# Preface

Resonance is one of the most important and well-known concepts in both mechanics and physics. In simple words, we deal with resonance conditions when "small reasons involve big consequences". These small reasons are actually the small forces or small periodic changes of the parameters. They can make big actions due to special coordination of their rhythms with the internal oscillation characteristics of the system.

The term "internal resonance" means that weakly coupled subsystems with close characteristics mutually support the oscillations. For linear systems this phenomenon is known as "beating", when periodic energy exchange between oscillators takes place. This phenomenon can also be observed in weakly nonlinear systems. However, the nonlinearity may lead to qualitatively new effects such as instability of cooperative modes and appearance of localized modes. Besides, the energy transfer up or down the frequency scale turns out to be possible. Such a transfer occurs at both subharmonic and superharmonic resonances, and involves two frequencies into the process.

We show that "internal resonance" may be considered as a relatively wide concept manifesting itself in broad series of phenomena — from secondary resonances in 1DOF models to localization of vibrations in rings, shells, and finite or infinite oscillatory chains.

Dealing with internal resonances we should take into account such factors as small differences between frequencies of linearized systems or closeness to secondary resonance conditions. We may call these factors imperfections. The simplest way to explain this concept is closely connected with analysis of symmetry properties of the system. Laws of nature, as well as technological and optimization requirements, determine the predominant role of the structures possessing clearly seen symmetry properties (molecules, crystals, homogeneous strings, bars, rings, plates, shells, and so on).

Presence of such symmetry plays a double role. On the one hand, the symmetry essentially simplifies the solving of many physical problems, which are related to calculation, for example, of normal modes or traveling waves as well as instability conditions in linear theories of vibrations and stability. On the other hand, nonlinearity may lead to bifurcations of different types accompanied by symmetry breaking. In particular, the symmetry breaking can be seen in instability of certain modes and appearance of essentially nonlinear modes (e.g., localized ones). Another consequence of symmetry is high sensitivity of the system's important characteristics to small defects (in Physics) and imperfections (in Mechanics). That is why the account of such defects or imperfections is one of the key problems in analysis and design of highly symmetrical physical and mechanical systems.

In the case of weak nonlinearity, where a linear approach is commonly considered a good approximation, the role of the defects and imperfections is exhibited in violation of more or less distinct degeneration. Such violation is a consequence of certain resonance and selection of the vibration modes that can be really observed in the systems under consideration.

It is worth noting that we should distinguish the long wavelength and short wavelength processes. In the former the characteristic wavelength strongly exceeds the spatial scale for imperfections so that the corresponding wave "does not feel" the imperfections and freely propagates along the system. In the latter case imperfections are really important and have to be taken into account.

Internal resonances are evinced in the simplest 2DOF system (two weakly coupled identical oscillators). In such a case the weak linear coupling between the oscillators plays the role of imperfection. Here we observe many nonlinear effects mentioned above:

- bifurcations of normal (linear) modes accompanied by the appearance of supplementary modes, more or less localized;
- dependence of bifurcation parameters on imperfection;
- complex resonances in the presence of external periodic forces.

For the chain of coupled nonlinear oscillators (this is the natural model for elastic systems on nonlinear supports) the "imperfections" are caused by linear coupling along the chain. The coupling is effectively weak if the wavelength is large enough. This is also true for modulation waves with large lengths in the case of short wavelength dynamics. Due to the presence of many degrees of freedom in these cases principally new phenomena, such as breathers, arise.

For homogeneous oscillatory chains (e.g., discrete models of strings, bars, beams without elastic supports) the situation is rather different. In the case of long wavelength dynamics internal resonances are not exhibited, and coupling between particles cannot be considered as a small imperfection. Here main nonlinear effects, such as appearance of soliton-like excitations, can be adequately understood if they are considered "from the viewpoint of observer moving with the sound wave". On the contrary, in the case of short wavelength dynamics a coupling between particles may be identified again with imperfection, and internal resonances arise.

Now when we compare the results of nonlinear dynamics study with the case of internal resonances in the two-degrees-of-freedom systems, on the one hand, and with more complicated models having many degrees of freedom (especially strings, bars, beams, plates and shells), on the other hand, a noteworthy gap is seen. One of the main objectives of our book is to fill this gap considering not only reduced models with several-degrees-of-freedom but also models with many DOF and infinite number of DOF.

In distinction to other books, we do not present many well-known methods and restrict ourselves to one of them beginning with simple examples. The reason is that corresponding procedure can be successfully extended to more complicated cases including infinite and finite chains as well as to continuous models.

Because the natural models of main approximation for the systems with many DOF are mostly formulated in complex variables, we use complex representations of the equations of motion from the very beginning. Such a presentation turns out to be adequate for application of multiple scales techniques.

This book does not intend to cover all fields of dynamics of systems with internal resonance. We consider only some principal problems to demonstrate possibilities of the method and main features of behavior of the systems in internal resonance.

We try also to carry out the analytical investigation of all problems under consideration to the greatest extent possible (using generalized dimensionless parameters) and to explain the physical essence of many nonlinear effects caused by internal resonances.

The content of this book is presented in eight chapters.

Chapter 1 plays a double role. Firstly, it prepares the important notions and techniques needed for further investigation. Secondly, it demonstrates that well-known secondary resonances in 1DOF systems may be considered within the framework of internal resonance conception.

Chapter 2 is devoted to one-to-one internal resonance in 2DOF autonomous symmetric cubic systems. A complete analysis of coupled steady-state modes (CSMs), which can be of two kinds – normal and elliptic, and their bifurcations and stability is presented, along with studying non-stationary motions and effects caused by damping.

Non-autonomous 2DOF symmetric cubic systems in 1:1 internal resonance are studied in Chapter 3. The interaction of the primary external resonance with the internal resonance is considered. A complete analytical investigation of coupled steady-state modes has been performed for undamped and damped systems, and conditions of existence of the CSMs have been derived. The topological structure of the CSM paths in reduced phase spaces is studied in connection with the behavior of corresponding autonomous systems. In particular, it is shown that in damped systems, in distinction from undamped ones, the CSMs are not exact normal or elliptic modes, but they approach normal or elliptic modes at increasing amplitudes.

In Chapter 4 we consider a problem of great practical importance — nonlinear dynamics of circular ring (or infinite cylindrical shell). In reasonable approximation this problem with account of interaction of conjugate modes and "splitting" of their natural frequencies can be reduced to somewhat modified problem for cubic 2DOF systems. An analytical investigation of free oscillations with special attention to coupled steady-state modes (traveling waves) and of forced oscillations in the vicinity of the primary resonance is carried out.

The next four chapters of the book are devoted to nonlinear wave problems for the oscillatory chains and systems of oscillators (they model different elastic chains supported by nonlinear springs).

Primary attention is paid to specific nonlinear phenomenon — spatial localization of vibrations and waves. We show that relatively simple description of such localization is possible due to the resonance conditions which arise naturally in the systems of coupled nonlinear oscillators and in the short wavelength nonlinear dynamics of oscillatory chains.

Consequently we consider longitudinal dynamics of the homogeneous system of coupled nonlinear oscillators, similar problems for oscillatory chain and non-homogeneous system of nonlinear oscillators, and transversal nonlinear dynamics of the coupled oscillators. In all cases we obtain conditions of localized vibrations and waves existence as well as their universal description on the basis of nonlinear Schrödinger equation and analytical solutions, which are envelope solitons (breathers).

The book is supplemented by an appendix — a translation of paper by A. I. Manevich "Inertial Forces and Methodology of Mechanics" published (in Russian) in "Reports of National Academy of Science of Ukraine", 2001, No 12.

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