ERRATUM/COMPACTNESS/RECENT PROGRESS IN CONFORMAL GEOMETRY

A. Bahri

The notations are the ones used in the book. The proof of Theorem 1 p290 and Proposition 31 p314 of Compactness pp279-384 contains two gaps. The first gap is located p 318 when we claim that the sum of the sizes of the $\pm v$-jumps Σc_i is $O(\epsilon)$. This estimate might not hold in full generality, the one which we are able to establish is $\Sigma c_i = O(\sqrt{\epsilon})$. The remainder of the arguments is correct, the results when the $*$'s are single $\pm v$-jumps are not affected, but the general result relies on this estimate.

The second gap is located p331, after the proof that the number of degenerate and non-degenerate ξ-pieces of x^∞ is bounded. This result holds, but we then claim that we can perturb $W_u(x_{2k+1}) \cap W_s(x^\infty_{2k})$ and exclude some flow-lines from this intersection. However the flow-lines which we claim to exclude cannot be gotten rid of by a simple perturbation argument in a level surface of the functional J above x^∞_{2k}. The claim would involve a global modification of $W_s(x^\infty_{2k})$ which we are not able to carry out.

These two gaps can however be overcome at the expense of a modification of the assumptions of Theorem 1 -Hypothesis (B)p290 is modified- and at the expense of some weakening in the conclusions. The result which we are able to prove is very close in spirit and in content, albeit more technical in its statement, to our earlier claims; the general result as we stated it before is still conjectured to hold. In the new version, Hypothesis (B) p290 is replaced by **one** of the following assumptions :

Hypotheses (B).

Hypothesis (1B):

The span of each family, measured in amount of v-rotation along the ξ-pieces running from the extreme left $\pm v$-jump of the family to the extreme right $\pm v$-jump of the same family, on a given characteristic ξ-piece is less than π by at least a fixed positive constant γ

Hypothesis (2B):

Given three consecutive $*$'s on a characteristic ξ-piece, the middle $*$ being a family, not a single $\pm v$-jump, the v-rotation on the ξ-pieces separating the $\pm v$-jumps between the extreme right edge of the first $*$ (starting from the left) and the extreme left edge of the last $*$ to the right is more than π by at least a fixed amount γ positive

This Hypothesis is a natural assumption, but we go beyond it and we introduce another set of independent assumptions:

Given a basic $\pm v$-jump of y^∞_{m-1} which is an edge to a characteristic ξ-piece, ξ transported from the bottom to the top of this $\pm v$-jump (the top being one of the ends of this characteristic piece) reads:

$$d\theta_i(\xi) = (1 + A^i_1)\xi + B^i_1[\xi, v] + C_1 v$$

$d\theta_i$ is the $\pm v$-transport from the bottom to the top of this $\pm v$-jump.

We introduce then:

Hypothesis(3B):

There exists a constant C positive depending only on the geometry of α, v such that:

i- Considering a characteristic ξ-piece of H^1_0-index larger than C and assuming that we are given a configuration such that all $*$'s on this ξ-piece are families, not single $\pm v$-jumps, we assume that the strengths of the $\pm v$-jumps of

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these various families which are **interior** *are comparable; that is, if* c_i^j *denotes the length along* $\pm v$ *of the* j^{th} $\pm v$*-jump of the family* i*, we assume that for any two of these families* i *and* j*:*

$$\frac{|\Sigma c_i^k|}{|\Sigma c_i^j|} \leq C$$

.

These sums are taken over the interior $\pm v$*-jumps, that is the* $\pm v$ *of the edges are discarded.*

The above condition is assumed to hold not only on the families of this characteristic piece but also on the families of the characteristic pieces of H_0^1*-index less than* C *and more than* 1*. Given a configuration, we consider among them those such that all* $*$*'s living on them are families, not single* $\pm v$*-jumps and we assume that they are all comparable among themselves, across different characteristic pieces.*

ii-The transport map Φ_s *of* ξ *along the various characteristic* ξ *pieces of* y_{m-1}^∞ *verify the inequalities:*

$$\frac{1}{C} \leq |D\Phi_s| \leq C$$

iii- for every $i \neq j$*,*

$$|\frac{A_1^i}{B_1^i}| \leq C|\frac{A_1^j}{B_1^j}|$$

We conjecture that the assumptions ii- and iii- can be warranted after manipulating the transport maps of ξ and v near y_{m-1}^∞ ([3]). If we further assume that the number of $\pm v$-jumps of the family is a priori bounded, assumption i- is "reasonable" and can be replaced by the simpler(but also different) condition that

i'- the sizes of any couple of $\pm v$*-jumps which are not close(that is the* v*-rotation on the* ξ*-pieces separating them is sizable) are comparable (on these special characteristic pieces where all* $*$*'s are families, not single* $\pm v$*-jumps)*

Some thought shows that we can expect that, as we decrease J and we deform our curves in the $\Gamma'_{2s}s$ and if the curves under deformation are not in the vicinity of a critical point at infinity, the relative sizes of the various $\pm v$-jumps should be bounded above and below by constants C and $\frac{1}{C}$, unless the ξ-distance between two of them becomes very small (then, one of them might disappear if it has been created as a companion of the other one). As we approach a critical point at infinity, we cannot expect such estimates to hold uniformly since some of the $\pm v$-jumps become large and contribute to the edges while other ones live on the ξ-pieces and are small. However, one can expect that such estimates hold for interior $\pm v$-jumps. We are also assuming that such comparison estimates hold across the characteristic when their H_0^1-index is bounded. This has to be checked.

Hypothesis (3B), with i- replaced by i'- is denoted **Hypothesis (3B)'**.

i-, i-', ii- and iii- are here introduced in a uniform way; that is all the families, all along the characteristic pieces and for all the basic $\pm v$-jumps associated to the characteristic pieces of y_{m-1}^∞. In fact, this uniformity is only needed on "enough of them" (measured by a number $\bar{m}$ of consecutive families starting from a given edge of a characteristic piece and by a number $\bar{m}$ of characteristic pieces which are either of strict H_0^1-index larger than 1 or are of H_0^1-index zero but have reverse edge orientations, to which iii-applies. Then the inequality $\frac{1}{C} \leq |D\Phi_s| \leq C$ has to be verified for $|s \leq \bar{m}\pi$, starting from the given edge of the characteristic piece).

We assume in Theorem 1 below, when m is odd that Hypothesis (3B)/(3B)' "holds through transmutations". This will mean the following:

Hypothesis (3B)/(3B)' holds "through transmutations"

When $m = 2k+1$, under Hypothesis (3B)/(3B)', the proof of Theorem 1 below contains two steps: in a first step, we prove that the number ℓ of full(half) unstable manifolds of characteristic ξ-pieces used once at a time in order to define the cycle of the dominated critical point at infinity y_{m-1}^∞ (ℓ is n p139 of [3]) is bounded below by $m_1 - C(m_0)$ where m_1 is the total number of characteristic ξ-pieces of the curve supporting y_{m-1}^∞. In a second step, we use when m is large Hypothesis (A) and we complete on this supporting curve "transmutations", see [1] pp 81-102. These transmutations increase the number m_1 of characteristic ξ-pieces by a large amount $m_2 \gg C(m_0)$. The number ℓ

remains unchanged through these transmutations (because of the special way in which we complete them) and this provides the contradiction and establishes Theorem 1.

y^∞_{m-1} thus changes. We assume though, in Theorem 1 below, that Hypothesis (3B)/(3B)' holds for the new y^∞_{m-1}, on one hand for the m_1 previous characteristic pieces of the supporting curve (of y^∞_{m-1}), and on the other hand for the new characteristic ξ-pieces of this curve. We assume that Hypothesis (3B)/(3B)' holds for the ξ-pieces of each of these groups, taken one at a time. This is the content of the assumption that Hypothesis (3B)/(3B)' "holds through transmutations".

While we conjecture that such an assumption is verified, see [3], we also conjecture that it can be removed and replaced by the simpler Hypothesis (3B)/(3B)'.

Under Hypothesis (A) and one of the various Hypotheses of type (B), Theorem 1 is modified as follows:

Theorem 1. *Let y^∞_{m-1} be a critical point at infinity of index $m-1$ having at least one characteristic piece. Assume that the maximal number of zeros of b on its unstable manifold is $2[\frac{m}{2}]$Let y_m be a periodic orbit of ξ of index m.*

i- Under Hypothesis (A) and Hypothesis (1B):

If m is even the intersection number $i(y_m,y^\infty_{m-1})$ is zero. This also holds true when m is odd if the number of $\pm v$-jumps of y^∞_{m-1} is large enough depending on the geometry of α and v.

ii-Under Hypothesis (A) and either of Hypothesis (2B), (3B) or (3B)'(which we assume to hold "through transmutations" when m is odd):

When m is even, the intersection number $i(y_m,y^\infty_{m-1})$ is zero if y^∞_{m-1} has more than one characteristic ξ-piece of a large enough H^1_0-index or if the number of characteristic ξ-pieces of y^∞_{m-1} which are either of strict H^1_0 index ≥ 1 or are of strict H^1_0-index zero with reverse edge orientation is large (depending on the geometry α and v). When m is odd, the intersection number $i(y_m,y^\infty_{m-1})$ is zero if y^∞_{m-1} has a large number (depending again on the geometry of α and v) of characteristic ξ-pieces which are either of strict H^1_0-index ≥ 1 or are of H^1_0-index zero but have reverse edge orientations

.

Observation: The proof of Theorem 1 does not require any restriction on the H^1_0-index of the ξ-pieces if the flow-lines from y_m to a neighborhood of y^∞_{m-1} do not involve companions, see 2.5.5 p 305 for the definition of this notion. Hypothesis (A) and Hypotheses (B) are discussed in [3]. Flow-lines from y_m to y^∞_{m-1} with y^∞_{m-1} having only non characteristic ξ-pieces have already been studied and ruled out for m large under Hypothesis (A), in [1].

The proof of Theorem 1 under these new assumptions follows the general outline of our former "proof", only that new arguments are introduced at the location of the gaps. We refer the reader to [2] for these modifications.

1. Bahri, A., *Flow-lines and Algebraic invariants in Contact Form Geometry PNLDE*, vol. 53, Birkhauser, Boston, 2003.
8. Bahri, A.,Compactness (2007) (to appear).
3. Bahri, A., (to appear).