Preface

Orthodox quantum mechanics is an extremely successful physical theory. The conceptual foundations and the mathematical formalism of the theory are so rich that after 80 years there are still many fundamental issues to be explored. The rapid development of technology leads to the construction and discovery of exciting new physical systems with quantum properties such as free atom Bose-Einstein condensate at very low temperatures and various low dimensional systems, nanostructures and quantum circuits. The advance of nanotechnology opens up the possibility of designing and assembling structures atom by atom. These systems differ from the traditional microscopic systems described by orthodox quantum theory in a number of ways. They could be macroscopic in dimensions, spatially confined in a circuit geometry, and more importantly they may possess classical properties as well as quantum properties. Orthodox quantum mechanics has a very rigid structure making it very difficult to accommodate new and novel properties. A generalization of orthodox quantum mechanics is proposed. We shall present a flexible quantum formalism to provide a unified theory of physical systems, from microscopic and macroscopic quantum to classical. Our aim is not to produce an all embracing general theory in a highly abstract form; the objective is to generalize orthodox quantum theory in a concrete form and to an extent that it can be directly applied to describe a wide range of physical systems.

The basic mathematical language used here is that of Hilbert space and operators. The relationship between classical and quantum quantities is made transparent by adopting a geometric method for quantizing basic physical quantities. The book is divided into four parts.

Part one presents a study of mathematical preliminaries. We shall concentrate on topics which are seldom discussed in conventional exposition of quantum theory. Firstly, there is the geometric language used for quantization. The central ideas involve the definition of vector fields in a manifold as operators, the concept of completeness of vector fields, and the geometric formulation of classical dynamical systems. Secondly, there are the less familiar aspects of operator theory essential for this book. These include a systematic discussion of symmetric operators and their maximal symmetric extensions, local operators, and a study of direct integrals of Hilbert spaces and operators. Spectral functions and spectral measures are presented via their direct link to classical probability functions and classical probability measures. To make this book self-contained, we shall devote two chapters to these discussions. We have kept the mathematics to a minimum, summarizing only those of immediate relevance to physical discussions later on. For a better understanding of the mathematics and its applications, we have included a large number of comments and explicit examples which are numbered and are often referred to in later chapters. This enables us to concentrate on the discussions of physical ideas in subsequent chapters avoiding many interruptions and digressions into mathematical technicalities. The presentation takes the form of brief summaries of definitions and theorems together with comments and examples to demonstrate their relevance. Readers familiar with these mathematical preliminaries can skip most of these discussions and go straight to part two of the book.

To facilitate cross reference in later chapters, we have numbered these definitions, theorems, comments and examples according to the sections they appear in. For instance, we have:

> Definition 2.12.5(1) indicates the first definition in §2.12.5, i.e., in Chapter 2, Section 12.5.
>
> §1.6.3E(1) E3 indicates the third example in the first set of examples presented in §1.6.3.
>
> §2.5.1C(2) C4 indicates the fourth comment in the second set of comments presented in §2.5.1.

We shall also follow the convention used in most physics texts of putting a "hat" over a symbol, e.g., $\widehat{A}$, to denote an operator. A new symbol, ɨ, to be referred to as *ibar*, is introduced to denote the ratio $i/\hbar$ which appears all too frequently.

Part two presents the main mathematical and theoretical framework for orthodox and generalized theories, in three chapters. Chapter 3 is on orthodox quantum mechanics. After a brief presentation of the postulate on quantum statics, this chapter launches into the quantization problem in order to establish basic quantum observables. The quantization methods introduced here will also be used in later chapters. A postulate on quantum dynamics based on the conventional unitary time evolution is then introduced. This is followed by a discussion on the asymptotic behaviour of quantum dynamics which leads to the concepts of asymptotic localization and separation. A general theory for state preparation is then presented. The chapter ends with a measurement theory. Chapter 4 sets out to generalize orthodox quantum mechanics. The generalization is based mathematically on direct integrals of Hilbert spaces and operators and physically on the notion of superselection rules. This leads to a unified and flexible theory which reduces to orthodox quantum mechanics as a special case, and is capable of describing non-orthodox systems from macroscopic quantum to classical. Chapter 5 begins a further generalization by incorporating strictly maximal symmetric operators as quantum observables.

Part three investigates the implications and applications of our generalized theory to demonstrate its relevance, in two chapters. Chapter 6 lays the mathematical foundations by reviewing the theory of selfadjoint extensions of symmetric operators. This is followed by a detailed discussion of point interactions essential in the construction of any model theory of quantum systems in a circuit geometry. Chapter 7 applies our generalized theory and point interactions to describe superconducting systems in various circuit geometry, especially those having a Josephson junction. The Josephson equation in superconductivity is seen to be derivable in a rigorous manner within our theory. Strictly maximal symmetric operators are seen to be necessary for certain circuit configurations.

The final part of the book is devoted to some topical issues arising from previous discussions. Chapter 8 investigates Schrödinger's cat states, dynamic and asymptotic decoherence, entanglement, chronological disordering and the formulation of an asymptotically separable quantum mechanics. Chapter 9 presents a path space formulation of quantum mechanics which lends further support for the emergence of superselection rules.

This book is not a comprehensive review of various theories and formulations existing in the vast literature on quantum mechanics. The materials presented in this book reflect the author's view and interest over a number of years working in the foundations of quantum mechanics. These materials have not previously been fully and systematically discussed in an accessible form. The author aims to demonstrate that quantum theory together with its mathematical structure and physical interpretation is capable of restrictions as well as generalizations. It is this flexibility which enables the theory to be so applicable. The richness of the formalism is likely to allow the theory to adapt and to cope with future demands arising from the discoveries of new physical phenomena for years to come.

No attempt is made to present a comprehensive review of other more familiar theories and formalism, since these are fully discussed by many existing monographs. We have not endeavoured to set out a grand scheme to encompass everything such as various interpretations, environmental influences and gravity. Our aim is modest. We shall keep close to the orthodox formalism and we do not claim that the theories presented are universal; they are designed to be applicable to specific types of physical systems. We do believe that the fundamental structure of quantum theory should not be rigid and set in stone. It must be allowed to evolve in order to keep abreast with technological development and the discovery of new physical phenomena. We have therefore devoted much space in this monograph to developing a flexible quantum theory and to the treatment of some typical non-orthodox quantum systems. This may help to demonstrate the relevance of fundamental studies of quantum theory to the understanding and the exploration of a rapidly expanding set of novel quantum systems.

There are monographs on fundamental issues of quantum theory motivated by mathematical or conceptual and philosophical considerations. This book is motivated mainly by physical considerations with an eye on possible applications to non-orthodox quantum systems. Although we have formulated our theories and various concepts in rigorous mathematics terms our primary interest is in understanding and developing applicable physical theories, not mathematics. Our analysis is motivated by simple and intuitive physical ideas. One can appreciate the physical ideas involved, e.g., on state preparation, measurement, asymptotic superselection rules and asymptotic notion of decoherence, without having to delve too deeply into mathematics. However, it is pleasing and reassuring to know that physical ideas can be formulated axiomatically and treated in a mathematically vigorous manner. There is a bibliographical list at the end of each chapter which is for immediate reference and not meant to be complete and exhaustive. The author apologizes for inevitable omissions.

This monograph aims at a readership of theoretical physicists, mathematical physicists, mathematicians and philosophers of science with an interest in the foundations of quantum mechanics and its applications. Hopefully the self-contained nature of the presentation will render this book useful to a wide range of readers.

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