Preface

After the mountain,
another mountain
(traditional Korean saying)

As my friend and colleague Luca Migliorini once wrote to me, the topology of algebraic varieties is a mystery and a miracle.

These lectures are an attempt to introduce the reader to the Hodge theory of algebraic varieties.

The geometric implications of Hodge theory for a compact oriented manifold become progressively richer and more beautiful as one specializes from Riemannian, to complex, to Kähler and finally to projective manifolds.

The structure of these lectures tries to reflect this fact.

I delivered eight one-hour lectures at the July 22 – July 27, 2003 Summer School on Hodge Theory at the Byeonsan Peninsula in South Korea. The present text is a somewhat expanded and detailed version of those lectures.

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experience. In particular, I would like to thank Professor Dong-Kwan Shin for showing me with great humor some aspects of the Korean culture.

Beyond the choice of topics, exercises and exposition style, nothing in these written-up version of the lectures is original. The reader is assumed to have some familiarity with smooth and complex manifolds. These lectures are not self-contained and at times a remark or an exercise require knowledge of notions and facts which are not covered here. I see no harm in that, but rather as an encouragement to the reader to explore the subjects involved. The location of the exercises in the lectures is a suggestion to the reader to solve them before proceeding to the theoretical facts that follow.

The table of contents should be self-explanatory. The only exception is §8 where I discuss, in a simple example, a technique for studying the class map for homology classes on the fibers of a map and one for approximating a certain kind of primitive vectors. These techniques have been introduced in [de Cataldo and Migliorini, 2002] and [de Cataldo and Migliorini, 2005].

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