

PREFACE

Neurons in the brain communicate with each other by transmitting sequences of electrical spikes or action potentials. One of the major challenges in neuroscience is to understand the basic physiological mechanisms underlying the complex spatiotemporal patterns of spiking activity observed during normal brain functioning, and to determine the origins of pathological dynamical states such as epileptic seizures and Parkinsonian tremors. A second major challenge is to understand how the patterns of spiking activity provide a substrate for the encoding and transmission of information, that is, how do neurons compute with spikes? It is likely that an important element of both the dynamical and computational properties of neurons is that they can exhibit bursting.

Bursting is a relatively slow rhythmic alternation between an active phase of rapid spiking and a quiescent phase without spiking. It is exhibited by a wide range of nerve and endocrine cells, including pancreatic β -cells, respiratory pacemaker neurons in the pre-Bötzinger complex, dopaminergic neurons of the mammalian midbrain, thalamic relay cells, and pyramidal neurons in the neocortex. Following the seminal work of John Rinzel and collaborators, a general framework for understanding the biophysical origins of bursting has been developed using mathematical and computational modeling. The fundamental idea is to dissect the full dynamics of a neuron into a fast subsystem coupled to a slowly varying subsystem. Typically, the fast subsystem has a time-scale of milliseconds and models the membrane potential and spike generation process, whereas the slow subsystem operates on a time-scale of seconds and models slow trans-membrane ionic currents. For many types of bursting, as exemplified by square wave bursting in pancreatic β -cells, the fast subsystem exhibits bistability between a quiescent fixed point and an active oscillatory state. At least one slow variable is then required to alternate back and forth through the region of bistability such that the neuron periodically jumps between the active

and quiescent phases along a hysteresis loop. It is also possible to generate bursting without bistability in the fast subsystem, provided that there are at least two variables in the slow subsystem that periodically sweep the fast subsystem through a sequence of bifurcations. This is exemplified by parabolic bursting in the abdominal ganglion neuron of *Aplysia*.

In recent years, several classification schemes of the various forms of bursting behavior have been developed by investigating the detailed bifurcation structure of the fast and slow subsystems. From a mathematical perspective, these classifications characterize the bursting behavior in terms of the topologies of the invariant sets (fixed points, periodic orbits etc.) that the solutions visit, and the various transitions between these sets. From a phenomenological perspective, they describe various properties of bursting such as the amplitude and shape of spikes, the evolution of the spiking frequency during the active phase, and the duration of the quiescent phase. These phenomena can also be measured experimentally and used to classify biological bursters. Although most studies of bursting mechanisms have been based on isopotential point neuron models, it is also possible for bursting to arise through interactions between different parts of the cell, namely the soma and dendrites, or through interactions between cells. In the latter case individual cells may themselves not be intrinsic bursters.

This book provides a detailed overview of the current state-of-the-art in the mathematical and computational modeling of bursting, with contributions from many of the leading researchers in the field. It is a natural complement to the more pedagogical work of Eugene M. Izhikevich (*Dynamical Systems in Neuroscience: The Geometry of Excitability and Bursting*, The MIT Press, to appear), and is divided into two parts. Part I (chapters 1–6) covers bursting at the single cell level and Part II (chapters 7–14) covers bursting at the network level. The first few chapters describe some specific examples of conductance-based models of bursting: the Hindmarsh-Rose model of a bursting neuron (chapter 1), the Chay-Keizer model of insulin secreting pancreatic cells (chapter 2), and a recent model of bursting in endocrine cells of the hypothalamus (chapter 3). This is followed by a bifurcation analysis of a reduced bursting model that highlights some important features regarding the decomposition into fast/slow subsystems (chapter 4). The final two chapters of Part I concern bursting in two-compartment models, consisting of a soma electrically coupled to an active dendritic compartment. The first shows how geometric methods can be used to study the well known Pinsky-Rinzel model of a hippocampal CA3 pyramidal neuron (chapter 5), and the second reviews a recent model of “ghostbursting”,

which is a novel type of bursting observed in sensory processing neurons of weakly electric fish (chapter 6). In the latter case, some possible computational advantages of bursting are discussed. Part II begins with an analysis of phase-locking in small networks of coupled bursting neurons using phase resetting curves (chapter 7). This is followed by an analysis of bursting in two-cell systems using symmetric bifurcation theory (chapter 8), which illustrates how local singularity theory rather than global geometric methods can also be used to classify different bursting mechanisms. The next four chapters consider the effects of coupling on bursting at the network level. First, a computationally efficient method for studying networks of cells is described, based on coupled bursting maps (chapter 9). This is followed by an analysis of diffusively coupled square-wave bursters (chapter 10). Two examples of bursting in excitatory networks are then considered. The first concerns a mean field model of spontaneous episodic activity in developing spinal cord networks, in which the individual cells are not themselves intrinsic bursters and the slow process arises from synaptic depression (chapter 11). The second involves a model of oscillatory bursting in an excitatory network of synaptically coupled respiratory neurons of the pre-Bötzinger complex (chapter 12). The last two chapters of Part II consider wave propagation in bursting networks: a geometric analysis of traveling waves in models of thalamic networks (chapter 13) and the effects of depolarization block on wave propagation in a model of disinhibited cortical slices (chapter 14).

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