

## Preface

Over the last sixty years control theory has gone through enormous development beyond the recognition of the old masters of the subject. The phenomenal development of system and control sciences is due to the interaction of scientists, mathematicians, engineers, economists and experts from industry and military. The subject is one of the most remarkable confluence of applied and pure mathematics and the very real world applications. It is amazing that some of the scientific and engineering problems would require the very abstract mathematics that used to be considered beyond the reach of applied scientists and engineers.

There are many prominent areas of systems and control theory that include systems governed by linear and nonlinear ordinary differential equations, systems governed by finite dimensional stochastic differential equations, systems governed by partial differential equations including their stochastic counter parts and, above all, systems governed by abstract differential and functional differential equations and inclusions on Banach (even metric and topological) spaces including their stochastic counterparts. This remarkable advance of the field is due to the unprecedented interest and contribution of pure and applied mathematicians and engineering sciences. There is no question that this interaction would continue simply because there are many unsolved challenging problems and emerging new ones. Such problems are of interest to mathematicians, scientists and engineers.

In recent years significant applications of systems and control theory have been witnessed in diverse areas such as physical sciences, social sciences, engineering, management, finance etc. In particular the most interesting applications have taken place in areas such as aerospace (civilian, military), buildings and space structures, suspension bridges, artificial heart, chemotherapy, power system, hydrodynamics, plasma, magneto hydrodynamics, computer communication network, flow control of internet traffic, optical network. Importance of applications whereby theory is tested and new theories are developed is clearly recognized.

The subject is so vast in scope and applications, it is impossible to cover all these in one single book. There are several dozens of books written for engineers and mathematicians on systems governed by ordinary differential equations, half a dozen books on systems governed by partial differential equations, and similar number of books on stochastic differential equations, and a number of books on systems governed by deterministic and stochastic abstract differential equations on Banach spaces.

The objective of this book is to present a small segment of theory and applications of systems and control governed by ordinary differential equations and inclusions. It is expected that any reader who has absorbed the materials presented here would have no difficulty to reach the core of current research directions.

The book includes 11 chapters and an appendix. The first chapter is mainly for those who have very little mathematical background. The second and the third chapter deal with linear and nonlinear systems respectively, covering questions of existence, uniqueness and regularity properties of solutions. In addition to classical models, these chapters also include some aspects of impulsive systems. Chapter 4 is elementary; it introduces the reader to basic Lyapunov stability theory for ordinary and impulsive systems as they are required for later chapters. Chapters 5 and 6 cover observability, identification, controllability and stabilizability of systems governed by ordinary linear and nonlinear differential equations subject to regular (measurable) as well as impulsive inputs. Chapter 7 presents calculus of variations as the precursor to optimal control theory. Here both existence of solutions and necessary conditions of extremality for variational problems are presented. Chapter 8 deals with optimal control theory. Both existence of optimal controls and necessary conditions of optimality are presented in details covering some elements of uncertain systems. For non convex control problems relaxed controls are used to develop necessary conditions of optimality. From these general results Pontryagin's minimum principle follows as a special case. Chapter 9, presents linear quadratic regulator theory with and without control constraints covering some aspects of disturbance rejection. Chapter 10 deals with time optimal control both for linear and nonlinear systems including impulsive systems. Chapter 11 deals with stochastic systems driven by Brownian and fractional Brownian motions including counting processes. Stochastic dynamic models recently developed for fluid approximation of network traffic are presented and their ramifications discussed. The appendix contains Lebesgue integration and some prominent results from measure theory and abstract functional analysis often used in the text.

**Target Audience:** The book is targeted to first and second year graduate students of Engineering, Mathematics, Management, Finance and Researchers in the field of systems theory, control, optimization and their applications. Selected parts of the book have been taught to first year graduate students of Systems Science and a select group of senior undergraduate students of engineering at the university of Ottawa for over last 25 years. Students who have taken this course were better prepared for graduate studies and research.

**Reading Guide:** Materials presented in Chapters 1-6 can be taught at the graduate levels of engineering under the title “Linear and Nonlinear Systems” without requiring any special prerequisite. The materials presented in Chapter 1 and some of the materials from the appendix are sufficient. If required, Chapter 3 on nonlinear systems can be skipped and recalled only when needed in the study of stability, observability and controllability in Chapters 4, 5 and 6. Materials presented in Chapters 1, 2, 4 and parts of Chapter 9 can be chosen for a one semester course on “Linear Systems and Optimal Control”. Materials presented in Chapters 7, 8, 10, 11 can be taught at the second year graduate level. A good prerequisite would be real analysis provided in Chapter 1, and the materials provided in the appendix. This course should lead the students to the current trends of research in the fascinating field of modern systems and optimal control theory. For Chapter 11, some knowledge of stochastic processes including Brownian motion and Poisson process is required.

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The author hopes that this book will inspire young mathematicians and engineers to discover the beauty of mathematics blended with and enriched by applications.