Preface

On May 20-22, 2005, a workshop was held at Roskilde University in Denmark to honour Krzysztof P. Wojciechowski on his 50th birthday. This volume collects the papers of that workshop.

The purpose of the volume is twofold. The more obvious one is to acknowledge and honour Krzysztof Wojciechowski's contributions over the last 20-25 years to the theory of elliptic operators. Lesch's write up goes over many of Krzysztof's achievements, highlighting those insights that were particularly influential in shaping the direction of the theory. It is supplemented by Park's review of recent work pioneered by Wojciechowski. As our second purpose, we also hope to offer younger researchers and graduate students a snapshot of the current state of affairs. The proceedings contain a mix of review and research papers, both reflecting on the past and looking into the future. We obviously do not attempt to speak for the whole, vast area of the theory of elliptic operators. Most papers in these proceedings are, in one way or another, studying objects and techniques that have interested Krzysztof: spectral invariants, cutting and pasting, boundary value problems, heat kernels, and applications to topology, geometry and physics.

The modern theory of elliptic operators, or simply elliptic theory, has been shaped by the Atiyah-Singer index theorem created some 40 years ago. The Atiyah-Singer index theory expanded the scope of ellipticity to consider relations with and applications to topology. The notion of index acquired a dual personality, both analytical and topological. Consequently, wherever topological invariants appear, one is now tempted to see if the analytical aspects can be developed to interpret the invariant. In other words, analysts are always on the lookout for topological or geometrical invariants hoping to find operators behind them. Developments in topology are therefore of special interest to elliptic theorists. Bleecker's paper revisits some aspects of the so called embedding proof of the Atiyah-Singer index theorem. The contributions of Bunke and Schick on T-duality and Nicolaescu's survey of singularities of complex surfaces detail some topological theories of potential interest to analysts and possible applications of analytical methods.

Heat kernel techniques are at the heart of another one of the several proofs of the Atiyah-Singer index theorem. Different tools and techniques have been developed and are continuing to be developed to understand heat kernels and related spectral functions in a variety of situations. Two problems stand out: to describe and compute variations of heat kernels with respect to parameters and to calculate asymptotics of heat kernels – like functions of operators. These have been the central technical issues for much of Krzysztof Wojciechowski's work. As the scope of elliptic theory increases, so is the variety of contexts for heat kernel calculations which will undoubtedly occupy the interest of people in the future. The papers of Avrimidi on heat kernels of non-Laplace operators, of Furutani on heat kernels on nilpotent Lie groups, of Grubb on expansions of zeta-like functions, and of Paycha and Rosenberg on canonical traces all fall into this category.

Since the original papers on index theory, elliptic theory has continued to develop. More areas of mathematics, other than topology, have started influencing its progress. More and more objects of a similar nature to index have been investigated. For one thing, index is a very simple spectral invariant, and an important branch of elliptic theory looks at other spectral invariants and their geometrical and topological significance. We need to mention here some invariants that have particularly interested Krzysztof: the eta invariant, spectral flow, analytic torsion and infinite dimensional determinants. But there are many other invariants such as Seiberg-Witten invariants and elliptic genus. We expect that this list is not complete and that the future will bring more analytic invariants with topological and geometrical applications.

In the spirit of topological surgery theory, a major effort was undertaken to study elliptic operators and their spectral invariants using "cutting and pasting". This naturally leads to the problem of how to set up an elliptic theory on manifolds with boundary. This is the subject that Krzysztof has devoted most of his mathematical efforts shaping. The papers of Dai on eta invariants, of Ma and Zhang on L^2-torsion, and Park's review of gluing formulas for zeta determinants, as well as the contribution of Lesch, give the state of the art for at least some of the questions in this area.

Beside topology, the operator theory and operator algebras have been and will in the future be a driving force in the development of elliptic theory. What started with the analysis of a single Fredholm operator on a manifold, acquired greater depth and importance by considering whole spaces of operators. With the invention of operator K-theory, elliptic theory is evolving in a more abstract, algebraic fashion. Ellipticity is now defined not just for (pseudo) differential operators and not just on manifolds with or without boundary or even with corners. The proper context for the study of ellipticity is noncommutative differential geometry. Noncommutative geometry aims to consider discrete spaces as well as noncommutative objects on equal footing with topological spaces. Moreover, there is a duality which runs even deeper with the modern interpretation of an elliptic operator as a K-cycle over a C^*-algebra. It seems quite possible, and even likely, that such more algebraic trends will constitute the mainstream of elliptic theory in the future. Operator-theoretic contributions to this volume include papers by Benameur et al. on spectral flow in von Neumann algebras, by Douglas on a new kind of index theorem, by Klimek on a noncommutative disk, by Mickelsson on star products and central extensions and by Wurzbacher on homotopy calculations for some spaces of operators, while Dodziuk explores elliptic theory in a discrete setting.

Theoretical particle, string and membrane physics have and will continue to provide major motivation for elliptic theory. As the world of elementary particles continues to expand, one naturally suspects that the so-called elementary particles are not so elementary any more. Some of the current theories develop the idea that the basic structures of the universe are not point-like but rather stringy– or membrane-like. Such objects would naturally live in dimensions higher than our 4 dimensional world. To write down laws for such objects one is lead to modern global analysis involving arbitrary dimensional manifolds and operators on them. Of course new structures and new ideas also appear, such as supersymmetry, conformal symmetry, mirror symmetry and anomalies. Many exciting new mathematical questions arise. Several papers in this volume follow this line of reserach: Bunke and Schick on very general mirror symmetry, Esposito et al. on quantum gravity, Paycha and Rosenberg on conformal anomalies, Paycha and Scott on superconnections, Zhu on symplectic functional analysis and Hamiltonian dynamics.

With its intricate theory, powerful methods and variety of applications, the theory of elliptic operators should stay in the forefront of mathematics for long years to come. The fact that this has been the case in the recent past, is due in a nontrivial way to the work and insights of Krzysztof Wojciechowski.

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The Editors