

Chapter 1

Introduction

In classical dynamics the second law of motion is used both as a definition of the force and also as the equation for predicting the position and the momentum of the particle as a function of time. This dual role of the equation of motion has been criticized by a number of eminent scientists [1]. In order to investigate the nature of the force law in many cases we are helped by the so called “theories of forces” [2], where theories like gravitational and electrodynamics specify the force function. Based on these and other well-defined theories we can derive the interaction between complicated system of particles and fields from the inter-particle forces. For instance we can determine the dipole-dipole interaction from the Coulomb force, or the radiation reaction force from the coupling of an electron to the electromagnetic field.

The idea that the force function is dependent on the position of the particle and not on time nor on its velocity has been suggested by a group of philosophers of science [3]. Let us quote Nigel’s observation in this regard [4]:

“In point of fact, the force-function employed in many of the familiar applications of the equations of motion is specified in a manner analogous to the Newtonian hypothesis, in so far as it does not contain the time-variable explicitly. Indeed, though there are numerous cases for which the time-variable enters explicitly into the force function (as in the case of damped vibrations), it is commonly assumed that the explicit presence of the time-variable can in principle be eliminated if the initial system of interacting bodies is suitably enlarged by including other bodies into it. For reasons that will be presently apparent, what is called the “principle of causality” (as distinguished from special causal laws) in fact usually construed in classical physics as the maxim that should be force-function for a given physical system contain the time-variable explicitly, the system is to be enlarged in such a manner as to allow a specification of the force-function in which the time-variable does not appear. And it is a matter

of historical fact that in the main the search for such enlarged systems that do not coincide with the entire cosmos has been successful.”

Thus certain forces such as the damping force exerted on a particle while it is moving in a viscous medium can be postulated as a velocity-dependent force proportional to a given power of the velocity of the particle. The dependence of the force law on velocity in this case is determined from the observed state of the motion. On the other hand one may consider the damping to be due to the collision of the particle in question with the smaller particles forming the environment, e.g. gas molecules. In the latter case the force of friction can be derived and its dependence on the momentum of the particle can be determined.

This idea of dividing a large conservative system into two parts, one part being the heat bath and the other forming the dissipative system is a useful one, particularly in quantum theory. In this way we can avoid certain difficulties in the formulation of the problem, but in turn, nearly in all cases we can only solve the problem approximately.

Usually we assume that the interaction between the two parts causes the transfer of energy from the dissipative system to the heat bath. However we can generalize the models of dissipation to include those cases where the coupling between the two parts requires the transfer of mass or the number of particles from one part to the other.

In quantum theory we borrow the idea of force (or preferably the potential) from the classical dynamics. When the force is conservative and derivable from a potential function the formulation of the quantum analogue of the classical motion is straightforward. But if we try to quantize a non-conservative classical system we find inconsistencies between the equations of motion and the canonical commutation relations. In addition the result depends on our choice of the Hamiltonian or Lagrangian, since for non-conservative systems the Hamiltonian, in general, does not represent the energy of the system.

We define dissipative forces in classical dynamics as any and all types of interaction where the energy is lost when the motion takes place (usually in the form of heat to a heat bath) [5]. Frequently the magnitude of the force, f , on a particle or a body may be closely represented, over a limited range of velocity by a power law, $f_d = av^\nu$, where v is the velocity of the particle or body and a and ν are constants.

Depending on the value of ν we have the following types of dissipative forces [5]:

- (1) Frictional force. This work is usually required to slide one surface over another, and once the motion is started the magnitude of the force is independent of the speed. Thus in this case $\nu = 0$.
- (2) Viscous force. When the force is proportional to the speed of the particle, i.e. $\nu = 1$ we call the force to be viscous force (see also § 2.1).
- (3) Newtonian dissipative force. For high speed motion of an object in the air, the force is proportional to the square of the velocity, i.e. $\nu = 2$.

In this book we will follow the present usage of these terms and occasionally use frictional force and viscous force to indicate dissipative forces of

the general type av^ν with ν , a positive but not necessarily an integer, taking different values.

Bibliography

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