

Preface

A general integration theory based on the concept of Riemann-type integral sums was initiated around 1960 by Jaroslav Kurzweil and independently by Ralph Henstock.

Much of this theory can be presented at the level of undergraduate courses and this fact is reflected in the growing number of university textbooks or elementary expositions which include at least elementary facts about the Henstock-Kurzweil theory. This concerns especially the last two decades and the publications [Ba01], [BS00], [DPS87], [G94], [H88], [H91], [KS04], [K80], [LM95], [LPY89], [LV00], [M97], [McL80], [McS83], [Pf93], [S99], [S01].

The main virtue of the presentation of the Henstock-Kurzweil integral of real-valued functions is that no measure theory is required and that even sophisticated convergence results can be derived using merely elementary tools from the calculus without advanced topology.

The relatively new concepts of the Henstock-Kurzweil and McShane integrals based on Riemann type sums are an interesting challenge also in the study of integration of Banach space-valued functions. The advantage of a relatively transparent and easy definition is undoubtedly an invitation to do so.

The investigations started around 1990 by the work of R. A. Gordon and since then attention has been paid to this field. One of the crucial problems is the comparison of the new concepts with the classical ones of the Bochner and Pettis integral. It

should be mentioned at this point that some results concerning the basic facts of integration of Banach space-valued functions using Riemann type sums are also included in the early book [K80] of Jaroslav Kurzweil.

This text presents an overview of the concepts and results achieved during the past 15 years. The Henstock-Kurzweil and McShane integrals play the central role in this text.

In Chapter 1 elementary facts concerning the definition and properties of the Bochner integral are presented, Chapter 2 is devoted to the Dunford and Pettis integrals.

In Chapter 3 we present the McShane and Henstock-Kurzweil integrals and Chapter 4 gives an overview of some special properties of the McShane integral. In this parts of the book special attention is paid to convergence theorems and the results are compared with the general Vitali Convergence Theorem.

Chapter 5 is devoted to the interrelations of the Bochner and McShane integrals while in Chapter 6 the more delicate problem of the relation between McShane and Pettis integrability is studied.

Properties of the indefinite integrals (primitives) for integration theories introduced based on Riemann-type integral sums are investigated in Chapter 7, some other convergence results (controlled convergence) are also presented. In the final Chapter 8 Denjoy and Henstock-Kurzweil extensions of the classical Bochner, Dunford and Pettis integrals are presented and a short overview of known results is given.

An appendix at the end of the book collects basic facts from functional analysis, function spaces, etc. for the reader's convenience with references to the respective sources.

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