

Contents

Preface

vii

I Aspects of Geometric and Operator Theories

1	Manifolds and Dynamical Systems	3
1.1	Topological Spaces and Topological Equivalence	4
1.1.1	Basic concepts and definitions	4
1.1.2	Topological equivalence	9
1.2	Euclidean Spaces	11
1.2.1	Basic concepts and definitions	11
1.2.2	Coordinate systems and coordinate transformations	13
1.2.3	Contravariant and covariant vectors in \mathbb{E}^n	15
1.2.4	Contravariant, covariant and mixed tensors	17
1.3	Differential Operators, Vectors and Fields	20
1.3.1	Differential operators and derivations	21
1.3.2	Tangent vectors, tangent vector fields and their integral curves	26
1.3.3	Transformation groups and complete vector fields	35
1.4	Cotangent Vectors and Differential Forms	40
1.4.1	Cotangent vectors, differentials and one-forms	41
1.4.2	Tensor fields and two-forms	47
1.4.3	Exterior differentiation	51
1.4.4	Interior products, closed and exact forms	53
1.5	Differentiable Manifolds	56
1.5.1	Definition and examples	56
1.5.2	Riemannian manifolds	60
1.5.3	Hamiltonian manifolds	63
1.6	Classical Dynamical Systems	67
1.6.1	Classical systems of finite order	67
1.6.2	First-order systems	68

1.6.3	Second-order Hamiltonian systems	69
1.6.4	Momentum observables, vector fields and operators	73
1.6.5	Concluding remarks	77
	References	78
2	Operators and their Direct Integrals	81
2.1	Hilbert Spaces	81
2.2	Operators: Basic Definitions	87
2.2.1	Boundedness, adjoints, extensions and restrictions, continuity and closure	87
2.2.2	Convergence of a family of bounded operators	92
2.2.3	Tensor products of Hilbert spaces and operators	94
2.3	Types of Operators and their Reductions	97
2.4	Unitary Operators and Unitary Transforms	107
2.5	Extensions of Symmetric Operators	113
2.5.1	Selfadjoint and maximal symmetric extensions	113
2.5.2	Von Neumann's formula for selfadjoint extensions	128
2.6	Probability and Expectation Values	130
2.6.1	Borel sets, measures and measurable functions	132
2.6.2	Probability measures and probability functions	137
2.6.3	Expectation values, variances and uncertainties	140
2.7	Spectral Measures and Probability	142
2.8	Selfadjointness and Spectral Decomposition	148
2.8.1	Spectral theorem	148
2.8.2	Functions of a selfadjoint operator	154
2.8.3	Spectra of selfadjoint operators	157
2.8.4	Spectral representation spaces and spectral representations of selfadjoint operators	161
2.9	Generalized Spectral Measures and Probability	167
2.10	Spectral Functions of Symmetric Operators	170
2.10.1	Symmetric operators and their spectral functions	170
2.10.2	Strictly maximal symmetric operators and their spectral functions	173
2.10.3	The square of maximal symmetric operators	175
2.10.4	Spectra of symmetric operators	178
2.11	Probability and Operators	180
2.11.1	Probability measures, spectral measures and selfadjoint operators	180
2.11.2	Probability measures, generalized spectral measures and strictly maximal symmetric operators	183

2.12	Local Operators in Coordinate Space	185
2.12.1	Definitions	185
2.12.2	Localization of bounded operators	187
2.12.3	Local operator algebras	188
2.12.4	Localization of unbounded operators 1	191
2.12.5	Localization of unbounded operators 2	192
2.12.6	Local momentum and local Hamiltonian	195
2.13	Direct Integrals of Hilbert Spaces	195
2.13.1	Discrete composition of Hilbert spaces	196
2.13.2	Continuous composition of Hilbert spaces	198
2.14	Direct Integrals of Operators	209
2.14.1	Direct sums of operators	209
2.14.2	Direct integrals of operators	213
2.14.3	Density operators	218
2.14.4	Statistical operators	221
2.15	Direct Integrals of Tensor Products	224
2.15.1	Direct integrals of tensor product Hilbert spaces	224
2.15.2	Direct integrals and tensor product of operators	225
	References	226

II Orthodox and Generalized Quantum Mechanics

3	Orthodox Quantum Mechanics	231
3.1	Introduction	231
3.1.1	Structure of physical theories	231
3.1.2	Mathematical framework of quantum mechanics	234
3.2	Orthodox Quantum Statics	236
3.2.1	Postulate on orthodox quantum statics	236
3.2.2	Pure and mixed states	239
3.2.3	Correlation between states	246
3.2.4	Discretization of bounded and unbounded observables	248
3.2.5	Approximate nature of measurements	250
3.3	Quantization in \mathbb{E}^n	252
3.3.1	Preliminaries on quantization	252
3.3.2	Failure of general schemes	256
3.3.3	Complete momentum observables	259
3.3.4	Observables linear in momenta	269
3.3.5	Incomplete momentum observables	272
3.3.6	Kinetic energy and the Hamiltonian	275
3.3.7	Constraint and quantization in circuit geometry	282

3.4	Orthodox Quantum Dynamics	286
3.4.1	Postulate on orthodox quantum dynamics	286
3.4.2	Asymptotic localization and separation: Free systems	290
3.4.3	Asymptotic localization and separation: Scattering systems	294
3.5	Quantum State Preparation	300
3.5.1	The problem	300
3.5.2	Mathematical preliminaries	302
3.5.3	Ideal particle source	303
3.5.4	Random particle source	305
3.5.5	Extension to spin- $\frac{1}{2}$ particles	307
3.6	Quantum Measurement	310
3.6.1	Local position observables and their measurability	310
3.6.2	Reduction to local position measurements	313
3.6.3	Spectral separation for spinless particles	314
3.6.4	Spectral separation for spin- $\frac{1}{2}$ particles	319
3.6.5	Local position measurement as an ionization process	320
3.6.6	A model ionization propagator	324
3.6.7	Projection postulate, local position measurements and uncertainty relations	328
3.6.8	Concluding remarks	329
	References	331
4	Physical Theory in Hilbert Space	337
4.1	Introduction	337
4.2	Unified Statics in Direct Integral Space	338
4.2.1	A unified postulate on quantum statics	339
4.2.2	Discrete and continuous direct integral decompositions	339
4.3	States and Superposition Principle	341
4.3.1	Regular and singular states, pure and mixed states	341
4.3.2	Coherence and superposition principle	344
4.3.3	Superselection rules, their origins and classical observables	345
4.4	Unified Dynamics in Direct Integral Space	351
4.4.1	Preliminaries	351
4.4.2	Preserving dynamics	352
4.4.3	Non-preserving dynamics 1: Motivation	356
4.4.4	Linear functionals for state description	358
4.4.5	Extensions and restrictions of linear functionals	361
4.4.6	Non-preserving dynamics 2: A general scheme	364
4.4.7	Non-preserving evolution and environments	367

4.5	Classical Systems of Finite Order	368
4.5.1	First-order systems in Hilbert space	368
4.5.2	Second-order Hamiltonian systems in Hilbert space . . .	373
4.6	Mixed Quantum Systems	378
4.6.1	A model system	378
4.6.2	Classification of physical systems	379
4.6.3	Quantum/Classical divide 1	381
4.6.4	Equilibrium and mixed quantum systems	383
4.7	Coupling of Systems of Different Types	384
4.7.1	Measuring devices	384
4.7.2	Coupling of orthodox quantum and classical systems . .	385
4.7.3	Coupling of orthodox and mixed quantum systems . . .	388
4.7.4	Coupling of classical and mixed quantum systems . . .	390
4.8	Concluding Remarks	390
	References	392
5	Generalized Quantum Mechanics	395
5.1	Introduction	395
5.2	Maximal Symmetric Operators and Observables	400
5.2.1	Observables: Concept and description	400
5.2.2	Measurement of intrinsically unsharp observables	406
5.3	Approximate and Related Observables	407
5.3.1	Approximate observables	407
5.3.2	Related family of observables	408
5.4	Implications on Quantization	409
5.5	Time Operators and Uncertainty Relation	409
5.6	Local Values in Coordinate and in Phase Spaces	413
5.6.1	Expectation values in terms of local values	413
5.6.2	Local values and semi-local observables	415
5.6.3	Local values in generalized phase space	418
5.7	Appendix on Maximal Probability Families	420
5.8	Appendix on Time Operators	423
5.9	Concluding Remarks	425
	References	426
III Point Interactions, Macroscopic Quantum Systems and Superselection Rules		
6	Point Interactions	431
6.1	Introduction	431

6.2	Extensions of Symmetric Operators	433
6.3	Extensions of Direct Sum Operators	435
6.3.1	Direct sums and their selfadjoint extensions	435
6.3.2	Selfadjoint extensions in terms of boundary conditions	439
6.4	Quantization by Parts and Point Interactions	443
6.5	Classification of Point Interactions in \mathcal{IE}	446
6.5.1	Type 1 (BC1): The step potential	450
6.5.2	Type 2 (BC2): δ -interaction as high-pass filters	451
6.5.3	Type 3 (BC3): δ' -interaction as low-pass filters	455
6.5.4	Type 4 (BC4): Perfect reflector	463
6.5.5	Type 5 (BC5): Elastic reflectors	464
6.5.6	Type 6 (BC6): Open end	464
6.5.7	Type 7 (BC7): Ideal π -phase shifters	465
6.5.8	Type 8 (BC8): High-pass π -phase shifters	467
6.5.9	Type 9 (BC9): Low-pass π -phase shifters	469
6.5.10	Type 10 (BC10): Ideal mid-pass $\frac{1}{2}\pi$ -phase shifters	471
6.5.11	Type 11 (BC11): Partial mid-pass filter	473
6.5.12	Type 12 (BC12): Ideal tunable phase shifters	476
6.6	Remarks on Quantization by Parts	478
6.7	Charged Particles in Circular Motion	480
6.7.1	Charged particles constrained to move in a circle	480
6.7.2	Charged particles in 3-dimensions	486
6.8	Point Interactions in a Circle	489
6.8.1	Momentum operators	490
6.8.2	Hamiltonians with reflection symmetry	491
6.9	Classification of Point Interactions in \mathcal{C}	495
6.9.1	Type 1 (BCC1): Free motion	495
6.9.2	Type 2 (BCC2): δ -interaction	495
6.9.3	Type 3 (BCC3): δ' -interaction	497
6.9.4	Type 4 (BCC4): Perfect reflector	498
6.9.5	Type 5 (BCC5): Elastic reflector	498
6.9.6	Type 6 (BCC6): Open end	499
6.9.7	Type 7 (BCC7): Ideal dynamic π -phase shifter	500
6.9.8	Type 8 (BCC8): Static π -phase shifter	500
6.9.9	Type 9 (BCC9): Gradient π -phase shifter	501
6.9.10	Type 10 (BCC10): Ideal $\frac{1}{2}\pi$ -phase shifter	502
6.9.11	Type 11 (BCC11): Static junction correlator	503
6.9.12	Type 12 (BCC12): Ideal tunable phase shifters	504
6.10	Current and Stationary States in a Circle	505
	References	506

7	Macroscopic Quantum Systems	509
7.1	Single-Particle Representation	509
7.2	Macroscopic Wave Function Hypothesis	512
7.3	Uniformly Thick Superconducting Rings	513
7.3.1	Physical properties	513
7.3.2	Superconducting rings: Preliminaries	514
7.3.3	Superconducting rings as equilibrium mixed quantum systems	520
7.4	Superconducting Rings with a Junction	522
7.4.1	Josephson junction and dc Josephson effect	522
7.4.2	Supercurrent and magnetic flux operators	524
7.4.3	The Hamiltonian: Preliminary results	525
7.4.4	Superconducting ring with a Josephson junction as an equilibrium mixed quantum system	528
7.4.5	Superconducting ring with a π -junction	530
7.4.6	Superconducting ring with a $\frac{1}{2}\pi$ -junction	530
7.4.7	Superconducting ring with a Josephson junction in an external magnetic field	531
7.5	Feynman's Derivation of Josephson's Equation	533
7.6	Superconducting Wire with a Junction	535
7.6.1	Point interactions	535
7.6.2	Momentum and supercurrent operators	535
7.6.3	Hamiltonian operator 1: π -junction	536
7.6.4	Hamiltonian operator 2: $\frac{1}{2}\pi$ -junction	536
7.6.5	Hamiltonian operator 3: Josephson junction	537
7.6.6	Superconducting wire with a Josephson junction as a mixed equilibrium quantum system	539
7.7	Y-Shape Circuits	542
7.7.1	Momentum and supercurrent operators: Special cases	542
7.7.2	Hamiltonian operators: Special cases	544
7.7.3	Physics of strictly maximal symmetric operators	544
7.7.4	Momentum and supercurrent operators: General cases	545
7.7.5	Hamiltonian operators: General cases	547
7.7.6	Correlation	547
7.7.7	Superselection rules	548
7.7.8	Condensate in a pure or in a mixed state	549
7.8	Continuous Y-Shape Circuit	551
7.9	Superconducting Quantum Interference Devices	552
7.10	Non-Equilibrium Mixed Quantum System	554
7.11	BCS Theory and Superselection Rules	558
7.12	Conceptual Analyses	562

7.12.1	Non-uniqueness of quantization	562
7.12.2	Y-shape circuits, equilibrium mixed quantum systems and non-locality	562
7.12.3	Equilibrium states, globalization and non-locality	566
7.12.4	Quantum/Classical divide 2	568
7.13	Orthodox Quantum Systems	569
7.14	Prospects and Other Approaches	573
	References	575

IV Asymptotic Disjointness, Asymptotic Separability, Quantum Mechanics on Path Space and Superselection Rules

8	Separability and Decoherence	581
8.1	Introduction	581
8.2	Scattering Systems and de Broglie Paradox	586
8.2.1	Scattering systems	586
8.2.2	de Broglie paradox	587
8.3	Schrödinger's Cat States	589
8.3.1	Classical-like states	589
8.3.2	Classical cats and their states	592
8.3.3	Quantum cats and their states	593
8.3.4	Disjointness and Schrödinger's cat states	594
8.3.5	Scattering systems and Schrödinger's cat states	595
8.3.6	Quantized oscillator and Schrödinger's cat states	595
8.3.7	Weak Schrödinger's cat states	597
8.3.8	Periodic Schrödinger's cat states	599
8.3.9	Double-well potentials and chiral molecules	601
8.3.10	Dynamic and asymptotic decoherence	607
8.4	Superconducting Schrödinger's Cat States	608
8.4.1	Breakdown of superselection rules and capacitive junction	608
8.4.2	Schrödinger's cat states in superconducting systems	618
8.5	Asymptotically Separable Quantum Theory	620
8.5.1	Motivation	620
8.5.2	Asymptotically separable quantum mechanics	620
8.6	Entanglement and Decoherence	622
8.6.1	Distinguishable particles	623
8.6.2	Identical Fermions and Pauli exclusion principle	626
8.7	Chronological Disorder	629

8.7.1	The concept of chronological disordering	629
8.7.2	Two-particle correlation and conservation laws	630
	References	633
9	Quantum Mechanics on Path Space	637
9.1	Introduction	637
9.2	Physical Space and Path Space	638
9.3	Functions on Path Space	644
9.4	Quantum Mechanics on Path Space	649
9.4.1	Hilbert spaces $\mathcal{H}_\gamma(\mathbf{\Pi}(\mathcal{C}))$ on path space $\mathbf{\Pi}(\mathcal{C})$	649
9.4.2	Comparing $\mathcal{H}_\gamma(\mathbf{\Pi}(\mathcal{C}))$ and $L^2(\mathcal{C}_c)$	651
9.4.3	Position operators in $\mathcal{H}_\gamma(\mathbf{\Pi}(\mathcal{C}))$	654
9.4.4	Momentum operators in $\mathcal{H}_\gamma(\mathbf{\Pi}(\mathcal{C}))$	654
9.5	Josephson Effect and Superselection Rules	655
9.6	Concluding Remarks	657
	References	657
	Bibliography	659
	Index	675