

Contents

1. First-order Partial Differential Equations	1
1. Introduction	1
2. Linear First-order Equations	4
3. The Cauchy Problem for First-order Quasi-linear Equations	11
4. General Solutions of Quasi-linear Equations	22
5. Fully-nonlinear First-order Equations	28
2. Second-order Partial Differential Equations	37
6. Linear equations	37
7. Classification and Canonical Forms of Equations in Two Independent Variables	44
8. Classification of Almost-linear Equations in \mathbf{R}^n	56
3. One-Dimensional Wave Equation	65
9. The Wave Equation on the Whole Line. D'Alembert Formula	65
10. The Wave Equation on the Half-line. Reflection Method	75
11. Mixed Problem for the Wave Equation	80
12. Inhomogeneous Wave Equation	84
13. Conservation of the Energy	88
14. Weak Derivatives and Weak Solutions	91
4. One-Dimensional Diffusion Equation	99
15. Maximum-minimum Principle for the Diffusion Equation	99
16. The Diffusion Equation on the Whole Line	105
17. Diffusion on the Half-line	116
18. Inhomogeneous Diffusion Equation on the Whole Line	119
5. Shock Waves and Conservation Laws	125
19. Conservation laws	125

20. Burgers' Equation	135
21. Weak Solutions. Riemann Problem	147
22. Discontinuous Solutions of Conservation Laws. Rankine-Hugoniot Condition	155
6. The Laplace Equation	163
23. Harmonic functions. Maximum-minimum Principle	163
24. Green's Identities	167
25. Green's Functions	176
26. Green's Functions for a Half-space and Sphere	179
27. Harnack's Inequalities and Theorems	187
7. Fourier Series and Fourier Method for PDEs	193
28. Fourier Series	193
29. Orthonormal Systems. General Fourier series	211
30. Fourier Method for the Diffusion Equation	222
31. Fourier Method for the Wave Equation	231
32. Fourier Method for the Laplace Equation	237
8. Diffusion and Wave Equations in Higher Dimensions	249
33. Diffusion Equation in Three Dimensional Space	249
34. Fourier Method for the Diffusion Equation in Higher Dimensions	256
35. Kirchoff's Formula for the Wave Equation. Huygens' Principle	263
36. Fourier Method for the Wave Equation on the Plane. Nodal Sets	270
References	281
Answers and Hints to Selected Exercises	285
Index	293