Chapter 1

Introduction of MIMO Channel and Space-Time Block Code

In the past few years, there has been a phenomenal increase in consumers' as well as manufacturers' interest in wireless communications. This is due to the advances of wireless communication technology providing the advantages of wide area coverage without wires, and most importantly, allowing mobility while communicating. Beyond the success of the established technologies such as mobile telephony, a wide range of new wireless communications services are being developed. For example, there has been growing interest in providing broadband wireless Internet services with rich multimedia contents at near wire-line data rates. However, the wireless channel suffers from random signal attenuation and phase distortion due to the destructive superposition of multiple received signals in a multipath propagation environment, a phenomenon commonly called *fading*. To mitigate fading and push the capacity of wireless channel to a higher limit, the use of multiple transmitting and/or receiving antennas, or the so-called multiple-input multiple-output (MIMO) concept, has recently been proposed.

1.1 MIMO Channel for Wireless Communications

Fading makes it extremely difficult for the receiver to recover the transmitted signal unless the receiver is provided with some form of *diversity*, i.e. replicas of the same transmitted signal with uncorrelated attenuation. In fact, diversity combining technology has been one of the most important contributors to reliable wireless communications. Ways to achieve diversity include:

- *Temporal Diversity*: In this scheme, channel coding in conjunction with time interleaving is used. Thus replicas of the transmitted signal are provided to the receiver in the form of redundancy in the temporal domain. However, in slow fading channels, temporal diversity is not an option for delay-sensitive applications.
- *Frequency Diversity*: In this scheme, the fact that signals that are transmitted on different frequencies tend to experience different fading effects is exploited. Thus replicas of the transmitted signal are provided to the receiver in the form of redundancy in the frequency domain. However, this scheme is not bandwidth-efficient.
- *Spatial Diversity*: In this scheme, spatially separated antennas are used to provide diversity in the spatial domain. Diversity combining technique is then used to select or combine the signals that have been transmitted or received on different antennas.

Spatial diversity is attractive as diversity can be obtained with no penalty in bandwidth efficiency. It can be implemented by deploying multiple antennas at the transmitter and/or the receiver. Depending on the location of the antennas, we can classify wireless communication system employing spatial diversity into the following three configurations:

- *Single Input Multiple Output* (SIMO): When there are single transmit antenna but multiple receive antennas, i.e. *receive diversity.*
- *Multiple Input Single Output* (MISO): When there are multiple transmit antennas but one receive antenna, i.e. *transmit diversity.*
- *Multiple Input Multiple Output* (MIMO): When there are multiple transmit antennas and multiple receive antennas, i.e. both transmit and receive diversity are used.

Besides providing spatial diversity, it has been shown in [1,2] that the capacity of a wireless channel grows linearly with the number of transmit and receive antennas, hence a MIMO system can be used to boost the capacity of wireless channel too.

Considering the fact that mobile receivers are typically required to be small and cost-effective, it may not be practical to deploy receive diversity at the mobile terminal. This motivates many researchers to consider transmit diversity by deploying multiple antennas at the base station. Moreover, in economic terms, the cost of multiple transmit antennas at the base station can be amortized over numerous mobile users. Hence transmit diversity has been identified as one of the key contributing technologies to the downlinks of 3G wireless systems such as W-CDMA and CDMA2000 [3]. There are generally three categories of transmit diversity:

- *Feedback Scheme*: This involves the feedback of channel state information (CSI, typically including channel gain and phase information) from the receiver to the transmitter in order to adapt the transmitter to the channel during the next transmission epochs. It is also commonly known as the "closed-loop" system.
- *Feedforward Scheme*: This involves the receiver making use of feedforward information sent by the transmitter, such as pilot symbols, to estimate the channel, but no channel feedback information is sent back to the transmitter. It is also commonly known as the "open-loop" or "coherent" system.
- *Blind Scheme*: This requires no feedback of CSI or feedforward of pilots, and the receiver simply makes use of the received signal to attempt data recovery without the knowledge of CSI. It is also commonly known as the "non-coherent" system.

To demonstrate the benefit of transmit diversity under the feedforward scheme, the bit error rate (BER) performance versus bit energy to noise spectral density ratio (E_b/N_o) of a typical transmit diversity scheme with various number of transmit antennas and one receive antenna at a spectral efficiency of 2 bits/sec/Hz (bps/Hz) is illustrated in Fig. 1.1. These results are achieved by using *Space-Time*

Block Code (STBC), a type of feedforward transmit diversity coding scheme that will be the main focus of this monograph. It can be seen that when there is only one transmit antenna, more than 15dB increase in E_b/N_o is required to achieve a BER of 10^{-3} in a Rayleigh faded wireless channel over an additive white Gaussian noise (AWGN) channel. By employing multiple transmit antennas to provide transmit diversity, the BER can be significantly reduced, such that the BER curve decays faster with E_b/N_o. This is due to the multiple transmit antennas providing higher spatial diversity level. However, unlike receive diversity that can be achieved by simply performing the diversity combining at the receiver side, transmit diversity requires some form of signal processing, generally known as space time coding, on the transmitted signals in order to achieve signal enhancement at the receiver.

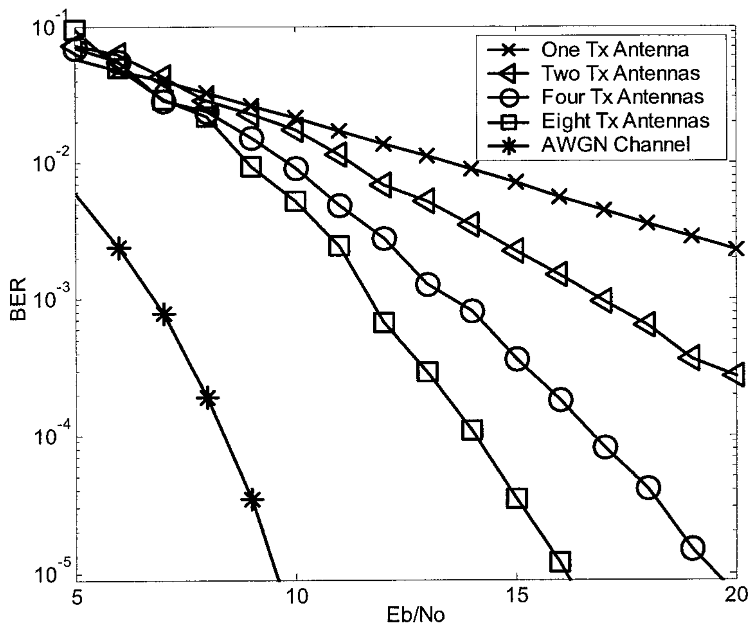

Fig. 1.1 Feedforward transmit diversity with various diversity levels.

Space-Time Coding (STC) is a technique that combines coding, modulation and signal processing to achieve transmit diversity. The first

STC proposed in the literature is *Space-Time Trellis Code* (STTC) [4], which has a good decoding performance but decoding complexity that increases exponentially with the transmission rate. In addressing the issue of decoding complexity of STTC, *Space-Time Block Code* (STBC) was subsequently proposed. Alamouti [5] discovered a remarkable STBC scheme for two transmit antennas. This scheme supports linear decoding complexity for maximum-likelihood (ML) decoding, which is much simpler than the decoding of STTC. It can achieve the same diversity gain as a corresponding STTC for two transmit antennas, though with a shortfall in coding gain. Despite the lower coding gain, Alamouti's scheme is very appealing in terms of implementation simplicity. Hence it motivates a search for similar schemes for more than two transmit antennas, to achieve diversity level higher than two. As a result, *Orthogonal Space-Time Block Code* (O-STBC) was introduced by Tarokh *et al.* in [6]. O-STBC is a generalization of the Alamouti's scheme to an arbitrary number of transmit antennas. It retains the property of having linear maximum-likelihood decoding with full transmit diversity.

Although O-STBC can provide full diversity at low computational cost, [7] showed that it suffers a loss in capacity when (1) there are multiple receive antennas, (2) the code rate is less than one. As rate-1 O-STBC with complex constellation is not possible for more than two transmit antennas [6], O-STBC design for more than two transmit antennas will always suffer capacity loss.

To address the issue of capacity loss, various non-orthogonal STBC designs have been proposed. An interesting one among them is the *Quasi-Orthogonal STBC* (QO-STBC) [8,9,10], which is designed to achieve a higher code rate than O-STBC by partially (instead of fully, as in the case of other non-orthogonal STBCs) relaxing the orthogonality of an O-STBC. For example, the ML decoding of the full-rate QO-STBC in [8] for four transmit antennas can be achieved by jointly detecting two out of four complex symbols in the codeword, and separately doing the same for the remaining two complex symbols. Due to this low decoding complexity advantage of QO-STBC, as well as its ability to achieve full transmit diversity, we seek to provide a complete study on QO-STBC in this monograph, and to seek further improvements in its design.

The focus of this monograph is on the spatial diversity for MISO or MIMO channel with feedforward and blind configurations by using QO-STBC in non-frequency selective channel and uncoded system. We focus on the fundamental code design issues of QO-STBC, as they serve as the basic element for extension to closed-loop MIMO system, coded MIMO system and MIMO systems for frequency selective fading channels, which will be briefly discussed in the last chapter of this monograph.

1.2 Transmit Diversity with Space-Time Block Code

Before we introduce the transmit diversity scheme based on Space-Time Block Code (STBC), we first review the traditional receive diversity scheme with *maximal ratio combining* (MRC) for one transmit antenna and two receive antennas, using Fig. 1.2 as an example.

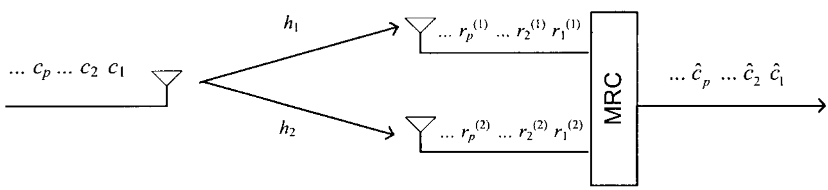

Fig. 1.2 Receive diversity with one transmit and two receive antennas

Denoting the transmitted signal at time p as c_p, and the received signals on the first and second receive antennas as $r_p^{(1)}$ and $r_p^{(2)}$ respectively, we obtain the following expressions:

$$\begin{aligned} r_p^{(1)} &= h_1 c_p + \eta_p^{(1)}, \\ r_p^{(2)} &= h_2 c_p + \eta_p^{(2)}, \end{aligned} \tag{1.1}$$

where h_1 and h_2 are the CSI or path gain from the transmit antenna to the first and second receive antennas respectively, and $\eta_p^{(1)}$ and $\eta_p^{(2)}$ are additive white Guassian noises (AWGN) at the respective received antennas at time instant p.

If only one receive antenna is available, the transmitted symbols can be estimated as follows, by assuming that the CSI is known accurately:

$$\begin{aligned}\hat{c}_p &= h_i^* r_p^{(i)} \qquad\qquad i=1,2 \\ &= |h_i|^2 c_p + h_i^* \eta_p^{(i)},\end{aligned} \tag{1.2}$$

where * denotes the complex conjugate and | . | denotes the magnitude of a complex element.

When multiple receive antennas are available, to retrieve the data symbols utilizing the diversity signals provided by multiple receive antennas, we perform MRC as follows:

$$\begin{aligned}\hat{c}_p &= h_1^* r_p^{(1)} + h_2^* r_p^{(2)} \\ &= \left(|h_1|^2 + |h_2|^2\right) c_p + h_1^* \eta_p^{(1)} + h_2^* \eta_p^{(2)}.\end{aligned} \tag{1.3}$$

We can see from both equations (1.2) and (1.3) that the signal estimate $\hat{c}_p$ consists of the actual signal c_p weighted by a factor related to the fading magnitude, then summed with a noise term. We can say that (1.3) gives a better estimate than (1.2) because the chance that both h_1 and h_2 in (1.3) fade simultaneously is much smaller than the chance that h_1 in (1.2) fades. Statistically, if h_1 and h_2 are Rayleigh-distributed and uncorrelated, $|h_1|^2$ will have a Chi-Square distribution with two degrees of freedom, while $|h_1|^2+|h_2|^2$ will have a Chi-Square distribution with four degree of freedom, hence a lower probability of deep fade. This explains the diversity gain of a MRC receive diversity scheme over a non-diversity scheme.

However, the deployment of multiple receive antennas at the mobile station may not be feasible due to size and cost constraints, this has therefore motivated the research of transmit diversity to provide spatial diversity for the downlink channel using multiple transmit antennas at the base station. In [5], Alamouti proposed a simple two-antenna transmit diversity scheme which up to today remains the only O-STBC that achieves the same diversity gain as the two-antenna receive MRC diversity scheme at full rate for any complex constellation.

The Alamouti O-STBC scheme is described as follows. Considering a system with two transmit antennas and one receive antenna as shown in Fig. 1.3, at a given symbol period, two signals are simultaneously transmitted from two antennas using the same bandwidth. At time $2p$-1, the signal c_{2p-1} is transmitted from the first antenna, while the signal c_{2p} is transmitted from the second antenna. During the next symbol period $2p$, signal $-c_{2p}^*$ is transmitted from the first antenna, and signal c_{2p-1}^* is transmitted from the second antenna. In order to normalize the total transmission power to be the same as the receive diversity scheme in Fig. 1.2, the transmission power from each transmit antenna is halved.

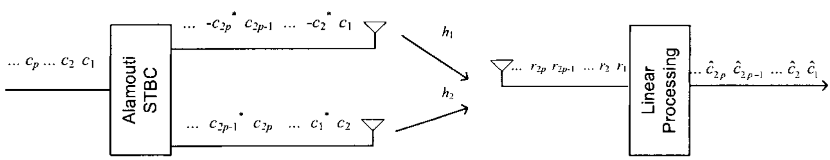

Fig. 1.3 Transmit diversity with two transmit and one receive antenna

In this case, h_1 and h_2 are the CSI from the first and second transmit antenna to the single receive antenna respectively, and they are assumed to remain unchanged for two symbol periods. The received signals at time $2p$-1 and $2p$ can be expressed respectively as:

$$\begin{aligned} r_{2p-1} &= \frac{1}{\sqrt{2}}\left(h_1 c_{2p-1} + h_2 c_{2p}\right) + \eta_{2p-1}, \\ r_{2p} &= \frac{1}{\sqrt{2}}\left(h_2 c_{2p-1}^* - h_1 c_{2p}^*\right) + \eta_{2p}, \end{aligned} \tag{1.4}$$

where the factor $1/\sqrt{2}$ accounts for the power normalization, η_{2p-1} and η_{2p} are AWGN at the receiver at time $2p$-1 and $2p$ respectively.

Assuming that perfect CSI is known to the receiver, the transmitted data symbols can be recovered by linear combining as shown below:

$$\begin{aligned} \hat{c}_{2p-1} &= h_1^* r_{2p-1} + h_2 r_{2p}^* = \frac{1}{\sqrt{2}}(|h_1|^2 + |h_2|^2)c_{2p-1} + h_1^* \eta_{2p-1} + h_2 \eta_{2p}^*, \\ \hat{c}_{2p} &= h_2^* r_{2p-1} - h_1 r_{2p}^* = \frac{1}{\sqrt{2}}(|h_1|^2 + |h_2|^2)c_{2p} + h_2^* \eta_{2p-1} - h_1 \eta_{2p}^*. \end{aligned} \tag{1.5}$$

Each of the resultant signals in (1.5) are similar to those in (1.3). Therefore, the diversity order of the above O-STBC system with simple linear receiver processing is the same as the corresponding receive diversity with MRC. This makes O-STBC very attractive. A detailed study of transmit diversity based on STBC will be presented in Chapter 2, while we continue to introduce the signal models for MIMO channel and STBC transmission in this chapter.

1.3 Notations and Abbreviations

Major notations employed in this monograph are:

$*$	Hadamard product
$\otimes$	Kronecker product
$\mathbf{0}_n$	zero matrix of dimension n-by-n
A, a	scalar
$\mathbf{a}$	column vector
$\mathbf{A}$	matrix
$\mathbf{I}_n$	identity matrix of dimension n-by-n
j	square root of -1
$(\,.\,)^*$	complex conjugate
$(\,.\,)^H$	complex conjugate transpose / hermitian
$(\,.\,)^I$	imaginary part of a complex element, vector or matrix
$(\,.\,)^R$	real part of a complex element, vector or matrix
$(\,.\,)^T$	transpose
$\lvert\,.\,\rvert$	magnitude of complex element
$\lVert\,.\,\rVert^2$	Frobenius norm
$\lceil x \rceil$	smallest integer larger than x
Det(**M**)	determinant of a matrix **M**
E(.)	expectation operator
max(.)	maximization operator
min(.)	minimization operator
Rank(**M**)	rank of a matrix **M**
Tr(**M**)	trace of a matrix **M**

Major abbreviations are:

AOD	amicable orthogonal design
AWGN	additive white Gaussian noise
BER	bit error rate
BLER	block error rate
bps/Hz	bits per sec per hertz
CSI	channel state information
CR	constellation rotation
DSTM	differential space-time modulation
GCLT	group-constrained linear transformation
JD	joint detection
MDC-QOC	minimum–decoding–complexity quasi-orthogonality constraints
MDC-QOSTBC	minimum–decoding–complexity quasi-orthogonal STBC
MIMO	multiple input multiple output
ML	maximum-likelihood
MSD	maximal symbol-wise diversity
OD	orthogonal design
O-STBC	orthogonal space-time block code
PSK	Phase shift keying
QAM	quadrature amplitude modulation
QOC	quasi-orthogonality constraints
QO-STBC	quasi-orthogonal space-time block code
SD	sphere decoding
SNR	signal-to-noise ratio
STBC	space-time block code

1.4 Signal Model of MIMO Channel and STBC

1.4.1 Signal model of MIMO channel

We consider a general MIMO wireless communication system with N_T transmit antennas at the base station and N_R receive antennas at the mobile, as shown in Fig. 1.4. At each time slot p, the signal $x_p^{(i)}$ is transmitted from the i^{th} transmit antennas, where i = 1, 2, …, N_T. The channel is assumed to be a flat fading channel and the path gain from transmit antenna i to receive antenna k is denoted as $h_{i,k}$. It is assumed that $h_{i,k}$ and $h_{l,q}$ are independent for $1 \le i,\ l \le N_T$, $1 \le k\ ,\ q \le N_R$, and for different i, k and l, q pairs. This condition is satisfied if the antennas are well separated by more than half of the wavelength of the transmitted wave, or by using antennas with different polarization. Since we shall focus on transmit diversity with just one receive antenna in our study, the receive antenna index k will later be omitted when not used, i.e. $h_{i,k}$ will just be written as h_i for simplicity. The flat fading path gains are modeled as independent complex Gaussian random variables with variance 0.5 per real dimension, i.e. $h_{i,k} = \alpha_{i,k} \exp(j\theta_{i,k})$, where $\alpha_{i,k}$ follows the Rayleigh distribution and $\theta_{i,k}$ is uniformly distributed. The channel fading is assumed to be quasi-static, i.e. the path gains are assumed to be constant over a frame of length F and only vary from frame to frame.

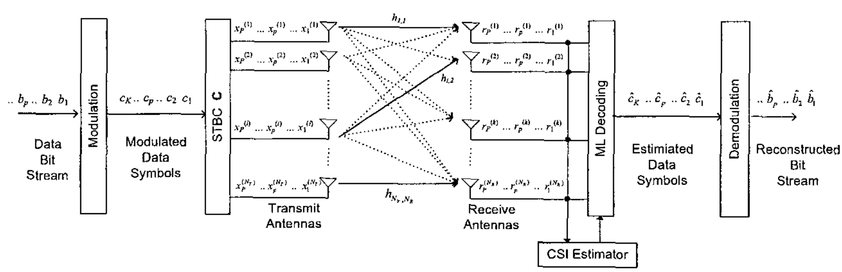

Fig. 1.4 MIMO wireless communication model

We assume that the transmitted signal $x_p^{(i)}$ has unit power and E_s is the total energy transmitted from all antennas. Therefore, the energy transmitted from each transmit antenna is E_s / N_T. Signals arriving at different receive antennas undergo independent fading. The signal at each receive antenna is a noisy superposition of different faded versions of the N_T transmitted signals. At time p, the signal $r_p^{(k)}$ received at the k^{th} antenna, where $k = 1, 2, \ldots, N_R$, is given by

$$r_p^{(k)} = \sqrt{\frac{E_s}{N_T}} \sum_{i=1}^{N_T} \left(h_{i,k} x_p^{(i)} + \eta_p^{(k)} \right), \tag{1.6}$$

where the noise samples $\eta_p^{(k)}$ are independent samples of a zero-mean complex white Gaussian random variable with variance $N_o / 2$ per real dimension. Since the total energy of the symbols transmitted from all transmit antennas is normalized to be E_s, the average energy of the received signal at each receive antenna is E_s and the SNR ρ is E_s / N_o per receive antenna. Without loss of generality, we may assume E_s to be equal to one unless it is stated otherwise.

The signals (1.6) collected by all the receive antennas may be expressed compactly in matrix form as follows:

$$\begin{bmatrix} r_p^{(1)} \\ r_p^{(2)} \\ \vdots \\ r_p^{(k)} \\ \vdots \\ r_p^{(N_R)} \end{bmatrix} = \sqrt{\frac{E_S}{N_T}} \begin{bmatrix} h_{1,1} & h_{2,1} & \cdots & h_{i,1} & \cdots & h_{N_T,1} \\ h_{1,2} & h_{2,2} & \cdots & h_{i,2} & \cdots & h_{N_T,2} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ h_{1,k} & h_{2,k} & \cdots & h_{i,k} & \cdots & h_{N_T,k} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ h_{1,N_R} & h_{2N_R} & \cdots & h_{i,N_R} & \cdots & h_{N_T,N_R} \end{bmatrix} \begin{bmatrix} x_p^{(1)} \\ x_p^{(2)} \\ \vdots \\ x_p^{(i)} \\ \vdots \\ x_p^{(N_T)} \end{bmatrix} + \begin{bmatrix} \eta_p^{(1)} \\ \eta_p^{(2)} \\ \vdots \\ \eta_p^{(k)} \\ \vdots \\ \eta_p^{(N_R)} \end{bmatrix}, \tag{1.7}$$

where the N_R–by–N_T MIMO channel $\mathcal{H}$ is defined as:

$$\mathcal{H} = \begin{bmatrix} h_{1,1} & h_{2,1} & \cdots & h_{i,1} & \cdots & h_{N_T,1} \\ h_{1,2} & h_{2,2} & \cdots & h_{i,2} & \cdots & h_{N_T,2} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ h_{1,k} & h_{2,k} & \cdots & h_{i,k} & \cdots & h_{N_T,k} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ h_{1,N_R} & h_{2N_R} & \cdots & h_{i,N_R} & \cdots & h_{N_T,N_R} \end{bmatrix}. \tag{1.8}$$

The information capacity achieved by such an open-loop MIMO channel is [1,2]:

$$C_{\text{channel}} = \mathrm{E}_{\mathcal{H}}\left\{\log\det\left(\mathbf{I}_{N_{\mathrm{R}}} + \frac{\rho}{N_{\mathrm{T}}}\mathcal{H}\mathcal{H}^{\mathrm{H}}\right)\right\} \text{ bps/channel use,} \quad (1.9)$$

where E{.} represents the expectation operation and bps stands for bits per sec.

1.4.2 Signal model of STBC

Suppose that a generic STBC codeword is transmitted from N_{T} transmit antennas to N_{R} receive antennas over an interval of P symbol periods. The propagation channel condition is time-invariant within a frame length of F symbol periods ($F \geq P$) and is known to the receiver. The transmitted codeword can be written as a $P \times N_{\mathrm{T}}$ matrix $\mathbf{C}$ that contains K complex constellation symbols. Its code length is P, and its code rate is defined as $R = K / P$. Following the model in [11], $\mathbf{C}$ can be expressed as:

$$\mathbf{C} = \sum_{i=1}^{K}\left(c_i^{\mathrm{R}}\mathbf{A}_i + jc_i^{\mathrm{I}}\mathbf{B}_i\right), \quad (1.10)$$

where the information symbols are $c_i = c_i^{\mathrm{R}} + jc_i^{\mathrm{I}}$, and c_i^{R} and c_i^{I} are the real (I) and the imaginary (Q) components of c_i. Matrices $\mathbf{A}_i$ and $\mathbf{B}_i$, both of dimension $P \times N_T$, are called the "*dispersion matrices*" of the STBC.

To illustrate all the above definitions, we again use the Alamouti STBC in Fig. 1.3 as an example. Using the signal model in (1.10), we can write the Alamouti STBC codeword as follows:

$$\mathbf{C}_{\text{Alamouti}} = \begin{bmatrix} c_1 & c_2 \\ -c_2^* & c_1^* \end{bmatrix}, \quad (1.11)$$

where the corresponding dispersion matrices are:

$$\mathbf{A}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \mathbf{A}_2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \mathbf{B}_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \mathbf{B}_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}. \quad (1.12)$$

From the above example, it should be clear that the matrix $\mathbf{A}_i$ "disperses" the real part of the symbol c_i on different antenna and time positions, while the matrix $\mathbf{B}_i$ does the same for the imaginary part of c_i, hence the name "dispersion matrix" for them.

The row of the STBC codeword represents the signal to be transmitted at a particular time slot, while the column of the STBC codeword represents the signal to be transmitted at a particular transmit antennas. One can also easily see that for $N_T = 2$ transmit antennas, the Alamouti STBC takes $P = 2$ symbols period to transmit $K = 2$ complex symbols, hence it has a code rate R of $K / P = 1$.

For a given number of transmit antennas, the design of a STBC depends crucially on the code parameters P, K, and the dispersion matrices $\{\mathbf{A}_i, \mathbf{B}_i\}$. With the representation of STBC in (1.10), the transmitted and received signals are related by [11]:

$$\tilde{\mathbf{r}} = \sqrt{E_s/N_T}\,\mathbf{H}\tilde{\mathbf{c}} + \tilde{\boldsymbol{\eta}}, \tag{1.13}$$

where

$$\tilde{\mathbf{r}} = \begin{bmatrix} \mathbf{r}_1^R \\ \mathbf{r}_1^I \\ \vdots \\ \mathbf{r}_{N_R}^R \\ \mathbf{r}_{N_R}^I \end{bmatrix}, \tilde{\mathbf{c}} = \begin{bmatrix} c_1^R \\ c_1^I \\ \vdots \\ c_K^R \\ c_K^I \end{bmatrix}, \tilde{\boldsymbol{\eta}} = \begin{bmatrix} \boldsymbol{\eta}_1^R \\ \boldsymbol{\eta}_1^I \\ \vdots \\ \boldsymbol{\eta}_{N_R}^R \\ \boldsymbol{\eta}_{N_R}^I \end{bmatrix},$$

$$\mathbf{H} = \begin{bmatrix} \mathcal{A}_1\tilde{\mathbf{h}}_1 & \mathcal{B}_1\tilde{\mathbf{h}}_1 & \cdots & \mathcal{A}_K\tilde{\mathbf{h}}_1 & \mathcal{B}_K\tilde{\mathbf{h}}_1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathcal{A}_1\tilde{\mathbf{h}}_{N_R} & \mathcal{B}_1\tilde{\mathbf{h}}_{N_R} & \cdots & \mathcal{A}_K\tilde{\mathbf{h}}_{N_R} & \mathcal{B}_K\tilde{\mathbf{h}}_{N_R} \end{bmatrix},$$

$$\mathcal{A}_q = \begin{bmatrix} \mathbf{A}_q^R & -\mathbf{A}_q^I \\ \mathbf{A}_q^I & \mathbf{A}_q^R \end{bmatrix}, \mathcal{B}_q = \begin{bmatrix} -\mathbf{B}_q^I & -\mathbf{B}_q^R \\ \mathbf{B}_q^R & -\mathbf{B}_q^I \end{bmatrix}, \tilde{\mathbf{h}}_i = \begin{bmatrix} \mathbf{h}_i^R \\ \mathbf{h}_i^I \end{bmatrix}.$$

In the above equation, $\mathbf{r}_i$ and $\boldsymbol{\eta}_i$ ($1 \le i \le N_R$) are $P \times 1$ column vectors which contain the received signals and AWGN noises for the i^{th} receive antenna respectively, over P symbol periods. $\mathbf{H}$ of dimension $2PN_R \times 2K$ is called the *equivalent channel matrix*, $\mathbf{h}_i$ is a $N_T \times 1$ column vector that contains the fading coefficients of the spatial sub-channels between the N_T transmit antennas and i^{th} receive antenna. The normalization factor

$\sqrt{E_s/N_T}$ in (1.13) ensures that the SNR $\rho = E_s/N_o$ is the same at each receive antenna, regardless of what N_T is.

Using the Alamouti STBC in (1.12) as an example, its equivalent channel matrix **H** can be computed using the following four matrices:

$$\mathcal{A}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \mathcal{A}_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix},$$
$$\mathcal{B}_1 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}, \mathcal{B}_2 = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}. \tag{1.14}$$

Basic design requirements on the dispersion matrices include the three *Power Distribution Constraints* [11]:

$$\begin{aligned} &\text{(i)} \quad \sum_{i=1}^{K}\left[\mathrm{Tr}\left(\mathbf{A}_i^{\mathrm{H}}\mathbf{A}_i\right)+\mathrm{Tr}\left(\mathbf{B}_i^{\mathrm{H}}\mathbf{B}_i\right)\right]=2PN_{\mathrm{T}}; \\ &\text{(ii)} \quad \mathrm{Tr}\left(\mathbf{A}_i^{\mathrm{H}}\mathbf{A}_i\right)=\mathrm{Tr}\left(\mathbf{B}_i^{\mathrm{H}}\mathbf{B}_i\right)=\frac{PN_{\mathrm{T}}}{K}, \qquad 1\le i\le K; \\ &\text{(iii)} \quad \mathbf{A}_i^{\mathrm{H}}\mathbf{A}_i=\mathbf{B}_i^{\mathrm{H}}\mathbf{B}_i=\frac{P}{K}\mathbf{I}_{N_{\mathrm{T}}}, \qquad 1\le i\le K. \end{aligned} \tag{1.15}$$

And Tr(.) denotes the trace, or sum of all diagonal elements, of a matrix.

The constraints (i)–(iii) in (1.15) govern the distribution of transmission power across space and time in the following ways: Constraint (i) ensures that the total average transmitted power is normalized to PN_T, i.e. $E[\mathrm{Tr}(\mathbf{CC}^{\mathrm{H}})] = PN_T$. Constraint (ii) is more restrictive and ensures that each of the I and Q signals (c_i^{R} and c_i^{I}) is transmitted with the same overall power over P symbol durations from all antennas. Constraint (iii) is the most stringent: it forces the symbols c_i^{R} and c_i^{I} to have equal energy in all spatial directions during *each* symbol duration. It is pointed out in [11] that codes satisfying the more stringent constraint in (1.15) (iii) generally give lower error rates in a feedforward or blind MIMO scheme. This agrees with the idea of [12] which states that since the encoder of a feedforward or blind MIMO system does not have any information about the channel, it should transmit the code symbols uniformly in all space and time directions, assuming that all spatial channels are independent and identically distributed.

It is further proposed in [13] that constraint (1.15) (iii) is the *maximal symbol-wise diversity* (MSD) condition, which guarantees full diversity protection for one-symbol error events that have the smallest Euclidean distance. As suggested in [13], intuitively it is obvious that if a code cannot provide full diversity protection against the one-symbol error event, it cannot provide full diversity protection aginst all error events.

As mentioned earlier, besides the dispersion matrices **A** and **B**, the design of STBC also depends on the number of information symbols transmitted within a codeword K, and the code length P. These two values are generally limited by the following constraints in (1.16) and (1.17):

$$\text{transmit diversity level} \le \min(N_\mathrm{T}, P), \tag{1.16}$$

$$K \le P \min(N_\mathrm{T}, N_\mathrm{R}). \tag{1.17}$$

Constraint (1.16) arises because the diversity order of a STBC is determined by the rank of its codeword (this will be elaborated in Section 1.5). Since the STBC codeword has dimension $P \times N_\mathrm{T}$, its rank is limited by the minimum value of N_T and P.

To understand constraint (1.17), let us refer to the signal model in (1.13). The K transmitted data symbols is represented by $2K$ unknown real values in $\tilde{\mathbf{c}}$. In order to solve for $\tilde{\mathbf{c}}$ at the receiver without ambiguity, there must be at least $2K$ linearly independent equations in (1.13). This implies that the equivalent channel matrix **H**, which has dimension $2PN_\mathrm{R} \times 2K$, must have $PN_\mathrm{R} \ge K$. In addition, the dispersion matrices **A** and **B**, each of size $P \times N_\mathrm{T}$, must have K independent basis, this requires $PN_\mathrm{T} \ge K$. Combining this and the earlier $PN_\mathrm{R} \ge K$ condition gives (1.17).

In order to achieve *full transmit diversity* of order N_T from the N_T transmit antennas, (1.16) requires that $P \ge N_\mathrm{T}$. Hence a square code design with $P = N_T$ gives the minimum possible code length for achieving full diversity, and STBC with such property is commonly called STBC with *minimum delay*. Furthermore, if only one receive antenna is used, i.e. $N_R = 1$, we get $K \leq P$ from (1.17). Hence the maximum achievable code rate $R = K / P$ of a transmit diversity scheme with only one receive antenna is one.

The capacity, C_{STBC}, achievable by a STBC is [11]:

$$C_{\mathbf{STBC}} = \frac{1}{2P}\mathrm{E}_{\mathbf{H}}\left[\log\det\left(\mathbf{I}_{2N_R P} + \frac{\rho}{N_T}\mathbf{H}\mathbf{H}^T\right)\right] \quad \text{bps/channel use.} \quad (1.18)$$

This is different from (1.9) as (1.9) is the capacity formula for a MIMO channel, while (1.18) is the capacity achievable by a STBC.

1.5 Design Criteria and Performance Measure of STBC

Guey *et al.* pointed out in [14] that the critical parameter for evaluating the performance of a space-time code in slow flat fading channel is the rank of the codeword difference matrix. In [4], Tarokh *et al.* further showed that the minimum rank of the codeword difference matrix quantifies the *diversity gain*, while the minimum product of the non-zero eigenvalues of the codeword distance matrix with the minimum rank quantifies the *coding gain*, of the space-time code. They will be reviewed in the following.

We assume that all the codewords have equal transmission probability and let P($\mathbf{C} \rightarrow \mathbf{E}$) denote the probability that the codeword **C** is transmitted but the receiver decides erroneously in favor of another codeword **E**. This probability term is commonly called the *pair-wise error probability* (PEP). With ideal CSI, PEP is well approximated as follows [4]:

$$P\left(\mathbf{C} \rightarrow \mathbf{E} \mid h_{i,k}, i = 1,2,\ldots,N_T; k = 1,2,\ldots,N_R\right) \leq \exp\left(-d^2\left(\mathbf{C},\mathbf{E}\right)\frac{E_s}{4N_o}\right), \quad (1.19)$$

where

$$d^2(\mathbf{C},\mathbf{E}) = \sum_{k=1}^{N_R}\sum_{p=1}^{P}\left|\sum_{i=1}^{N_T} h_{i,k}(c_{p,i} - e_{p,i})\right|^2 ,$$

$c_{p,i}$ and $e_{p,i}$ are the entries in the p^{th} row and i^{th} column of **C** and **E**.

To simplify the PEP expression, we first define two matrices, the first is the *codeword difference matrix* $\mathbf{B}_{\mathbf{CE}}$ of size $P \times N_T$, which is defined as [4]:

$$\mathbf{B}_{\mathbf{CE}} = \mathbf{C} - \mathbf{E}, \tag{1.20}$$

and the second is the *codeword distance matrix* $\mathbf{A}_{\mathbf{CE}}$ of size $N_T \times N_T$, which is defined as [15]:

$$\mathbf{A}_{\mathbf{CE}} = \mathbf{B}_{\mathbf{CE}}^{\mathrm{H}}\mathbf{B}_{\mathbf{CE}}. \tag{1.21}$$

Furthermore, we also define $\lambda_1, \lambda_2, \ldots, \lambda_D$ as the non-zero eigenvalues of $\mathbf{A}_{\mathbf{CE}}$, where D denotes the rank of $\mathbf{A}_{\mathbf{CE}}$, which is the same as the rank of $\mathbf{B}_{\mathbf{CE}}$. At high SNR, the upper bound of the PEP in (1.19) can be simplified as [4]:

$$P(\mathbf{C}\to\mathbf{E}) \le \left[\left(\prod_{i=1}^{D}\lambda_i\right)\left(\frac{E_s}{4N_o}\right)^{-D}\right]^{-N_R}. \tag{1.22}$$

From (1.22), we can define the following two quantities to account for the decoding performance of a STBC:

- *Transmit Diversity Gain*: the minimum rank D of the matrix $\mathbf{A}_{\mathbf{CE}}$ over all pairs of distinct codewords. It accounts for the slope of the bit error rate (BER) or block error rate (BLER) curve of the STBC. A STBC is said to achieve *full diversity* if $D = N_T$.
- *Diversity Product*: defined as follows in [16] for a full diversity code,

$$\begin{aligned}\varsigma &= \frac{1}{2\sqrt{N_T}}\min_{\forall \mathbf{C}\neq\mathbf{E}}\left|\operatorname{Det}\left(\mathbf{A}_{\mathbf{CE}}\right)\right|^{1/(2P)} \\ &= \frac{1}{2\sqrt{N_T}}\min_{\forall \mathbf{C}\neq\mathbf{E}}\left(\prod_{i=1}^{D}\lambda_i\right)^{1/(2P)},\end{aligned} \tag{1.23}$$

where the factor $1/\left(2\sqrt{N_T}\right)$ guarantees that $0 \leq \zeta \leq 1$, and Det(.) denotes the determinant of a matrix. Diversity product accounts for the *coding gain*, which determines the left or right shift of the BER or BLER curve of the STBC.

Based on the above, a set of code design criteria for space-time codes, commonly called the *Rank & Determinant Criteria*, can be stated as follows [4]:

- *Rank Criterion*: To maximize the diversity gain of the STBC, maximize the minimum rank D of the matrix $\mathbf{A}_{CE}$ over all distinct pairs of codewords. Hence, in order to achieve the maximum diversity order, the matrix $\mathbf{A}_{CE}$ has to be full rank, i.e. $D = N_T$, for any codeword pair **C** and **E**.
- *Determinant Criterion*: To maximize the coding gain of the STBC, maximize the minimum product of non-zero eigenvalues of the matrix $\mathbf{A}_{CE}$ which has the minimum rank. When a code achieves full diversity, this determinant criterion implies the maximization of the diversity product in (1.23).

Based on the channel and STBC models presented in this chapter, we will give a detailed overview on the design of O-STBC and QO-STBC in the next chapter.