

Contents

<i>Preface</i>	vii
1. Conjugacy Classes, Characters, and Clifford Theory	1
1.1 Class Functions and Characters	1
1.2 Induced and Tensor-induced Modules	3
1.3 Schur's Lemma	4
1.4 Brauer's Permutation Lemma	6
1.5 Algebraic Conjugacy	7
1.6 Coprime Actions	9
1.7 Invariant and Good Conjugacy Classes	10
1.8 Nonstable Clifford Theory	12
1.9 Stable Clifford Theory	13
1.10 Good Conjugacy Classes and Extendible Characters	18
2. Blocks of Characters and Brauer's $k(B)$ Problem	19
2.1 Modular Decomposition and Brauer Characters	19
2.2 Cartan Invariants and Blocks	21
2.3 Defect and Defect Groups	23
2.4 The Brauer–Feit Theorem	25
2.5 Higher Decomposition Numbers, Subsections	26
2.6 Blocks of p -Solvable Groups	28
2.7 Coprime $\mathbb{F}_p X$ -Modules	31
3. The $k(GV)$ Problem	32
3.1 Preliminaries	32
3.2 Transitive Linear Groups	34
3.3 Subsections and Point Stabilizers	36
3.4 Abelian Point Stabilizers	41

4. Symplectic and Orthogonal Modules	45
4.1 Self-dual Modules	45
4.2 Extraspecial Groups	47
4.3 Holomorphs	49
4.4 Good Conjugacy Classes Once Again	54
4.5 Some Weil Characters	56
4.6 Symplectic and Orthogonal Modules	60
5. Real Vectors	63
5.1 Regular, Abelian and Real Vectors	63
5.2 The Robinson–Thompson Theorem	66
5.3 Search for Real Vectors	68
5.4 Clifford Reduction	71
5.5 Reduced Pairs	74
5.6 Counting Methods	74
5.7 Two Examples	77
6. Reduced Pairs of Extraspecial Type	82
6.1 Nonreal Reduced Pairs	82
6.2 Fixed Point Ratios	84
6.3 Point Stabilizers of Exponent 2	86
6.4 Characteristic 2	90
6.5 Extraspecial 3-Groups	92
6.6 Extraspecial 2-Groups of Small Order	96
6.7 The Remaining Cases	103
7. Reduced Pairs of Quasisimple Type	110
7.1 Nonreal Reduced Pairs	110
7.2 Regular Orbits	112
7.3 Covering Numbers, Projective Marks	115
7.4 Sporadic Groups	119
7.5 Alternating Groups	121
7.6 Linear Groups	125

7.7	Symplectic Groups	129
7.8	Unitary Groups	136
7.9	Orthogonal Groups	145
7.10	Exceptional Groups	147
8.	Modules without Real Vectors	148
8.1	Some Fixed Point Ratios	148
8.2	Tensor Induction of Reduced Pairs	149
8.3	Tensor Products of Reduced Pairs	155
8.4	The Riese–Schmid Theorem	156
8.5	Nonreal Induced Pairs, Wreath Products	160
9.	Class Numbers of Permutation Groups	170
9.1	The Partition Function	170
9.2	Preparatory Results	171
9.3	The Liebeck–Pyber Theorem	172
9.4	Improvements	174
10.	The Final Stages of the Proof	180
10.1	Class Numbers for Nonreal Reduced Pairs	180
10.2	Counting Invariant Conjugacy Classes	182
10.3	Nonreal Induced Pairs	185
10.4	Characteristic 5	186
10.5	Summary	194
11.	Possibilities for $k(GV) = V $	195
11.1	Preliminaries	195
11.2	Some Congruences	197
11.3	Reduced Pairs	199
12.	Some Consequences for Block Theory	202
12.1	Brauer Correspondence	202
12.2	Clifford Theory of Blocks	203
12.3	Blocks with Normal Defect Groups	207

13. The Non-Coprime Situation	209
Appendix A: Cohomology of Finite Groups	213
Appendix B: Some Parabolic Subgroups	217
Appendix C: Weil Characters	221
Bibliography	225
List of Symbols	230
Index	231