

# Contents

<i>Preface</i>	v
1. Markov and Semi-Markov Processes	1
1.1 Preliminaries . . . . .	1
1.2 Markov Processes . . . . .	2
1.2.1 Markov Chains . . . . .	2
1.2.2 Continuous-Time Markov Processes . . . . .	6
1.2.3 Diffusion Processes . . . . .	10
1.2.4 Processes with Independent Increments . . . . .	11
1.2.5 Processes with Locally Independent Increments . . . . .	14
1.2.6 Martingale Characterization of Markov Processes . . . . .	15
1.3 Semi-Markov Processes . . . . .	19
1.3.1 Markov Renewal Processes . . . . .	19
1.3.2 Markov Renewal Equation and Theorem . . . . .	21
1.3.3 Auxiliary Processes . . . . .	23
1.3.4 Compensating Operators . . . . .	24
1.3.5 Martingale Characterization of Markov Renewal Processes . . . . .	25
1.4 Semimartingales . . . . .	25
1.5 Counting Markov Renewal Processes . . . . .	28
1.6 Reducible-Invertible Operators . . . . .	31
2. Stochastic Systems with Switching	35
2.1 Introduction . . . . .	35
2.2 Stochastic Integral Functionals . . . . .	36
2.3 Increment Processes . . . . .	40

2.4	Stochastic Evolutionary Systems . . . . .	43
2.5	Markov Additive Processes . . . . .	46
2.6	Stochastic Additive Functionals . . . . .	47
2.7	Random Evolutions . . . . .	50
2.7.1	Continuous Random Evolutions . . . . .	50
2.7.2	Jump Random Evolutions . . . . .	54
2.7.3	Semi-Markov Random Evolutions . . . . .	56
2.8	Extended Compensating Operators . . . . .	59
2.9	Markov Additive Semimartingales . . . . .	61
2.9.1	Impulsive Processes . . . . .	61
2.9.2	Continuous Predictable Characteristics . . . . .	63
3.	Stochastic Systems in the Series Scheme . . . . .	67
3.1	Introduction . . . . .	67
3.2	Random Evolutions in the Series Scheme . . . . .	68
3.2.1	Continuous Random Evolutions . . . . .	68
3.2.2	Jump Random Evolutions . . . . .	72
3.3	Average Approximation . . . . .	74
3.3.1	Stochastic Additive Functionals . . . . .	74
3.3.2	Increment Processes . . . . .	79
3.4	Diffusion Approximation . . . . .	81
3.4.1	Stochastic Integral Functionals . . . . .	81
3.4.2	Stochastic Additive Functionals . . . . .	84
3.4.3	Stochastic Evolutionary Systems . . . . .	88
3.4.4	Increment Processes . . . . .	89
3.5	Diffusion Approximation with Equilibrium . . . . .	90
3.5.1	Locally Independent Increment Processes . . . . .	90
3.5.2	Stochastic Additive Functionals with Equilibrium . . . . .	93
3.5.3	Stochastic Evolutionary Systems with Semi-Markov Switching . . . . .	97
4.	Stochastic Systems with Split and Merging . . . . .	103
4.1	Introduction . . . . .	103
4.2	Phase Merging Scheme . . . . .	104
4.2.1	Ergodic Merging . . . . .	104
4.2.2	Merging with Absorption . . . . .	110
4.2.3	Ergodic Double Merging . . . . .	112
4.3	Average with Merging . . . . .	116

4.3.1	Ergodic Average . . . . .	117
4.3.2	Average with Absorption . . . . .	119
4.3.3	Ergodic Average with Double Merging . . . . .	120
4.3.4	Double Average with Absorption . . . . .	121
4.4	Diffusion Approximation with Split and Merging . . . . .	122
4.4.1	Ergodic Split and Merging . . . . .	123
4.4.2	Split and Merging with Absorption . . . . .	126
4.4.3	Ergodic Split and Double Merging . . . . .	128
4.4.4	Double Split and Merging . . . . .	130
4.4.5	Double Split and Double Merging . . . . .	132
4.5	Integral Functionals in Split Phase Space . . . . .	134
4.5.1	Ergodic Split . . . . .	134
4.5.2	Double Split and Merging . . . . .	137
4.5.3	Triple Split and Merging . . . . .	138
5.	Phase Merging Principles . . . . .	139
5.1	Introduction . . . . .	139
5.2	Perturbation of Reducible-Invertible Operators . . . . .	140
5.2.1	Preliminaries . . . . .	140
5.2.2	Solution of Singular Perturbation Problems . . . . .	141
5.3	Average Merging Principle . . . . .	150
5.3.1	Stochastic Evolutionary Systems . . . . .	151
5.3.2	Stochastic Additive Functionals . . . . .	152
5.3.3	Increment Processes . . . . .	154
5.3.4	Continuous Random Evolutions . . . . .	156
5.3.5	Jump Random Evolutions . . . . .	157
5.3.6	Random Evolutions with Markov Switching . . . . .	159
5.4	Diffusion Approximation Principle . . . . .	160
5.4.1	Stochastic Integral Functionals . . . . .	161
5.4.2	Continuous Random Evolutions . . . . .	165
5.4.3	Jump Random Evolutions . . . . .	169
5.4.4	Random Evolutions with Markov Switching . . . . .	172
5.5	Diffusion Approximation with Equilibrium . . . . .	173
5.5.1	Locally Independent Increment Processes . . . . .	174
5.5.2	Stochastic Additive Functionals . . . . .	175
5.5.3	Stochastic Evolutionary Systems with Semi-Markov Switching . . . . .	176
5.6	Merging and Averaging in Split State Space . . . . .	182
5.6.1	Preliminaries . . . . .	182

5.6.2	Semi-Markov Processes in Split State Space . . . . .	184
5.6.3	Average Stochastic Systems . . . . .	186
5.7	Diffusion Approximation with Split and Merging . . . . .	188
5.7.1	Ergodic Split and Merging . . . . .	188
5.7.2	Split and Double Merging . . . . .	189
5.7.3	Double Split and Merging . . . . .	190
5.7.4	Double Split and Double Merging . . . . .	191
6.	Weak Convergence . . . . .	193
6.1	Introduction . . . . .	193
6.2	Preliminaries . . . . .	193
6.3	Pattern Limit Theorems . . . . .	196
6.3.1	Stochastic Systems with Markov Switching . . . . .	196
6.3.2	Stochastic Systems with Semi-Markov Switching . . . . .	201
6.3.3	Embedded Markov Renewal Processes . . . . .	205
6.4	Relative Compactness . . . . .	209
6.4.1	Stochastic Systems with Markov Switching . . . . .	209
6.4.2	Stochastic Systems with Semi-Markov Switching . . . . .	212
6.4.3	Compact Containment Condition . . . . .	213
6.5	Verification of Convergence . . . . .	216
7.	Poisson Approximation . . . . .	219
7.1	Introduction . . . . .	219
7.2	Stochastic Systems in Poisson Approximation Scheme . . . . .	220
7.2.1	Impulsive Processes with Markov Switching . . . . .	220
7.2.2	Impulsive Processes in an Asymptotic Split Phase Space . . . . .	225
7.2.3	Stochastic Additive Functionals with Semi-Markov Switching . . . . .	228
7.3	Semimartingale Characterization . . . . .	231
7.3.1	Impulsive Processes as Semimartingales . . . . .	232
7.3.2	Stochastic Additive Functionals . . . . .	237
8.	Applications I . . . . .	243
8.1	Absorption Times . . . . .	243
8.2	Stationary Phase Merging . . . . .	249
8.3	Superposition of Two Renewal Processes . . . . .	253
8.4	Semi-Markov Random Walks . . . . .	258

8.4.1	Introduction . . . . .	258
8.4.2	The algorithms of approximation for SMRW . . . . .	259
8.4.3	Compensating Operators . . . . .	262
8.4.4	The singular perturbation problem . . . . .	265
8.4.5	Stationary Phase Merging Scheme . . . . .	267
9.	Applications II . . . . .	269
9.1	Birth and Death Processes and Repairable Systems . . . . .	269
9.1.1	Introduction . . . . .	269
9.1.2	Diffusion Approximation . . . . .	270
9.1.3	Proofs of the Theorems . . . . .	272
9.2	Lévy Approximation of Impulsive Processes . . . . .	276
9.2.1	Introduction . . . . .	276
9.2.2	Lévy Approximation Scheme . . . . .	278
9.2.3	Proof of Theorems . . . . .	282
	<i>Problems to Solve</i> . . . . .	287
Appendix A	Weak Convergence of Probability Measures . . . . .	301
A.1	Weak Convergence . . . . .	301
A.2	Relative Compactness . . . . .	303
Appendix B	Some Limit Theorems for Stochastic Processes . . . . .	305
B.1	Two Limit Theorems for Semimartingales . . . . .	305
B.2	A Limit Theorem for Composed Processes . . . . .	308
Appendix C	Some Auxiliary Results . . . . .	311
C.1	Backward Recurrence Time Negligibility . . . . .	311
C.2	Positiveness of Diffusion Coefficients . . . . .	312
	<i>Bibliography</i> . . . . .	315
	<i>Notation</i> . . . . .	325
	<i>Index</i> . . . . .	329