conservation of energy in which the sum of the kinetic and gravitational energies is constant throughout. Initially, the incoming body is so far away that the gravitational energy is zero so the total energy throughout the encounter is the initial kinetic energy of the particle. As the distance between the body decreases the gravitation energy becomes stronger and the kinetic energy of the incoming body decreases. It has its smallest value when the two bodies are closest together. The kinetic energy is always greater than the gravitational energy, even when the bodies are closest together so the incoming body is not captured. It moves away with increasing speed as the distance between them increases once again. The outward path is a mirror image of the inward path and the body finally reaches a large distance away with the same energy that it had when the encounter started. The mathematical details will not be given here. The path followed can be shown to be a hyperbola and is sketched in Fig. 1.14. The passing body is B and is coming from below.

The star, presumed stationary, is S. The initial direction of B is changed, to move towards S by gravity. After the closest approach B passes off in a direction different from the initial one. The energy at the beginning and the end of the encounter is the same. The body B is moving with the same

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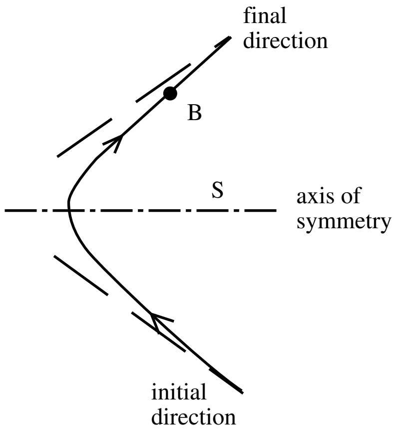

Fig. 1.14. The hyperbolic path of a close gravitational encounter between the body with the indicated initial direction and a star S. The angle between the initial and final directions will be greater the stronger the encounter.

speed but in a different direction. The same analysis will apply to a small satellite passing a planetary body.[9]

Although Kepler's laws of planetary motion are usually stated for a closed, orbiting, body it is true that a passing encounter can also be a planetary motion and can be included as a corollary. Kepler was concerned with captured bodies in orbit, and not with passing encounters which were not observed.

*

Summary

1. Of the five to six thousand stars visible in the night sky five are special in that they move rapidly across the sky. These are the planets known to antiquity.
2. Ptolemy in the 1st century AD set a model Solar System in which the Sun, Moon and planets orbited the Earth as the centre of the Universe.
3. He described the motions of a planet in terms of a circular path, certainly one primary one and perhaps one or more smaller, secondary epicycles.

[9]This also applies to the encounter between two charges bodies of opposite electrical charge, usually a central proton with positive charge and a passing electron with negative charge.

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4. Copernicus in the 15th century proposed the correct arrangement with the Sun at the centre, the Moon orbiting the Earth and the planets orbiting the Sun. He deduced correctly that Mercury and Venus orbit inside that of the Earth while the remaining planets orbit outside.
5. Kepler found the correct shapes of the orbits of Earth, Mars and Jupiter by observation and showed they are ellipses.
6. Kepler inferred the three laws controlling the motion of a planet in an elliptical orbit, his laws of planetary motion.
7. The motion of a body moving past another without capture is a hyperbola.
8. Other empirical discoveries at the time were the law of inertia by Galileo which led Isaac Newton to develop his three laws of particle motion.
9. The laws of motion can be applied to astronomical and planetary problems only if a simple law is known for the gravitational interaction between two bodies. Newton's empirical inverse square law provides such a simple expression.
10. The discoveries up to the 17th century, important for planetary science, have now been collected together.

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