Preface

The theory of stochastic processes is considered to be difficult to understand by those who do not have a sound background in advanced topics of mathematics and mathematical analysis. Indeed, this is true, although exaggerated: option pricing, for instance, is based on sophisticated theory of real and functional analysis, such as measures and integrations. However, to understand the basics of option pricing a relatively modest background in mathematics seems to be sufficient. This book tries to avoid high-level mathematical backgrounds and makes it possible to be accessible to readers from different fields of science, and the practitioners of financial markets.

Although there are several publications on similar subjects, this book mainly focuses on pricing of derivative securities, options in particular, through a variety of approaches in literature: *binomial trees, Monte Carlo simulations,* and *partial differential equations*. By no means is it complete nor does it contain striking new theoretical ideas. This book is based on the lectures I have given on *scientific computing* and *computational finance*. In fact, this book tries to fill the gap between *mathematical finance* and *numerical approaches* by collecting the most appreciated and up-to-date monographs and many other selected literature in these fields, such as [Brandimarte (2002); Higham (2004); Korn and Korn (2001); Seydel (2002); Wilmott *et al.* (1995)].

Not only is this book considered a textbook in related undergraduate or graduate courses, but it also aims to serve those who want to implement and/or learn pricing algorithms by themselves. Indeed, the contents as well as the algorithms with their implementations in MATLAB make this book self-contained on basic methods of option pricing. Hopefully, it will provide an introduction to the essential features of *computational finance* and help those improve their mathematical backgrounds for more advanced topics.

This book contains working MATLAB codes to illustrate the ideas as well as the algorithms presented in the text. These codes and MATLAB functions will hopefully help readers implement their own algorithms and build their own library for pricing options. For ease of access as well as flexibility these codes were written without using sophisticated features of MATLAB so that their implementations in any other programming languages could easily be achieved. Moreover, to ensure the accuracy of the material in this book the codes and functions are also made available on-line. They can be obtained via the world-wide web at

`http://www.metu.edu.tr/~ougur`

for those who are already familiar with MATLAB and want to implement the vectorised versions for better performance. The website will also include the *errata* of the book, as well as comments and other material.

The chapters in the book can be followed depending on the readers' interests and their backgrounds. For instance, many students in finance are familiar with *Monte Carlo* methods, while the students of scientific computing are accustomed to *numerical solutions of partial differential equations*, in general. To help readers, this book is organised as follows.

The book starts with *fixed-income* securities and an introduction to *portfolio optimisation* by means of the *mean-variance approach*. Such an introduction is expected to introduce some of the technical terms used in the financial world and to help those who are not familiar with programming in MATLAB. In fact, the introductory chapter is relatively easier and almost independent of the remaining parts of the book.

Binomial methods in Chapter 2 presumes to serve an introduction to options as well as the basics of option pricing. Using the *no-arbitrage* principle and the risk-free interest rate, risk-neutral valuation of *European* options is introduced by the help of binomial models. Other type of options, in particular *American* ones, are also introduced and priced in this chapter.

In Chapter 3, stochastic processes and simulation of stochastic differential equations by means of *numerical methods* are presented. In order to support ideas and methods, the *stochastic Itô integral* and its properties are introduced, but a deep mathematical background is avoided. The *Itô lemma* and its use in evaluating such integrals as well as its applications in finance completes the chapter.

The famous *Black-Scholes* formulae for option pricing are derived in Chapter 4 by solving the transformed *heat equation*. Having such closed-form formulae, the concept of *hedging* is then easily discussed and further illustrated with some implementations in MATLAB.

Chapter 5 starts with *pseudo-random numbers* and discusses the transformation of *random variables* in order to draw *variates* from a given distribution. In particular, the basic algorithms, the *Box-Muller* and the *Marsaglia*, are introduced so as to draw *samples* from a *normal* distribution. The main part of this chapter is devoted to the *Monte Carlo* methods and the use of *variance reduction* techniques for option pricing by simulation. This chapter also contains several examples to illustrate and compare the methods.

Finally, in Chapter 6, after an introduction to *numerical solutions of partial differential equations* (PDEs) by *finite difference* methods, the *Black-Scholes PDEs* in option pricing are solved numerically. First, a relatively easier *heat equation*, which is obtained by transforming the variables in the Black-Scholes setting, is solved numerically. The basic methods, such as *explicit, implicit* and the *Crank-Nicolson*, as well as their stability analysis are investigated. The methods are also illustrated by applications to European options. Then, the finite difference formulae are applied directly to the Black-Scholes PDEs to value several options on *uniform grids.* In this chapter, *American* options are treated by means of the *free-boundary problems*, too. In order to value American options by finite difference methods, the *projected successive over-relaxation* algorithm is introduced. Of course, all examples and topics are enhanced by the presentation of suitable MATLAB codes. Finally, in this last chapter, similarities between *binomial* and *trinomial* methods with finite difference ones are presented.

The book ends with an appendix: *a short introduction to* MATLAB for those who are not familiar to its basic usage, built-in functions and plotting utilities. In the appendix, the use of *m-files* in MATLAB is emphasised.