

# Preface

This book arose out of a desire to investigate the relationship between certain algebraic properties of the curvature tensor and the underlying geometry of a pseudo-Riemannian manifold.

In Chapter 1, we discuss the geometry of the Riemannian curvature tensor. Basic geometrical notions are presented in Section 1.2. In Section 1.3, a passage to the algebraic context is given by introducing algebraic curvature tensors which are algebraic objects with the same symmetries as that of the Riemann curvature tensor. One says that a pseudo-Riemannian manifold is *curvature homogeneous* if the curvature tensor “looks the same at each point”. This notion is made precise in Section 1.4. Section 1.5 presents some results from linear algebra and Section 1.6 provides concepts from differential geometry that will be needed subsequently. In Section 1.7, the geometry of the Jacobi operator is discussed and in Section 1.8, corresponding results for the curvature operator are given. Chapter 1 concludes in Section 1.9 with some general facts concerning the spectral geometry of the curvature tensor.

Chapter 2 deals with curvature homogeneous generalized plane wave manifolds. This is a class of manifolds that are geodesically complete, whose exponential map is surjective, and which have a number of other properties. In Section 2.2, we present the main geometrical results concerning this class of manifolds. The remainder of Chapter 2 deals with specific examples. Sections 2.3, 2.4 and 2.5 deal with manifolds of signature  $(2, 2)$ ,  $(2, 4)$ , and  $(p, p)$ , respectively. Section 2.6 treats generalized plane wave manifolds with flat factors, Section 2.7 discusses Nikčević manifolds, and Section 2.8 presents Dunn manifolds. Sections 2.9 and 2.10 deal with  $k$ -curvature homogeneous manifolds.

Chapter 3 discusses examples which are not generalized plane wave man-

ifolds. Section 3.2 discusses Lorentz manifolds, Section 3.3 treats Walker manifolds of signature  $(2, 2)$ , Section 3.4 deals with questions of geodesic completeness and Ricci blowup, and Section 3.5 presents Fiedler manifolds.

Chapter 4 is more algebraic in nature. In Section 4.2, we present various topological results. In Section 4.3, we use the Nash embedding theorem to provide generators for the space of algebraic curvature tensors and algebraic covariant derivative tensors. Sections 4.4 and 4.5 treat Jordan Osserman algebraic curvature tensors and Szabó covariant derivative algebraic curvature tensors, respectively. In Section 4.6, we study questions in conformal geometry. Section 4.7 deals with Stanilov models. Section 4.8 treats complex geometry.

Chapter 5 contains a discussion of complex models which are both Osserman and complex Osserman; the classification is complete except in a few exceptional dimensions and ranks. Chapter 6 contains an introduction to Stanilov–Tsankov theory; this is a discussion of Jacobi Tsankov manifolds, skew Tsankov manifolds, Stanilov–Videv manifolds, and Jacobi Videv manifolds. Chapters 5 and 6 are joint work with M. Brozos-Vázquez.

Each chapter is divided into sections; the first section of a chapter provides an outline to the subsequent material in the chapter. Many sections are further divided into subsections. Theorems, lemmas, corollaries, and so forth are labeled by section. Equations which are cited are labeled by section; equations which are not cited are not labeled.

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