Preface

Preface to the Second Edition

The second edition is revised, expanded and enhanced. This is now a more complete text in Stochastic Calculus, from both a theoretical and an applications point of view. Changes came about, as a result of using this book for teaching courses in Stochastic Calculus and Financial Mathematics over a number of years. Many topics are expanded with more worked out examples and exercises. Solutions to selected exercises are included. A new chapter on bonds and interest rates contains derivations of the main pricing models, including currently used market models (BGM). The change of numeraire technique is demonstrated on interest rate, currency and exotic options. The presentation of Applications in Finance is now more comprehensive and self-contained. The models in Biology introduced in the new edition include the age-dependent branching process and a stochastic model for competition of species. These Markov processes are treated by Stochastic Calculus techniques using some new representations, such as a relation between Poisson and Birth-Death processes. The mathematical theory of filtering is based on the methods of Stochastic Calculus. In the new edition, we derive stochastic equations for a non-linear filter first and obtain the Kalman-Bucy filter as a corollary. Models arising in applications are treated rigorously demonstrating how to apply theoretical results to particular models. This approach might not make certain places easy reading, however, by using this book, the reader will accomplish a working knowledge of Stochastic Calculus.

Preface to the First Edition

This book aims at providing a concise presentation of Stochastic Calculus with some of its applications in Finance, Engineering and Science.

During the past twenty years, there has been an increasing demand for tools and methods of Stochastic Calculus in various disciplines. One of the greatest demands has come from the growing area of Mathematical Finance, where Stochastic Calculus is used for pricing and hedging of financial derivatives, such as options. In Engineering, Stochastic Calculus is used in filtering and control theory. In Physics, Stochastic Calculus is used to study the effects of random excitations on various physical phenomena. In Biology, Stochastic Calculus is used to model the effects of stochastic variability in reproduction and environment on populations.

From an applied perspective, Stochastic Calculus can be loosely described as a field of Mathematics, that is concerned with infinitesimal calculus on non-differentiable functions. The need for this calculus comes from the necessity to include unpredictable factors into modelling. This is where probability comes in and the result is a calculus for random functions or stochastic processes.

This is a mathematical text, that builds on theory of functions and probability and develops the martingale theory, which is highly technical. This text is aimed at gradually taking the reader from a fairly low technical level to a sophisticated one. This is achieved by making use of many solved examples. Every effort has been made to keep presentation as simple as possible, while mathematically rigorous. Simple proofs are presented, but more technical proofs are left out and replaced by heuristic arguments with references to other more complete texts. This allows the reader to arrive at advanced results sooner. These results are required in applications. For example, the change of measure technique is needed in options pricing; calculations of conditional expectations with respect to a new filtration is needed in filtering. It turns out that completely unrelated applied problems have their solutions rooted in the same mathematical result. For example, the problem of pricing an option and the problem of optimal filtering of a noisy signal, both rely on the martingale representation property of Brownian motion.

This text presumes less initial knowledge than most texts on the subject (Métivier (1982), Dellacherie and Meyer (1982), Protter (1992), Liptser and Shiryayev (1989), Jacod and Shiryayev (1987), Karatzas and Shreve (1988), Stroock and Varadhan (1979), Revuz and Yor (1991), Rogers and Williams (1990)), however it still presents a fairly complete and mathematically rigorous treatment of Stochastic Calculus for both continuous processes and processes with jumps.

A brief description of the contents follows (for more details see the Table of Contents). The first two chapters describe the basic results in Calculus and Probability needed for further development. These chapters have examples but no exercises. Some more technical results in these chapters may be skipped and referred to later when needed.

In Chapter 3, the two main stochastic processes used in Stochastic Calculus are given: Brownian motion (for calculus of continuous processes) and Poisson process (for calculus of processes with jumps). Integration with respect to Brownian motion and closely related processes (Itô processes) is introduced in Chapter 4. It allows one to define a stochastic differential equation. Such equations arise in applications when random noise is introduced into ordinary differential equations. Stochastic differential equations are treated in Chapter 5. Diffusion processes arise as solutions to stochastic differential equations, they are presented in Chapter 6. As the name suggests, diffusions describe a real physical phenomenon, and are met in many real life applications. Chapter 7 contains information about martingales, examples of which are provided by Itô processes and compensated Poisson processes, introduced in earlier chapters. The martingale theory provides the main tools of stochastic calculus. These include optional stopping, localization and martingale representations. These are abstract concepts, but they arise in applied problems, where their use is demonstrated. Chapter 8 gives a brief account of calculus for most general processes, called semimartingales. Basic results include Itô's formula and stochastic exponential. The reader has already met these concepts in Brownian motion calculus given in Chapter 4. Chapter 9 treats Pure Jump processes, where they are analyzed by using compensators. The change of measure is given in Chapter 10. This topic is important in options pricing, and for inference for stochastic processes. Chapters 11-14 are devoted to applications of Stochastic Calculus. Applications in Finance are given in Chapters 11 and 12, stocks and currency options (Chapter 11); bonds, interest rates and their options (Chapter 12). Applications in Biology are given in Chapter 13. They include diffusion models, Birth-Death processes, age-dependent (Bellman-Harris) branching processes, and a stochastic version of the Lotka-Volterra model for competition of species. Chapter 14 gives applications in Engineering and Physics. Equations for a non-linear filter are derived, and applied to obtain the Kalman-Bucy filter. Random perturbations to two-dimensional differential equations are given as an application in Physics. Exercises are placed at the end of each chapter.

This text can be used for a variety of courses in Stochastic Calculus and Financial Mathematics. The application to Finance is extensive enough to use it for a course in Mathematical Finance and for self study. This text is suitable for advanced undergraduate students, graduate students as well as research workers and practioners.

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Fima C. Klebaner
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