

Preface

This work deals with a well-defined class of probabilistic models. The focus is on polymers. More precisely I should say that the focus is on the equilibrium statistical mechanics of a class of polymers: dynamical and non-equilibrium phenomena are not treated, reflecting the fact that these directions are at the moment under-developed, at least from the viewpoint of the so called *rigorous results*. Moreover, only a subset of the world of polymer models is taken into consideration.

If I try to characterize such a subset, keeping in mind the motivations (that come from physics, biology, chemistry, material science, etc.), I end up with a list: $(1+d)$ -dimensional pinning models, $(1+1)$ -dimensional wetting models, adsorption models, copolymer models, DNA denaturation models, etc. and to each model in this list one should probably add the adjective *disordered*, since this work is mostly focused on disordered systems, even if non-disordered systems do play a central role. But such a list may at first appear quite *disordered* in itself...

If instead I take a purely mathematical standpoint, there is a totally natural thread connecting the models in the list: take a (persistent or terminating) renewal process and modify its law by giving rewards or penalties at the renewal epochs, possibly depending on time lapsed since the previous renewal epoch, and do that by introducing exponential, or *Boltzmann*, weights. On page 685 of his celebrated review paper *Walks, Walls, Wetting, and Melting* [Fisher (1984)] Michael E. Fisher writes:

“In fact, there is a rather simple but general mathematical mechanism which underlies a broad class of exactly soluble one-dimensional models which display phase transitions. This mechanism does not seem to be as well appreciated as it merits and it operates in a number of applications we wish to discuss.”

The general mathematical mechanism exposed by Fisher is really the thread I mention previously, with the difference that in this book disorder is introduced and the soluble character of the models disappear. One of my aims is to look at this general mechanism from the perspective of Renewal Theory, making it (possibly) more appealing to mathematicians, linking it with a number of extremely sharp results developed in this beautiful branch of probability and setting up a framework in which disorder appears to be somewhat more friendly.

The book is organized as follows:

- Chapter 1 is a collection of (motivating, I hope) models, with partial solutions, that lead to the general framework given in Section 1.8. This general framework is definitely useful for proving some general results, but I found it more useful (and readable!) working out many of the results on more particular cases, even in the cases in which they hold in full generality.
- Chapter 2 and Chapter 3 deal with non-disordered models and the Renewal Theory approach is developed in detail: some of the arguments are relatively heavy. However the basic ideas of this approach are already in Chapter 1 and one can read Chapters 4 to 9, the chapters dealing with disordered models, almost skipping these two chapters. This may sound paradoxical, since in Chapters 5 to 8 the reader will find very many links (above all) to Chapter 2. But these parts of Chapter 2 can be read when the moment comes. I stress that I am only signaling this option and I am not advising against reading the book more sequentially.
- Chapter 4 deals with the problem of the existence of the free energy for disordered models. An elementary proof is worked out in detail and some *other* proofs are sketched. The reason for emphasizing *other* is simply that I find it troublesome talking about different proofs when they are all based on super-additivity.
- Chapter 5 and Chapter 6 focus, respectively, on the phase diagram of the pinning model and of the copolymer models, which are in a sense the two basic models (by combining them one finds the general framework). The two chapters can be read independently, but some of the arguments given for pinning are not repeated for copolymers (but of course their validity is stated in the bibliographic complements at the end of the chapter, with a sketch of how to modify the arguments).

- Chapter 7 and Chapter 8 deal with the properties of polymer paths. These are sharply different in the localized regime (Ch. 7) and in the delocalized one (Ch. 8). The two chapters are independent and they do not require a detailed knowledge of Ch. 5 and Ch. 6. In fact I believe that they can even be read directly after Ch. 4, but of course it may be somewhat reassuring knowing that a localized, or delocalized, regime does exist.
- Chapter 9 deals with computational approaches to analyze these systems. The focus is on both methods (algorithms, data analysis tools) and results.
- There are then three appendices. Appendix A is rather long and aims at making this work as self-contained as possible. There one will find a number of standard and not so standard results, mostly from probability theory (but not only). Appendix B collects some important technical estimates, and Appendix C is a very quick reminder that directed random walks in $(1 + 1)$ -dimension are also *effective* models of interfaces and that some of our polymer models can in fact be reinterpreted in a different light.

In the sections in which the results are fully worked out (and these sections constitute about ninety percent of the book), the reader will find very few references, in fact almost none: the references are collected in the last section of each chapter, when discussing who did what and when, more things that have been done and what has not been done yet. I truly apologize for the (inevitable) omissions.

I have tried to make this book as self-contained as possible and the different chapters independent. My impression is that the diagram of dependences is something like the one in the figure, but of course I may be too optimistic.

And now a bit of history: I first met polymer models (precisely: copolymers) during my first post-doc (at the University of Zürich), when I met Erwin Bolthausen who suggested to me a number of very interesting problems that (I quote) *cannot be so difficult* (the responsibility for such a statement is entirely on Erwin's shoulders: some of these problems are the open problems in this book). At that time I drifted toward *not so difficult* problems not involving directly polymers, but that was not a choice: I have simply been driven by what I could solve first. However I kept playing with polymer questions and about three years ago I decided to make a serious effort, taking as a pretext also the fact that I had been invited by Franco

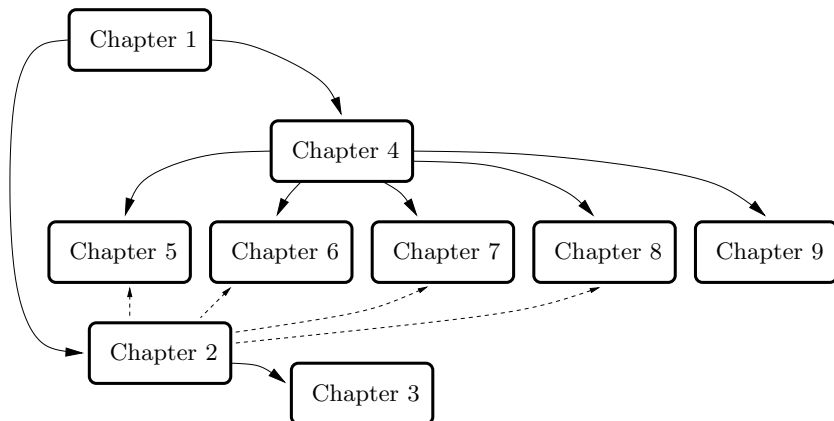


Diagram of possible reading orders of chapters. Dashed lines denote weaker links.

Flandoli to give a series of lectures at the Centro E. de Giorgi in Pisa during a trimester on *Interacting Particles and Computational Biology*. Polymer models and the *old questions* were ideal. On that occasion I started writing some notes, which then evolved through the years and through various graduate level mini-courses and courses I have given since. I have given full length math graduate courses at the University of Milano-Bicocca (I thank Daniela Bertacchi and Arrigo Cellina for the invitation) and at the Universities of Paris 6 and 7, where I work. I have also given a mini-course at the University of Pisa (I am grateful to Maurizio Pratelli and, again, to Franco Flandoli for inviting me) and right now, at the moment in which I am writing, at the Conference *Stochastic Processes in Mathematical Physics* that is held at Villa La Pietra (NYU, but in Florence). The latter event was an occasion to celebrate the sixtieth birthday of Michael Aizenman and Chuck Newman and this brought an extraordinary audience: I deeply thank the organizers and the scientific committee for having given me such an opportunity.

The lecture notes have gradually evolved thanks to the interaction with so many people through the years. However this book has been rewritten from scratch, starting about six months ago. The lecture notes still exist [Giacomin (2004)] and while they have, of course, a largely non-empty intersection with this book and contain substantially less material, a portion of them is not in here. This is due to choice of argument, and also line of presentation: in particular this book is really based on renewal processes

and not on random walks (and, in particular, not on the simple random walk). Apart from generalizing and shifting the approach a bit, another reason for writing the book from scratch was to rethink and rewrite every argument with a more mature spirit (more mature with respect to the notes and to what I have published up to now on the subject). This of course increases the chance of mistakes and, certainly, of typos. But I hope that the reader will appreciate that essentially all the arguments presented here are somewhat, and at times substantially, different from what one already finds in the literature.

I am greatly indebted to many people for their direct and indirect contributions. Let me first mention the two people that up to now have marked most my scientific perspectives: Joel Lebowitz, my PhD advisor and constant presence later on, and Erwin Bolthausen, to whom I went as a post-doc right after my thesis. Then I want to thank my very close collaborators over the last three years (as may be apparent from the list of references): Thierry Bodineau, Francesco Caravenna, Fabio Toninelli and Lorenzo Zambotti. They were also among the people who helped me the most in reviewing the preliminary versions of this book. But of course the list of people from which I greatly benefited is much longer, each of them for different reasons. In such a list certainly belong: Ken Alexander, Jean Bertoin, Francis Comets, Jean-Dominique Deuschel, Ron Doney, Thomas Garel, Massimiliano Gubinelli, Frank den Hollander, Dima Ioffe, Cécile Monthus, Enzo Orlandini, Nicolas Pétrélis, Jacques Portes, Julien Sohier, Herbert Spohn, Yvan Velenik and Ofer Zeitouni. Six members of my lab (LPMA) are already mentioned above: let me however thank all the people of LPMA and also all the colleagues and friends at Paris 7 and Paris 6 for the very nice atmosphere that I have always felt in Jussieu-Chevaleret.

While working on polymers I rediscovered my passion for computer programming that characterized my pre-university life (later, theoretical stuff took over). I also discovered the fantastic world of General Public License software (I now truly believe that research and education cannot go on without free software), so it is a pleasure for me to take this opportunity to thank all the people who contribute to this world scale enterprise.

And to the three persons closest to me while writing, and closest to me *tout court*, I dedicate the book.

Giambattista Giacomin
Paris, June 30th 2006