Chapter 1

Gödel Visits Escher's Studio

"I paint things as I think of them, not as I see them."

Picasso

In a rare break from his everyday occupation of proving theorems, Gödel decides to visit Escher's studio. He has heard rumors that this fellow is experimenting with painting paradoxical pictures. He arrives at the studio at about 11:00 am. It is quite a gloomy and foggy day in Baarn but this does not seem to bother Gödel. He rings the bell and very soon a serious, well-dressed bearded man opens the door to a small, frail-looking man wearing heavy rim glasses.

"Good morning sir, my name is Dr. Kurt Gödel. I am a mathematician and I would like, if you don't mind, to look at your work. I am not an art connoisseur. In fact I am not sure I understand modern art, but I understand you are on to something different. Is this a bad time?"

"Not at all. Please come in," replies Escher showing Gödel the way in.

Once inside the studio Escher discretely lets his visitor wander around. Gödel, his hands clasped behind his back, is silently moving around looking at the pictures, some of which are framed and hanging on the walls and some are laying flat on large tables. "If you have any questions please call me," Escher says after a while. Gödel does not reply. He seems quite fixed in front of a lithograph. Escher approaches saying: "Oh, this one! It is one of my favorites. I gave it the title *Print Gallery*."

Gödel remains motionless looking at the lithograph, his eyes wide open and charged as if they are about to produce sparks. In that painting, Gödel observes a young man who is standing in a gallery, looking at a painting of a ship in the harbor of a small town with many flat stone roofs. On top

Fig. 1. M. C. Escher's "Print Gallery" (© 2007 The M. C. Escher Company Holland).

of one of the stone roofs a little boy ponders something, while two floors below a woman gazes out of a window from her apartment. Her apartment sits directly above a gallery where a young man is standing and looking at a painting of a ship in the harbor of a small town … *ad infinitum*!

"This is a self-referential painting," replies Gödel, regaining his composure. "I thought only verbal self-referential statements existed."

"I actually call it a strange loop.[1] My intention with this painting is to draw the viewer into some kind of a ride where what you think is real soon appears to be an illusion. In order to achieve this we have to have a level followed by another level of greater reality. When these levels are connected in a loop what is real and what is an illusion is not clear. In strange loops one cannot know what is true and what is false."

"This is incredible …, how did you, technically speaking, achieve this?"

"I'll be happy to explain my procedure to you Dr. Gödel. One way to paint a scene is to lay a regular grid over the scene and the same grid on a piece of blank paper and then paint each square at a time. This way I would

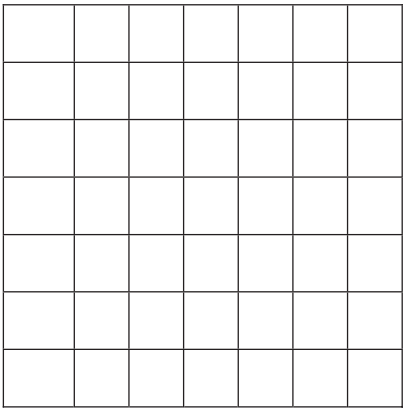

Fig. 2. Regular grid.

paint the scene exactly as I see it. But this is for amateurs, if I may say so. What I did was to transform the regular grid over the blank paper to a distorted grid. I hate to sound childish but let me start by showing you a regular grid where all squares are the same.

The distortion scheme I chose will bend the horizontal lines approximately like this

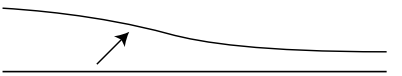

Fig. 3. Distortion for horizontal line.

and the vertical lines like this.

Fig. 4. Distortion for vertical line.

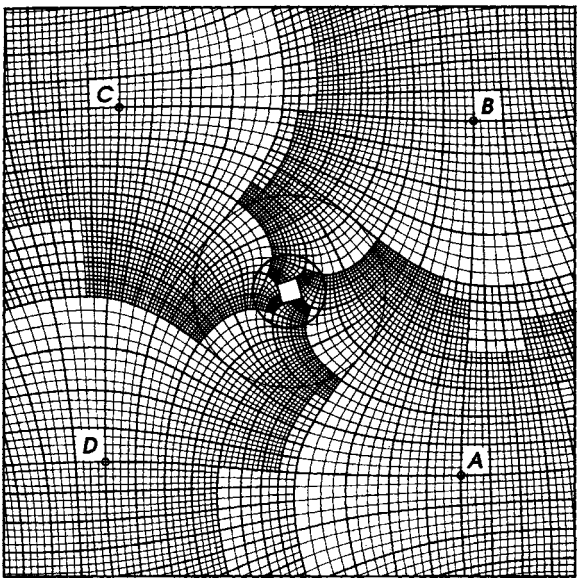

Fig. 5. M. C. Escher's "Grid: for Print Gallery" (© 2007 The M. C. Escher Company Holland).

I guess you mathematicians call this a nonlinear transformation. In any case, such a distortion scheme will cause the squares to be stretched in the vertical and in the horizontal directions with the expansion becoming more pronounced as we go from right to left and from bottom to top. As we go clockwise around the center the grid because of the expansion has to fold onto itself. After fiddling with it I finally arrived at the following grid (Fig. 5).

"Very interesting," said Gödel.

"So now Dr. Gödel, let me show you the original scene with a regular grid overlaid, which was used to produce "*Print Gallery*" (Fig. 6).

Here I have outlined the path ABCD. In producing *Print Gallery* I started by transferring the original squares along this particular path. As you can see, because of the transformation, the gallery appears to get larger as we move from right to left and then from bottom to top."

"Excuse me if this sounds naive, but clearly segment CD is much shorter than AB and it only spans the length of a small painting hanging in the gallery. Yet, in the final product, this small painting is expanded to span the whole breadth. I understand that according to your transformation the

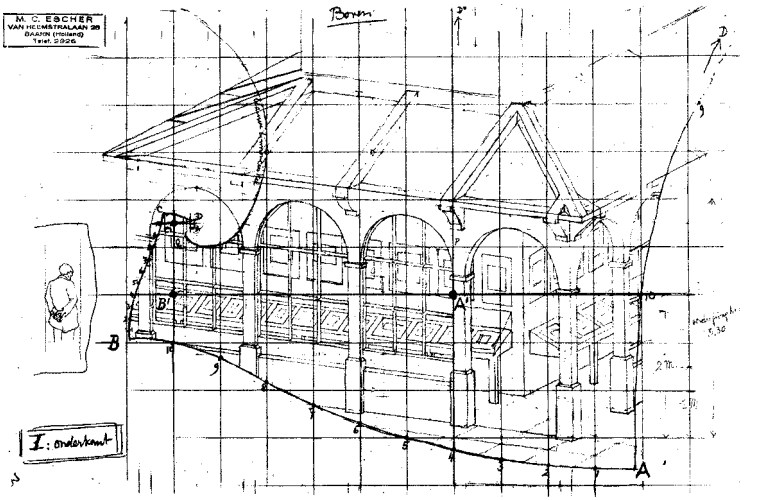

Fig. 6. M. C. Escher's "Print Gallery before the expansion" (© 2007 The M. C. Escher Company Holland).

pulling and stretching will enlarge this small painting, but I would not think by that much."

"You are very observant, Dr. Gödel. Remember that my intention was to create some kind of a visual strange loop. In order to achieve this, after the first level of expansion along ABCD, I concentrated only on the segment CD, which I repeatedly expanded until it filled the available space."

"Brilliant, I must say," replies Gödel, who now prepares to give Escher a little taste of his logic. "It is obvious that underlying the concept of your strange loops is the notion of infinity. Since a strange loop is an endless repetition of a finite process, the coexistence between finite an infinite produces a strong sense of paradox."

"I am impressed," concedes Escher.

"No, sir, I am impressed. I bow to your ingenuity. Your visual presentation of such a paradox is amazing."

Escher makes a gesture that appears to imply thanks. "What did you mean earlier that only verbal self-referential statements existed?"

"Have you ever heard of Epimenides?"

"I am afraid not," replies Escher.

"Epimenides lived in ancient Greece about 2,500 years ago. He was from the island of Crete and invented the first ever verbal realization of a self-referential statement, or as you call it, a strange loop."

"Really? What is it?"

"My version of it is '*Cretans are liars*,'" said Gödel[a].

Escher pauses for a moment pondering about this statement. Then he starts thinking out loud. "If the above statement is true, then Epimenides is telling the truth. But since he himself is a Cretan and according to the statement Cretans are liars, then he must be lying and thus the statement is false. If we instead assume that the statement is false, then Cretans are not liars, which means that Epimenides is telling the truth, meaning that the statement is true."

"Quite right. Like your painting this self-referential statement violates the usual dichotomy of statements into true and false. The minute you assume the statement to be true it backfires making you think it is false, and the minute you decide that it is false another backfiring brings you back to the idea that it must be true."

"Most amazing," replies Escher.

"Most amazing indeed. But, coming back to your painting, and if you forgive my question, why did you leave the painting incomplete?"

"Do you refer to the center of the painting?" replies Escher smiling.

"Yes," answers Gödel. "At the center of the picture I can see that there is some kind of a blemish with your signature. One would never know what the center would look like."

"Yes, yes, of course. I must explain. You see, Dr. Gödel, the grid used for *Print Gallery* was designed to create an annular or cyclic bulge that would allow me to produce my strange loop. The lines of the grid are arranged so that moving clockwise around the center they spread further apart. This "empty" spot at the center is, if I may use a silly analogy, like the eye of a hurricane. As the air spirals toward the center of the hurricane, natural laws prevent it from converging exactly at the center and force it to "hit a wall" near the center and rise higher leaving the area close to the center "empty". The "empty" spot can be made smaller by considering grids of

[a]It is worth pointing out that Epimenides' paradox (which is more accurately written The Cretans, always liars, evil beasts, idle bellies!) was quoted in the New Testament (Titus 1:12–13), but was not intended or interpreted as a logical paradox in ancient times.

higher resolution, but with the present tools there will always be an empty spot at the center. It may sound strange but I could completely finish the painting only if I had infinite resolution."

Gödel is speechless. He seems quite stirred up but distant. His mind is obviously somewhere else. Politely, Escher asks, "Would you like some tea?" Gödel snaps out of whatever state he is in and replies, "No thanks. I must be going. This has been an experience. I am grateful for your time."

"The pleasure was mine, sir. I hope we will have a chance to see each other and chat more some other time."

"A chance?" replies Gödel departing from the studio rather hastily.

Back on the street Gödel looks up in the sky. The fog is gradually dissipating as the sun begins to break through. He walks rather carelessly not aware that he is bumping into people who turn and sullenly look at him. "Where do these loops come from?" he murmurs. "Why do they exist if they can only cause trouble? It is logical to speculate that mathematics has something to do with it, but what is the connection between them and mathematics? Can these self-referential statements be connected to number theory? If they can, what will be the implications? Mathematical statements, propositions, theorems, axioms, formulas and what have you involve language as well as symbols and numbers. It is easy to see how language can talk about language and paintings about pictures, but how can mathematical statements talk about numbers when they don't involve only numbers? In order to talk about numbers, everything should be expressed as a number. Only then one can use mathematical reasoning to explore mathematical reasoning itself. If I can do this, I am half-way there."