Chapter 1

The Useful and the Beautiful

Any reader browsing this book should keep this in mind: that all illustrations have a mathematical significance. Moreover, some of the underlying equations are applicable in physics, chemistry, biology or the social sciences (Chapter 8). Other equations are of purely mathematical interest and possess properties that are of general significance, i.e. they are not dependent on the properties of a specific application (Chapter 9).

I will explain the technical details in Chapter 6. In all cases, the coordinates (abscissas and ordinates) correspond to parameters that control a given system (such as the amplitude and the period of electric pulses in Sec. 8.1, the light's phase shift and the mirrors' absorbance in Sec. 8.11, or the reproduction rate of predators and prey in Sec. 8.16). At a given point on the plane, i.e. at a pair of coordinates, these control parameters are set to be constant in time, while the so-called phase variables (called x_n and y_n in this work) may change in time. In Sec. 8.16, for example, the phase variables are the relative numbers of predators and prey. The shade of grey or the colour at a point on the plane indicates how the phase variables change in time, telling us if they change in a predictable or a chaotic way. The shade of grey or colour also tells us, if the system is predictable and how fast it recovers from perturbations; if the system is chaotic, meaning that its predictability is limited like the weather, to what extent can we make finite-time predictions.

Here I want to insert a clarifying note for the non-scientist: the diagrams in this book are comparable to maps in geography. For a given point on the plane, the colours in geographical maps tell us where there is a sea and how deep it is, or and where is land and how high it is. Mirroring the function of the longitude and latitude of geographical maps, the coordinates in this book can signify quantities that characterize a system and do not change in

time (such as the interest rate of a long-term bank deposit in an economical model). In an analysis of those quantities that do change in time (such as the amount of money on the bank account), the colours (or shades of grey) indicate (instead of sea or land), whether the system is predictable, and its stability when disturbed, or the extent to which it is unpredictable.

In addition to their scientific significance, the diagrams are aesthetically appealing. Note, however, that the fact that mathematical or scientific objects can stimulate us aesthetically is not a new phenomenon: simply consider the golden mean, the platonic solids and other polyedra, mazes, labyrinths, kaleidoscopic images [3] and especially fractals [4]. Moreover, there are living beings, such as radiolaria (see the impressive collection published by Ernst Haeckel [5]), as well as astronomical nebulae, which are a visual delight.

In part, this book contains diagrams which may be scientifically useful, but were nevertheless generated only with an aesthetic motivation. For example, this is the case for most diagrams in Sections 9.20, 9.21 and 9.22. However, the rules of mathematics were never broken.